Power distribution in the luminescent waveguide

Caizhi Cao, Harrie Kreuwel, and T. J. A. Popma

This paper describes how to calculate the power distribution emitted by luminescent particles which are mixed in the film of the waveguide, i.e., the percentage of the power transported by each guided mode, emitted to the cladding and the substrate. For our sample, they are ~20%, 25%, and 55% separately. Some calculated results have been proved: the powers of the TE and TM modes of the luminescent waveguide in the same order are almost the same; the order of the film mode is lower; its power is larger.

I. Introduction

The problem of luminescent dye molecules placed in the vicinity of optical waveguides has been investigated, and several patents have been granted that take advantage of luminescent light coupled into waveguides. In this paper a luminescent substance is mixed in polyvinyl film; when the film is embedded between substrate and cover, it becomes an activated luminescent waveguide (Fig. 1). After absorbing the energy from incident light, the luminescent particles will emit some energy. The power transported by each guided mode, emitted into the cladding or the substrate, is an important question.

These excited particles can be considered as dipoles. Their total radiated power and the angular radiation pattern which are close to a plane interface, are reviewed by Lukosz and Kunz. Based on that, the dipoles are placed between two interfaces. It can be developed into a dipole in a multilayered structure.

To resolve this problem, we use two methods at the same time:

(1) A ray optical model of light propagation. The power ratio between cladding and substrate can be obtained.

(2) The conception of the mode and electromagnetic theory. The power ratios of each mode to the substrate mode can be obtained.

These two methods have their own advantages in some areas and cannot completely solve this problem, but there is a common area in the substrate, and some results calculated by both methods are the same. We combine these two calculations, and thereby obtain the whole power distribution.

II. Method 1: Angular Power Distribution

The structure of a waveguide is shown in Fig. 2. The symbols \( n_1, n_2, \) and \( n_3 \) represent the refractive indexes of cladding, film, and substrate, while \( h \) is the thickness of the film. An electric dipole at distance \( x_0 \) from interface 2 emits an electromagnetic wave in all directions. If we consider only the right-hand side, two rays will be emitted at angles \( \alpha_1 \) and \( \alpha_2 \). After reflection and refraction one (or more) times, the rays will be radiated into the cladding and the substrate at angles \( \alpha_2 \) and \( \alpha_3 \), respectively.

If the electromagnetic field amplitude at \( x_0 \) is taken as unity, and the phase as zero, the amplitudes and phases after every reflection and refraction are shown in Fig. 2. The reflection coefficients on interfaces 1 and 2 are represented by \( r_{12} \) and \( r_{13} \), while the transmission coefficients on these interfaces are \( t_{12} \) and \( t_{13} \). These coefficients depend on the angle of incidence and polarization of light and are given by Fresnel formulas as follows:

\[
\begin{align*}
    r_{12} &= \frac{K_1 n_2 \cos \alpha_1 - n_2 \cos \alpha_2}{K_1 n_2 \cos \alpha_1 + n_2 \cos \alpha_2}, \\
    r_{13} &= \frac{K_1 n_2 \cos \alpha_1 - n_3 \cos \alpha_2}{K_1 n_2 \cos \alpha_1 + n_3 \cos \alpha_2}, \\
    t_{12} &= \frac{2K_1 n_2 \cos \alpha_1}{K_1 n_2 \cos \alpha_1 + n_2 \cos \alpha_2}, \\
    t_{13} &= \frac{2K_1 n_2 \cos \alpha_1}{K_1 n_2 \cos \alpha_1 + n_3 \cos \alpha_2},
\end{align*}
\]

where

\[
\begin{align*}
    K_1 &= K_2 = 1 \quad \text{for TE}, \\
    K_1 &= n_2^2/n_1^2, \quad K_2 = n_3^2/n_2^2 \quad \text{for TM}.
\end{align*}
\]
thickness of the film is so small that the rays going out to the cladding appear to originate at point $o'$. The transmittivity on interface 1 is

$$T_{12} = t^* \frac{n_r \cos \alpha_2}{K_2 n_r \cos \alpha_1}. \quad (8)$$

The power distribution is

$$P(a_2) = T_{12} P_w(a_1) d\Omega_1 / d\Omega_2 = T_{12} P_w(a_1) n_r^2 \cos \alpha_2 / (n_f^2 \cos \alpha_1). \quad (9)$$

Here $P(a_2)$ is the power radiated within a unit solid angle, $d\Omega = \sin \alpha_2 d\alpha_2 d\phi$, in direction $a_2$. $P_w(a_1)$ is the power radiated within a unit solid angle in direction $a_1$ in an unbounded medium.

The situation is rotationally symmetric about the $X$-axis so we only take account of the $XOZ$ plane ($\phi = 0$) to calculate the radiative power per unit $d\phi$ to represent the whole situation. The radiative power per unit $d\phi$ in the cladding is

$$P_\epsilon = 2 \int_{a_2} P(a_2) \sin \alpha_2 d\alpha_2. \quad (10)$$

We can consider three orientations of the dipoles separately and normalize by the total power emitted by one dipole. No matter how the dipole is placed, the total power emitted by one dipole is the same (see Appendix A).

(1) The dipole is placed along the $Y$-axis and excites the TE wave for the $XOZ$ plane. So

$$P_{w_0}(a_1) = \frac{3}{8\pi}, \quad (11)$$

and $K_1 = K_2 = 1$. We can calculate the angular power distribution and radiative power per unit $d\phi$ from Eqs. (7)–(10).

(2) The dipole is placed along the $X$-axis and excites the TM wave, now

$$P_w(a_1) = \frac{3}{8\pi} \sin^2 \alpha_1. \quad (12)$$

and $K_1 = n_r^2/n_f^2$, $K_2 = n_f^2/n_r^2$.

(3) The dipole is placed along the $Z$-axis and also excites the TM wave. The situation is similar to case (2), but

$$P_w(a_1) = \frac{3}{8\pi} \cos^2 \alpha_1. \quad (13)$$

B. Radiation in the Substrate

From Fig. 2 we can obtain the amplitude in the $a_3$ direction:

$$t = t_{12} \exp(i(h - x_0)k_1 \cos \alpha_1) [1 + r_{12} t_{12} \exp(iB) + \ldots] + t_{12} r_{13} \exp[(i(h + x_0)k_1 \cos \alpha_1) [1 + r_{12} r_{13} \exp(iB) + \ldots]$$

$$+ t_{12} r_{13} \exp[(i(h - x_0)k_1 \cos \alpha_1) [1 - r_{12} r_{13} \exp(iB),$$

$$tt^* = t_{12}^2 + t_{12} r_{13} + 2r_{13} \cos(2x_0h_1 \cos \alpha_1). \quad (7)$$

In fact, reflection and refraction only take place a few times because $r_{12}$ and $r_{13}$ are not equal to 1 and the

For $0 \leq a_3 \leq 90^\circ$, corresponding to $0 \leq a_1 < a_c = \arcsin(n_r/n_f)$, let $B = 2h k_1 \cos \alpha_1$, the amplitude in direction $a_3$ is

$$tt^* = t_{12}^2 + 2t_{12} r_{13} \cos(2x_0h_1 \cos \alpha_1) - 2t_{12} r_{13} \cos B. \quad (14)$$

For $0 \leq a_3 < \arcsin(n_r/n_f)$, corresponding to $0 < a_1 < \arcsin(n_r/n_f)$, there are internal reflections at both interfaces: $\delta_{12} = 0$ for TE; $\delta_{12} = 0, \pi$ for TM, so
For \( \arcsin(n_c/n_f) < a_3 < 90^\circ \), corresponding to \( \arcsin(n_c/n_f) < a_1 < \arcsin(n_c/n_f) \), there is total reflection at interface 1, so

\[
\tan(\theta_{11}/2) = (n_f^2 \sin^2 \alpha_1 - n_c^2)^{1/2}/(K \sin \alpha_1),
\]

(16)

\( K_1 = 1 \) for TE; \( K_1 = n_f^2/n_c^2 \) for TM,

(17)

\( r_{12} = 1 \).

(18)

The transmittivity on interface 2 is

\[
T_{13} = t^{tt*} \frac{n_f \cos \alpha_3}{K n_1 \cos \alpha_1},
\]

(19)

\( K_2 = 1 \) for TE; \( K_2 = n_f^2/n_c^2 \) for TM.

(20)

The angular power distribution in the substrate is

\[
P(a_3) = T_{13} P_{x*} \frac{d \beta}{d \alpha_3} = T_{13} P_{x*} n_f^2 \cos \beta / (n_c^2 \cos \alpha_1). \tag{21}
\]

The expressions for \( P_{x*}(a_3) \) are shown as Eqs. (11)–(13) for three orientations of the dipole. The radiative power per unit \( d \phi \) in the substrate is

\[
P_{x+1c} = 2 \int_{0}^{\pi/2} P(a_3) \sin \alpha_3 da_3. \tag{22}
\]

The reason we call it \( P_{x+1c} \) will be seen later. For the guided mode, the denominator in Eq. (15) is zero, so this method is useless.

We take an example to show the calculated results. This waveguide sample was made by dipping a fine solution of rhodamine 6G and polyvinyl in acetone and \( n_c = 1, n_f = 1.54867, n_s = 1.51, h = 5.30 \mu m \). We calculated the angular power distribution and power per unit \( d \phi \) for three cases. The angular power distribution in the cladding and in the substrate is plotted in Figs. 3 and 4. In the cladding we assumed that \( x_0 = 0; 0.1 h; \ldots; h \), then averaged the results. The distribution curves in Fig. 3 are smooth that eleven positions are enough. The distribution in the substrate fluctuates and is sensitive to dipole position \( x_0 \), so we needed to take more positions, for example, 1001 points, to calculate the average value.

Seen from another point of view, the field generated by a dipole of strength \( J \) in Fig. 2 can be expressed as

\[
E = \sum_{j} j \phi \exp(i \beta \phi) + \text{rad mod}, \tag{27}
\]

where \( j \) is the order of the guided mode. Here we omit the factor \( \exp(i \omega t) \) and abbreviate the substrate mode

\[
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\]
and substrate cover mode, because first we only consider the film modes.

We use the reciprocity theorem,\(^6\)

\[
\mathbf{V} \cdot (\mathbf{E}_1 \mathbf{E}_1' + \mathbf{E}_2 \mathbf{E}_2') = J_1 \cdot \mathbf{E}_1' + J_2' \cdot \mathbf{E}_1,
\]

(29)

for the derivation of Eq. (29), see Appendix B. We insert Eqs. (27) and (28) into Eq. (29) as \(\mathbf{E}_1, \mathbf{H}_1, \) and \(\mathbf{E}_2', \mathbf{H}_2'\) are equal to the fields of the \(j\)th film mode on the source-free waveguide, so \(J_2' = 0\), and

\[
\mathbf{E}_1' = \mathbf{E}_j \exp(i\beta_j z),
\]

(30)

\[
\mathbf{H}_1' = \mathbf{H}_j \exp(i\beta_j z).
\]

Using the orthogonality condition and the wave expression

\[
\frac{1}{2} \iint \mathbf{E}_j \times \mathbf{H}_k \cdot \delta dx \, dy
\]

(31)

and the formula

\[
\mathcal{S} \cdot \nabla \cdot \mathbf{E} = \mathcal{S} \cdot \mathbf{E} \cdot nds,
\]

(32)

we have

\[-4N_j e_j = \iint_0 \mathbf{J}_1 \cdot \mathbf{E}_j' \exp(-i\beta_j z) \, dv,
\]

(33)

so

\[e_j = -\frac{1}{4N_j} \iint_0 \mathbf{J}_1 \cdot \mathbf{E}_j' \exp(-i\beta_j z) \, dv.
\]

(34)

The power of the \(j\)th mode is

\[P = 2 \cdot \frac{1}{2} \text{Re} \left( \int_0 \mathbf{E} \times \mathbf{H}^* \cdot \delta ds \right)
\]

(35)

\[= 2 \cdot \frac{1}{2} \int \int |\mathbf{E}_j| \cdot |\mathbf{H}_j| \delta dx \, dy = 2 |\mathbf{E}_j|^2 |\mathbf{H}_j|,
\]

We multiply by two because the wave propagates backward and forward.

Now we can calculate the power emitted by three different oriented dipoles separately.

(1) **TE mode generated by the dipole oriented at the \(Y\)-axis.** From Eq. (34) we obtain

\[e_j = -\frac{1}{4N_j} J_{E_y}(x_0),
\]

(36)

where \(J_{E_y}(x_0)\) is the field of position \(x_0\) in the film. According to the formulas expressed in Ref. 5 (see Appendix C),

\[E_{yj}(x_0) = E_j \cos(k_j x_0 - \phi_j),
\]

(37)

\[N_j = \frac{1}{2} \int_{-\infty}^{+\infty} \mathbf{E}_j \cdot \mathbf{H}_j' \, dx = \frac{\beta}{4 \omega \mu} \mathbf{E}_j^2 h_{\text{eff}},
\]

(38)

where \(N_j\) is the power per unit guided width of the \(j\)th mode, and \(h_{\text{eff}}\) is the effective film thickness. We obtain power \(P\) of the mode whose effective refractive index is \(N\), carried by every unit guided width as follows:

\[P_j = \frac{\beta \omega \mu j^2}{2 \h_{\text{eff}}} \cos^2(k_j x_0 - \phi_j)
\]

\[= \frac{\beta \omega \mu j^2}{2 \h_{\text{eff}}} \cos^2(k_j x_0 - \phi_j)
\]

\[= \frac{\beta \omega \mu j^2}{2 \h_{\text{eff}}} \cos^2(k_j x_0 - \phi_j)
\]

(39)

where \(\h_{\text{eff}} = h_{\text{eff}}/\lambda\).

(2) **TM mode generated by the dipole oriented at the \(X\)-axis.** From Eq. (34) we obtain

\[e_j = -\frac{1}{4N_j} J_{E_z}(x_0).
\]

(40)

Since (see Appendix C)

\[E_z = (\beta/\omega \epsilon) H_y, \quad \epsilon \text{ is the dielectric constant of the film,}
\]

(41)

\[H_{yz}(x_0) = H_y \cos(k_j x_0 - \phi_j),
\]

(42)

\[N_j = \frac{1}{2} \int_{-\infty}^{+\infty} E_y H_z \, dx = \frac{\beta}{4 \omega \epsilon} \mathbf{H}_y^2 h_{\text{eff}},
\]

(43)

we can obtain

\[P_z = \frac{\beta \omega \mu j^2}{2 \omega \epsilon \h_{\text{eff}}} \cos^2(k_j x_0 - \phi_j)
\]

\[= \frac{\beta \omega \mu j^2}{2 \omega \epsilon \h_{\text{eff}}} \cos^2(k_j x_0 - \phi_j)
\]

\[= \frac{\beta \omega \mu j^2}{2 \omega \epsilon \h_{\text{eff}}} \cos^2(k_j x_0 - \phi_j)
\]

(44)

Here we use the relation

\[n_j^2 = \epsilon/\epsilon_0.
\]

(45)

(3) **TM mode generated by the dipole oriented at the \(Z\)-axis.** Now

\[e_j = -\frac{1}{4N_j} J_{E_z}(x_0).
\]

(46)

From Appendix C,

\[E_z = -(i\omega \epsilon) H_z/\partial x,
\]

(47)

so

\[E_{zj}(x_0) = (i\omega \epsilon) H_z \sin(k_j x_0 - \phi_j);
\]

(48)

insert Eq. (48) into Eq. (46), then insert Eqs. (46) and (43) into Eq. (35). Finally we obtain the power generated by the dipole oriented at the \(Z\)-axis:
From Appendix C we know that
\[ P_z = \frac{k_0E_0^2}{2\omega e_0} \cdot (n_2^2 - N^2) \sin^2(k_x x_0 - \phi) \]  
(49)

Comparing Eqs. (39), (44), and (49), the following factors have the same value:
\[ \omega \mu = \frac{k}{2\omega e_0} = \frac{\sqrt{\varepsilon_0}}{2\lambda c_0} = F, \]  
(50)
where \( \mu = \mu_0 \). We are only interested in the ratios, so the value of \( F \) is not important. We calculate the second factors in Eqs. (39), (44), and (49).

We can consider the substrate mode in the same way as the film mode; the only difference is that the effective refractive index \( N \) of the substrate mode is continuous from \( n_s \) to \( n_c \). We always choose the transverse propagation constant \( k_z \) in the substrate as an integration variable. The power of the guided width of every unit is integrated from \( k_z = 0 \) to \( k_z = k(n_s^2 - n_z^2)^{1/2} \).

Now the field in the waveguide can be expressed as
\[ E = \text{film mode} \]
\[ + \int_0^{(\pi n_s - \pi n_c)/2} e(k_z) \mathbf{E}(k_z) \exp[i\delta(k_z)z]dk_z + \text{sub cov mode}, \]  
(51)
\[ H = \text{film mode} \]
\[ + \int_0^{(\pi n_s - \pi n_c)/2} e(k_z) \mathbf{H}(k_z) \exp[i\delta(k_z)z]dk_z + \text{sub cov mode}. \]  
(52)

Here we also omit the factor \( \exp(iwt) \) and abbreviate the film mode and substrate cover mode.

Equation (31), the orthogonality condition, and the power expression change to
\[ \frac{1}{2} \int \mathbf{E}_x \times \mathbf{H}_y \cdot \delta z dx dy = \frac{1}{2} \int \mathbf{E}_x' \times \mathbf{H}_y' \cdot \delta z dx dy \]
\[ = N(k_z) \delta(k_z - k_z), \]  
(53)
where \( \delta(k_z - k_z) \) is the delta function.

The power of the total substrate mode is
\[ P' = 2 \cdot \frac{1}{2} Re\left( \int \mathbf{E} \times \mathbf{H}^* \cdot \delta z ds \right) \]
\[ = \int \int \int e(k_z)E_x(k_z)E_{x*}(k_z)H_y^*(k_z)dk_z \delta z dx dy \]
\[ = \int_{k_z} e(k_z) P_2 N(k_z) dk_z, \]  
(54)
We also discuss three different oriented dipoles.

(1) TE mode generated by the dipole oriented at the Y-axis. From Eq. (34) we now obtain
\[ e(k_z) = -\frac{1}{4N(k_c)} \cdot J_{E_y}(x_0), \]  
(55)
From Appendix C we know that
\[ E_y(x_0) = E_y \cos[k_z(x_0 - h) + \phi], \]  
(56)
\[ N(k_z) = \frac{\pi \beta}{4\omega \mu} E_y^2, \]  
(57)
\[ E_y^2 = E_y^2 \left[ 1 + \frac{[n_s^2 - n_z^2]/(n_s^2 - N^2)] \sin^2(\phi_x - k_z h) \right]. \]  
(58)
Inserting Eqs. (55)–(58) into Eq. (54), power \( P'_y \) of the substrate mode, which is carried by every unit guided width, can be obtained:
\[ P'_y = \frac{\pi(n_s^2 - N^2)^2}{2\omega e_0} \frac{\mu_0 J_{E_y}^2}{2\pi} \int_0^{(\pi n_s - \pi n_c)/2} \cos^2[k_z(x_0 - h) + \phi] \]
\[ \frac{1 + [n_s^2 - n_z^2]/(n_s^2 - N^2)] \sin^2(\phi_x - k_z h) \]
\[ dk_z. \]  
(59)

(2) TM mode generated by the dipole oriented along the X-axis. From Eq. (34) we now obtain
\[ e(k_z) = -\frac{1}{4N(k_c)} \cdot J_{E_x}(x_0) = -\frac{\beta}{4N(k_c)\omega e_0} J_{H_y}(x_0); \]  
(60)
here we use Eq. (41) again. From Appendix C we know that
\[ H_y(x_0) = H_y \cos[k_z(x_0 - h) + \phi], \]  
(61)
\[ N(k_z) = \frac{\pi \beta}{4\omega e_0} \frac{H_y^2}{N(k_c)}, \]  
(62)
\[ H_y^2 = H_y^2[1 + (q \cdot n_s^2)/n_f^2](n_s^2 - n_z^2)/(n_s^2 - N^2) \sin^2(\phi_x - k_z h)]. \]  
(63)
Inserting Eqs. (60)–(63) into Eq. (54), power \( P'_z \) is
\[ P'_z = \frac{\pi(n_s^2 - N^2)^2}{2\omega e_0} \frac{\mu_0 J_{E_z}^2}{2\pi} \int_0^{(\pi n_s - \pi n_c)/2} \frac{\beta \cos^2[k_z(x_0 - h) + \phi] \}
\[ \frac{1 + [n_s^2 - n_z^2]/(n_s^2 - N^2)] \sin^2(\phi_x - k_z h) \] \[ dk_z. \]  
(64)
where we have again used Eq. (45).

(3) TM mode generated by a dipole oriented along the Z-axis. Now
\[ e(k_z) = -\frac{1}{4N(k_c)} \cdot J_{E_z}(x_0), \]  
(65)
\[ E_z = -(i/\omega \epsilon_0) \delta H_y \delta x = (i/\omega \epsilon_0) k_x H_y \sin[k_z(x - h) + \phi]; \]  
(66)
Inserting Eq. (66) into Eq. (65), then inserting Eqs. (65), (62), and (63) into Eq. (54), we can obtain power \( P'_z \):
\[ P'_z = \frac{n_s^2 J_{E_z}^2}{2\pi \omega e_0} \frac{\beta}{2\pi} \int_0^{(\pi n_s - \pi n_c)/2} \frac{k_z^2 \sin^2[k_z(x_0 - h) + \phi] \}
\[ \frac{1 + (q \cdot n_s^2)/n_f^2)(n_s^2 - n_z^2)/(n_s^2 - N^2) \sin^2(\phi_x - k_z h) \] \[ dk_z. \]  
(67)
For simplicity, let \( l = k_x, \) and from Appendix C we can know that \( k_z^2 = n_s^2 k_x^2 - \beta^2 \) and \( \beta = kN \). Inserting these into Eqs. (59), (64), and (67), the following factors have the same value [compare with Eq. (50)]:
\[ \omega \mu = \frac{k^2}{2\pi \epsilon_0} = \frac{\sqrt{\varepsilon_0}}{2\lambda c_0} = F. \]  
(68)
So we can obtain expressions of the power of the substrate mode as follows:
\[ P'_z = 2F_{p'_z} \] \( (i = x, y, z), \) \[ P'_y = \int_0^{(\pi n_s - \pi n_c)/2} \frac{\cos^2[k_z(x_0 - h) + \phi] \}
\[ 1 + [n_s^2 - n_z^2]/(n_s^2 - N^2)] \sin^2(\phi_x - k_z h) \] \[ dl. \]  
(69)


Table II. Effective Refractive Indexes of the Sample

<table>
<thead>
<tr>
<th>Order</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>N (TE)</td>
<td>1.54766</td>
<td>1.54465</td>
<td>1.53966</td>
<td>1.53268</td>
<td>1.52381</td>
<td>1.51338</td>
</tr>
<tr>
<td>N (TM)</td>
<td>1.54764</td>
<td>1.54456</td>
<td>1.53945</td>
<td>1.53234</td>
<td>1.52331</td>
<td>1.51278</td>
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Table III. Power Distribution in the Film and the Substrate

<table>
<thead>
<tr>
<th>film mode</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>TE</td>
<td>0.0590</td>
<td>0.0595</td>
<td>0.0587</td>
<td>0.0582</td>
<td>0.0569</td>
<td>0.0519</td>
</tr>
<tr>
<td>TM</td>
<td>0.0596</td>
<td>0.0595</td>
<td>0.0592</td>
<td>0.0586</td>
<td>0.0572</td>
<td>0.0510</td>
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</tbody>
</table>

Table IV. Power Distribution in Percent

<table>
<thead>
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<th>film</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>substrate cladding lower cladding</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE</td>
<td>3.31</td>
<td>3.30</td>
<td>3.29</td>
<td>3.26</td>
<td>3.19</td>
<td>2.91</td>
<td>56.01</td>
</tr>
<tr>
<td>TM</td>
<td>3.33</td>
<td>3.33</td>
<td>3.31</td>
<td>3.27</td>
<td>3.20</td>
<td>2.85</td>
<td>55.91</td>
</tr>
</tbody>
</table>

We can say that

\[ p_i^* = \int_0^{(n_f^2 - n_i^2)\beta \frac{1}{12}} \frac{n_i^2(n_f^2 - \beta)\frac{1}{12} \cos^2[k_i(x_0 - h) + \phi_i] \text{d}l}{n_i^2(1 + [(q_i n_i^2/n_f^2) \cdot (n_i^2 - n_f^2)/(\beta)^2] \sin^2(\phi_i - k_i h))}, \]

(70)

\[ p_i = \int_0^{(n_f^2 - n_i^2)\beta \frac{1}{12}} f_i(l) \text{d}l, \quad (i = x, y, z), \]

(72)

where \( f_i(l) \) represents the integrand functions of Eqs. (69)–(71).

In the cladding, the value of \( \beta \) is from \( kn_c \) to zero. From Eqs. (59) and (67), \( P_x \) and \( P_z \) are infinity. So the method is useless for this area.

We take the same sample in which \( n_c = 1, n_f = 1.54867, n_s = 1.51, h = 5.30 \mu m \) to calculate its power for the film mode and the substrate mode. First we calculated the effective refractive indexes of the TE and TM film modes; results are given in Table II. There are six modes for TE or TM. These can be checked by experiment. 5,7

Then we calculate the average power of the dipole's position taken at \( x_0 = 0; 0.001h; \ldots h \) for each film mode and substrate mode. We still assume the power of the substrate mode to be 1 to normalize the film mode. The power distribution in film and substrate is given in Table III. The curves shown in Fig. 5 represent functions \( f_i(l) vs l \) for three cases. Comparing Fig. 5 with the left-hand side of Fig. 4, the curves of both plots are similar, and we come to the same conclusion: the power distribution in the substrate fluctuates.

Combining Tables I and III, we finally obtain the power distribution in each area in percent (see Table IV).

From this example we can see that

(a) Power in the film is \(~20\%\), in the substrate \(~55\%\), and in the cladding \(~25\%\).
(b) TE and TM power distributions in the same order film modes are nearly the same.
(c) The lower the order of film mode the larger is its power. If there are more film modes, there is not too much difference in the lower-order modes, but it is evident that the power of the highest one is much less than the others. Sometimes it will be difficult to find the highest one in the experiment.

If the parameters of the waveguide are changed, the percentage will be changed but not too much. When the film is thinner, the number of film modes is less and the power of each mode is larger.

IV. Experiments

A. Experiment 1: Comparing the Powers of the Same Order Film Mode of TE and TM

When we measured the luminescent spectrum of the above sample, we also compared the powers of TE and TM in the same order.

The experimental equipment is shown in Fig. 6. We used a green Ar laser to improve the Rh6G emittance, because the peak of the absorption spectrum of Rh6G
Fig. 5. $f(l)$ versus $l$.

Fig. 6. Power of TE compared with that of TM emitted by luminescent particles.

Fig. 7. Luminescent spectra of TE and TM in the same mode (Rh6G).

Fig. 8. Power of each mode is compared.

is near this wavelength. The angle of incidence on the prism can be adjusted at one of the coupling angles, so that more energy will be coupled into the film; this also strengthens the luminescent light coupling out from the prism. If a piece of paper is positioned at A, we see several mode lines. We then used a lens and mirror system to focus one line on the entrance slit of a monochromator. Behind the exit slit of the monochromator we used a lens to focus the light onto a photodiode as a detector. For this kind of sample, the same order mode lines of TE and TM overlap. We can measure the luminescent spectra of TE and TM in turn by turning the polarizer, which is placed in front of the monochromator, and we compare the peaks. The lock-in technique is used for improving the signal-to-noise ratio.

The curves, intensity vs $\lambda$, printed by the $X$-$Y$ recorder, are shown in Fig. 7. The TE and TM powers of the same order film mode, emitted by Rh6G particles are almost the same. This agrees with the calculated results.

B. Experiment 2: Comparing the Power of Each Film Mode

The experimental setup is shown in Fig. 8. We used cylindrical lens 1 as a coupling prism and adjusted the center of the cylinder to agree with the axis of the rotary table. A small focus cylindrical lens 2 is also mounted on the prism table, which is located above the rotary table. The Ar laser light passes through cylinder 2 to make a sharp yellow line on the film. This is the luminescent light emitted by the Rh6G dipoles after absorbing the Ar light.

A long black tube, with two pinholes and a detector connector, is fixed on the rotary table, and its axis is
adjusted through that of the rotary table. A filter is fixed in front of the tube to absorb the green light from the Ar source.

To make the gap between the bottom of cylinder 1 and the film uniform, we evaporate a film, for example, a glass film, on the Rh6G film. Positioning a piece of paper in front of the filter at θ ~ 60°, we can clearly see some mode lines. Therefore we can use the rotatable detector to measure intensity vs θ to show the power of each mode.

The condition of coupling is \( n_p \sin \theta = N \), where \( n_p \) is the refractive index of cylinder 1 and \( N \) of the lower-order mode is larger, corresponding to the larger \( \theta \). There are six modes of which the results are shown in Fig. 9. There are scattered light and substrate cover continuum besides the modes. In our experiment it was difficult to separate them. In the small area of the guided modes, it makes a little difference to the background. The resolution power of this measurement is not very good, nevertheless we can still say that the order of mode is lower, its power is larger. The intensity of the lower-order modes is higher, but it occupies a smaller space in the \( \theta \) direction. The powers of these lower-order modes are almost the same, but the highest one is much less than others.

C. Experiment 3: Measuring the Angular Power Distribution in the Cladding

Using the same experimental setup shown in Fig. 8, we removed cylinder 1. When the rotary table is turned, we can measure intensity vs \( \theta \). The result is shown in Fig. 10; we did not use the polarizer, so that is the total angular power distribution. The outline is similar to the curve in Fig. 3. The power in the perpendicular direction is the largest. The horizon dip angle is bigger, the power is larger.

V. Conclusions

Theoretical calculation shows that the power in the film is ~20%, in the substrate ~55%, and in the cladding ~25% for our sample. It has not yet been proved by the experiment, but according to our calculation, some results have been experimentally proved:

1. For a luminescent waveguide, the powers of TE and TM of the same order mode are almost the same.
2. The order of film mode is lower, its power is larger. If there are more modes, the powers of these lower-order modes are not much different.
3. In the cladding, the angular power distribution is: the horizon dip angle is bigger, the power is larger, and the power in the perpendicular direction is the largest.

This theoretical analysis may also be useful in considering other kinds of activated waveguide.

The authors wish to thank J. P. van Weers and J. Baxter for making part of the experimental equipment.

Appendix A: How to Normalize the Angular Power Distribution \( P_\alpha(\alpha) \) Emitted by an Electronic Dipole in an Unbounded Medium

The position of the dipole is taken as the origin of the coordinates, as shown in Fig. 11. We examine the time average Poynting vector \( S \) at an arbitrary point \( M(r, \theta, \phi) \), so \( x = r \cos \theta \), \( y = r \sin \theta \sin \phi \), \( z = r \sin \theta \cos \phi \), and the direction cosines are \( \cos x = x/r \), \( \cos y = y/r \), \( \cos z = z/r \). The total power \( P_{\text{tot}} \) through the surface of a sphere with radius \( r \) can be obtained by integrating \( dQ = \sin \theta d\theta d\phi \) over \( \theta \), \( 0 \) to \( \pi \) and \( \phi \), \( 0 \) to \( 2\pi \). Finally, the angular power distribution \( P_\alpha(\alpha) \) normalized can be obtained by dividing \( S \) by \( P_{\text{tot}} \). We also discuss three cases separately.

(a) If the dipole of strength \( J \) is placed along the \( X \)-axis, the time-average Poynting vector is

\[
S = \frac{\mu_0 \omega J^2}{32\pi^2 c r^2} \sin^2 \alpha = S_r.
\]

In short, let \( W = \mu_0 \omega J^2/(32\pi^2 c r^2) \), so \( S = W \sin^2 \alpha \). The total power through the surface of a sphere with radius \( r \) emitted by this dipole is

\[
P_{\text{tot}} = \int_0^{2\pi} \int_0^\pi S \sin \theta d\phi = 2W \int_0^\pi \sin^2 \theta d\alpha = (8\pi/3)W.
\]

So

\[
P_\alpha(\alpha) = S/P_{\text{tot}} = (3/8\pi) \sin^2 \alpha.
\]

In this case \( P_\alpha(\alpha) \) is rotationally symmetric about the \( X \)-axis, i.e., independent of the azimuth angle \( \phi \).

(b) If the dipole is placed along the \( Y \)-axis, then \( S = W \sin^2 \beta \), and
The total power emitted by this dipole is

\[ P_{\text{tot}} = \int_0^{2\pi} W(\cos^2 \theta + \sin^2 \theta \cos^2 \phi) \, d\theta \, d\phi = (8\pi/3)W. \]

So

\[ P_\alpha = S/P_{\text{tot}} = (3/8\pi)(\cos^2 \theta + \sin^2 \theta \cos^2 \phi). \]

If we only take account of the XOZ plane, then \( \phi = 0 \), and calculate the radiation power per unit \( \phi \) to represent the whole situation as in Fig. 2, \( P_\alpha(a_k) = 3/(8\pi) \).

(c) If the dipole is placed along the Z-axis,

\[ S = W \sin^2 \gamma, \]

\[ \sin^2 \gamma = \cos^2 \theta + \sin^2 \theta \sin^2 \phi, \]

\[ P_{\text{tot}} = \int_0^{2\pi} W(\cos^2 \theta + \sin^2 \theta \sin^2 \phi) \, d\theta \, d\phi = (8\pi/3)W. \]

### Table V. Some of the Formulas

<table>
<thead>
<tr>
<th>relation of field components</th>
<th>TE</th>
<th>TM</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_y = ( E_x ) = ( E_z ) = 0</td>
<td>( E_y = H_x = H_z = 0 )</td>
<td></td>
</tr>
<tr>
<td>( H_x = -(\beta/\omega \mu)E_y )</td>
<td>( E_x = (\beta/\omega \epsilon)H_y )</td>
<td></td>
</tr>
<tr>
<td>( H_z = (i/\omega \mu)\partial E_y / \partial x )</td>
<td>( E_z = -(i/\omega \epsilon)\partial H_y / \partial x )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>film mode</th>
<th>TE</th>
<th>TM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_y = E_c \exp[-r_c(x-h)], h &lt; x )</td>
<td>( H_y = H_c \exp[-r_c(x-h)], h &lt; x )</td>
<td></td>
</tr>
<tr>
<td>( E_y = E_f \cos(k_f x - \Phi_f), 0 &lt; x &lt; h )</td>
<td>( H_y = H_f \cos(k_f x - \Phi_f), 0 &lt; x &lt; h )</td>
<td></td>
</tr>
<tr>
<td>( E_y = E_s \cos(k_s x + \Phi), x &lt; 0 )</td>
<td>( H_y = H_s \cos(k_s x + \Phi), x &lt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>substrate mode</th>
<th>TE</th>
<th>TM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tan \Phi_s = r_s/k_f )</td>
<td>( \tan \Phi_s = (n_f/n_s)^2 r_s/k_f )</td>
<td></td>
</tr>
<tr>
<td>( P = -2(\omega \mu) E_c \left( \frac{n_f^2 - n_s^2}{n_s^2(n_f^2 - n_s^2)} \sin^2 (\Phi - k_f h) \right) )</td>
<td>( P = 2(\omega \epsilon) H_c \left( \frac{n_f^2 - n_s^2}{n_s^2(n_f^2 - n_s^2)} \sin^2 (\Phi - k_f h) \right) )</td>
<td></td>
</tr>
<tr>
<td>( E_s = E_f \left[ \frac{n_f^2 - n_s^2}{n_f^2 - n_s^2} \sin^2 (\Phi - k_f h) \right] )</td>
<td>( H_s = H_f \left[ \frac{n_f^2 - n_s^2}{n_s^2(n_f^2 - n_s^2)} \sin^2 (\Phi - k_f h) \right] )</td>
<td></td>
</tr>
<tr>
<td>( P = -2(\omega \mu) \int_0^\infty dx E_y(x) H_x(x) )</td>
<td>( P = 2(\omega \epsilon) \int_0^\infty dx E_y(x) H_x(x) )</td>
<td></td>
</tr>
<tr>
<td>( P = \frac{2\pi}{\omega \epsilon} E_f \bar{E}_s \bar{k}_f )</td>
<td>( P = \frac{2\pi}{\omega \epsilon} H_f \bar{H}_s \bar{k}_f )</td>
<td></td>
</tr>
</tbody>
</table>
In this case
\[ P_n(a) = S/P_{tot} = \frac{(3/8\pi)(\cos^2 \phi + \sin^2 \phi \sin^2 \phi)}{S} \]

In our situation \( \phi = 0 \),
\[ P_n(a) = \frac{(3/8\pi)(\cos^2 \phi + \sin^2 \phi \sin^2 \phi)}{S} \]

It is reasonable to assume that the total power emitted by a dipole is the same, no matter how it is placed. After normalizing we can compare these three cases.

**Appendix B: Derivation of the Reciprocity Theorem**

There are two solutions of Maxwell’s equations, \( \mathbf{E}_1, \mathbf{H}_1 \) and \( \mathbf{E}_2, \mathbf{H}_2 \), so,
\[
\nabla \times \mathbf{E}_1 = -i\omega \mu \mathbf{H}_1, \tag{B1}
\]
\[
\nabla \times \mathbf{H}_1 = J_1 + i\omega \mathbf{E}_1, \tag{B2}
\]
\[
\nabla \times \mathbf{E}_2 = i\omega \mu \mathbf{H}_2, \tag{B3}
\]
\[
\nabla \times \mathbf{H}_2 = J_2 - i\omega \mathbf{E}_2. \tag{B4}
\]

From [Eq. (B1)] \( \mathbf{H}_2 \) + [Eq. (B3)] \( \mathbf{H}_1 \), we obtain
\[
\nabla \times \mathbf{E}_1 + \nabla \times \mathbf{E}_2 = 0, \tag{B5}
\]
and [Eq. (B2)] \( \mathbf{E}_2 \) + [Eq. (B4)] \( \mathbf{E}_1 \),
\[
\nabla \times (\nabla \times \mathbf{H}_1) + \nabla \times (\nabla \times \mathbf{H}_2) = J_1 - J_2 + J_1 - J_2. \tag{B6}
\]

Then from Eqs. (B6) and (B5) and using the formula
\[
\nabla \cdot (a \times b) = b \cdot (\nabla \times a) - a \cdot (\nabla \times b), \tag{B7}
\]
we obtain
\[
\nabla \cdot (\mathbf{H}_1 \times \mathbf{E}_2 + \mathbf{H}_2 \times \mathbf{E}_1) = J_1 \cdot \mathbf{E}_2 - J_2 \cdot \mathbf{E}_1. \tag{B8}
\]

**Appendix C: Some Formulas from Ref. 5**

For convenience, some formulas from Chap. 2.3 in Ref. 5 are listed as follows: some symbols and their defining equations are \( k \) is the wave vector in free space; \( \lambda \) is the free-space wavelength; \( N \) is the effective refractive index; \( \beta \) is the propagation constant;
\[
k = 2\pi/\lambda, \quad \beta = kN; \tag{C1}
\]
\( k_i \) (\( i = s, f \)) is the wave vector in the cladding; film; substrate; \( \beta_i \) (\( i = c, s \)) is the attenuation constant in the cladding; substrate;
\[
k_i^2 = n_i^2k^2 - \beta_i^2 = -r_i^2, \tag{C2}
\]
\[
k_i^2 = n_i^2k^2 - \beta_i^2, \tag{C3}
\]
\[
k_i^2 = n_i^2k^2 - \beta_i^2 = -r_i^2. \tag{C4}
\]
\( \phi_i(\mathbf{a}) \) is the phase shift on the border of the cladding; substrate; \( h_{\text{eff}} \) is the effective thickness.

**Table V** shows some of the formulas concerned.

**References**