ABSTRACT

The force transmission of a flexible instrument through an endoscope is deteriorated due to friction between the contacting surfaces. Friction force along the axial direction can be reduced by combining the translation motion input with rotational motion input at the proximal end of the instrument. The effect of the combined motion on the force transmission is studied for a flexible instrument through a curved rigid tube. A mathematical formula is derived for the reduction in friction force along the axial direction due to the combined motion input. The force transmission of a flexible instrument through a curved rigid tube is analysed using the capstan equation. The ratio of the input and output forces is compared for the combined motion with that of the translation motion only. A ratio ζ is defined to measure the reduction in the friction force along the axial direction due to the combined motion input. The analytical result shows the reduction in the friction force for the combined motion input. A flexible multibody model is set up and various simulations are performed with different motion inputs. The simulation result showed a reduction in the value of ζ in accordance with the analytical result for the given velocity ratio. The results are further validated with the experimental results. The simulation and experimental results show an agreement with the analytical solutions. The knowledge of force transmission with a combination of motions can be used to increase the force fidelity of a flexible instrument in applications like robotic surgery with a flexible instrument.

Key words: friction alleviation, force transmission, flexible instrument, combined motion

INTRODUCTION

The application of flexible instruments in robotic surgical system becomes apparent and inevitable with new trend in surgery, i.e. flexible endoscopic surgery, Natural Orifice Transluminal Endoscopic Surgery (NOTES) and Single Incision Laparoscopic Surgery (SILS) [1, 2]. Nonetheless, the surgeons are deprived of their touch feedback, often referred as haptic or force feedback, while performing robotic surgery using these instruments [3–5]. The addition of force feedback capabilities into minimally invasive surgical procedures provides an excellent opportunity to improve the quality of surgical procedures by reducing surgical errors and potentially increasing patient safety.

However, the frictional force causes disturbances to haptic sensations. The amount of haptic feedback may be enhanced by reducing the frictional force [4]. Moreover, the reduction in friction force also leads to improved motion transmission [6]. A better estimation of instrument tip motion also helps in accurate control of thin flexible surgical instrument [7].

In our previous study [8, 9], we studied the translational and rotational behavior of a flexible instrument inside a curved rigid tube. The force transmission of a flexible instrument is deteriorated due to friction between the instrument and the
The ratio of the input and output forces is compared for the combined motion with that of the translational motion only. A flexible multibody is set up. Various simulations are performed combining the rotational motion with the axial motion.

The objective of this paper is to study the effect of combined motion on the force transmission of the flexible instrument along the axial direction through a curved rigid tube. A flexible wire of circular cross-section with diameter $d_w$ is used as a flexible instrument for the analysis. The wire is inserted through a curved rigid tube with a wrap angle $\alpha$. Figure 1 shows the configuration of the wire inside the tube. The translational motion input $x_{in}$ is combined with the rotational motion input $\theta_{in}$. The reaction force at the proximal end is the input force $F_{in}$. The distal end of the wire is attached to a spring. The force applied at the distal end is the output force $F_{out}$. The output motions at the distal end are—$x_{out}$ in translation and $\theta_{out}$ in rotation.

A mathematical formula is derived for the reduction in the friction force along the axial direction due to the combined motion input. The force transmission of a flexible wire through a curved rigid tube is analyzed using the capstan equation. The ratio of the input and output forces is compared for the combined motion with that of the translational motion only. A flexible multibody is set up. Various simulations are performed and the results are compared with the analytical results. For the experimental validation, we performed various experiments using flexible wire inside a curved rigid tube. The experimental results are discussed and the conclusion is presented at the end.

**ANALYTICAL FORMULATION**

The translational motion input at the proximal end results into sliding velocity $v_a$ along the axial direction. Likewise, the rotational motion input at the proximal end produces sliding velocity $v_c$ along the circumferential direction at the point of contact. If $\omega$ is the angular velocity along the axial direction and $d_w$ is the diameter of the instrument, the instantaneous velocity at the point of contact along the circumferential direction is given by $v_c = \omega d_w / 2$.

Assuming the input motions, translation and rotation, at the proximal end are conveyed uniformly along the length of the wire, the resultant sliding velocity at the point of contact on the tangent plane is given by

$$v_t = \sqrt{v_a^2 + v_c^2}$$

where the angle $\phi$ is the angle the resultant sliding velocity $v_t$ makes with the unit vector $e_a$ along the axial direction. The velocity components along the axial and circumferential directions on the tangent plane at the point of contact P are shown in Fig. 2.

**Reduction in Friction Force along the Axial Direction**

As the friction force acts in the opposite direction of the resultant sliding velocity, the resultant friction force also makes an angle $\phi$ with the unit vector $e_a$ along the axial direction. The friction force components along the axial and circumferential directions are shown in Fig. 2. It is evident from the figure that for a given velocity $v_t$ the friction force along the axial direction is the maximum for $\phi = 0$ and $F_a = F_t$. On the other hand, the friction force along the axial direction is the minimum for $\phi = \pi/2$ and $F_a = 0$. In general, the friction force along the axial
direction is given by
\[ F_a = F_t \cos \phi. \]  (3)

If we assume the Coulomb friction force model, for the given normal reaction force \( F_n \) acting at the contact point \( P \), the friction force \( F_t \) is given by
\[ F_t = \mu F_n \]  (4)

where \( \mu \) is the coefficient of friction between the two contacting surfaces.

Therefore, if the translational motion is combined with the rotational motion, the friction force along the axial direction can be rewritten as
\[ F_a = \mu F_n \cos \phi. \]  (5)

Hence, the effective coefficient of friction \( \mu_{\text{eff}} \) along the axial direction in the case of combined motion is
\[ \mu_{\text{eff}} = \mu \cos \phi. \]  (6)

The reduction in the friction force along the axial direction due to the combined motion is given by
\[ \Delta F_a = (1 - \cos \phi)F_t. \]  (7)

Therefore, by combining the motion, the friction force along the axial direction can be reduced. A reduction in the overall friction force along the axial direction also leads to reduced motion hysteresis in translation [6].

**Force Transmission Due to the Combined Motion**

For the wire in curved tube as shown in Fig. 1, the force transmission in the translational motion can be compared with the capstan equation. The well-known relationship between the output and input forces on the both sides of a flexible string wound around a circular body is given by the capstan equation [10]. If \( F_{\text{in}} \) and \( F_{\text{out}} \) are the forces on the input and output sides, the capstan equation can be written as
\[ \frac{F_{\text{in}}}{F_{\text{out}}} = e^{\pm \mu \alpha} \]  (8)

where \( \mu \) is the coefficient of friction between the two contacting surfaces, and \( \alpha \) is the wrap angle. Here, the sign is positive when the wire is retracting and it is negative when the wire is moving forward.

If there is only translational motion along the axial direction, the logarithm of the force ratio can be written as
\[ \ln \left( \frac{F_{\text{in}}}{F_{\text{out}}} \right)_{\text{trans}} = \pm \mu \alpha. \]  (9)

The absolute value of the logarithm of the force ratio remains constant for a given configuration of the wire and the tube and is equal to \( \mu \alpha \). The sign depends on the direction of motion of the wire.

In the case of the combined motion, the logarithm of the force ratio is given by
\[ \ln \left( \frac{F_{\text{in}}}{F_{\text{out}}} \right)_{\text{comb}} = \pm \mu_{\text{eff}} \alpha. \]  (10)

Let us define \( \zeta \) as a ratio of the logarithm of the force ratio for the combined motion and the logarithm of the force ratio for the translational motion to compare the effect of the combined motion on the force transmission along the axial direction, that is
\[ \zeta = \frac{\ln \left( \frac{F_{\text{in}}}{F_{\text{out}}} \right)_{\text{comb}}}{\ln \left( \frac{F_{\text{in}}}{F_{\text{out}}} \right)_{\text{trans}}}. \]  (11)

Here, we use the absolute value in the denominator for the comparison. From Eqn. (9), Eqn. (10) and Eqn. (6), Eqn. (11) can be rewritten as
\[ \zeta = \pm \frac{\mu_{\text{eff}} \alpha}{\mu \alpha}. \]  (12)

The ratio \( \zeta \) is used to calculate the force transmission along the axial direction. As \( \phi \to \pi/2 \), the value of \( \zeta \) approaches towards 0. Correspondingly, the force transmission along the axial direction will be the maximum. In this case, the friction force will be acting along the circumferential direction of the wire, and the force transmission along the axial direction will be maximized. However, As \( \phi \to 0 \), the value of \( \zeta \) approaches towards 1. The force transmission along the axial direction, in this case, will be the minimum. The friction force will act along the axial direction of the wire, and will limit the force transmission along the axial direction.

For all angles \( \phi \), it follows from Eqn. (2) and Eqn. (13) that \( \zeta \) can be expressed in terms of velocity ratio \( v_c/v_a \) as
\[ \zeta = \pm \frac{1}{\sqrt{1 + (v_c/v_a)^2}}. \]  (14)
It is apparent from Eqn. (14) that the magnitude of the ratio $\zeta$ decreases with increasing velocity ratio $v_c/v_a$ and, therefore, the force transmission improves with the increased velocity ratio.

**Analytical Results**

We analyzed the effect of the combined motion on the force transmission along the axial direction. From Eqn. (14), we observe that the ratio $\zeta$ depends on the velocity ratio $v_c/v_a$. In order to get different velocity ratio, a constant axial velocity is combined with a sinusoidal rotational input at the proximal end.

Let us consider a guide wire inserted into a curved rigid tube as illustrated in Fig. 1. The distal end of the guide wire is attached to a spring with a linear stiffness $k_p$. The input motion is described at the proximal end both in the translation and rotation. The coefficient of friction between the contacting surfaces is given by $\mu$.

It is assumed that the guide wire is loaded in tension all the time. A constant velocity $v_{in}$ is defined along the axial direction. A sinusoidal rotational motion of amplitude $A$ and the frequency $f$ is also defined at the proximal end such that the initial angular displacement and the velocity are zero, and is given by

$$\theta_{in} = A\cos(2\pi ft) - A$$
$$\dot{\theta}_{in} = -2\pi fA\sin(2\pi ft).$$

Therefore, the velocity components along the axial and circumferential directions are given by

$$v_a = \pm v_{in}$$
$$v_c = \dot{\theta}_{in}d_a/2.$$  \hspace{1cm} (17), (18)

The axial velocity has a constant magnitude, but the sign is reversed to limit the range of the axial motion. It is assumed that the guide wire can freely rotate along the axial direction and the spring does not constrain the guide wire in rotation.

**Constant $v_a$ with Sinusoidal $v_c$**

Input motion profile both in translation and rotation are defined to analyze the effect on the ratio $\zeta$. A constant translational velocity $v_a = 5$ mm/s is defined along the axial direction. Its direction is reversed every 1 s. A sinusoidal motion is defined in rotation with different amplitudes $A = \pi$, $2\pi$, and $3\pi$ radians and a frequency $f = 1$ Hz (given by Eqn. (16)). A guide wire of diameter $d_a = 0.8$ mm is considered. The corresponding velocities along the circumferential direction $v_c$ are calculated (Eqn. (18)). Figure 3 shows the velocity profiles in the axial and circumferential directions. For $t < 1.75$ s, there is no rotational motion defined. The velocity along the circumferential direction $v_c$ is triggered after $t = 1.75$ s.

The value of the ratio $\zeta$ is calculated for different values of the axial velocity $v_a$ and the circumferential velocity $v_c$ using Eqn. (14) and shown in Fig. 4. For $t < 1.75$ s, as there is no rotational motion, the value of the ratio $\zeta$ changes to $\pm 1$ as the direction of motion changes. However, after $t = 1.75$ s, the magnitude of the ratio $\zeta$ starts decreasing from 1 as the magnitude of the sinusoidal rotational motion starts increasing. The maximum decrease in the magnitude of the ratio $\zeta$ occurs when the circumferential velocity is maximum for the given axial velocity. As the circumferential velocity approaches zero, the magnitude of the ratio $\zeta$ also approaches towards 1. In the subsequent periodic behavior, the frequency of the sine function can be recognized.

Here, we observed that when the circumferential velocity...
$v_c$ is zero, the friction force acts completely along the axial direction and thus limits the force transmission. However, as the magnitude of the circumferential velocity $v_c$ increases, the magnitude of the ratio $\zeta$ decreases and it reaches to the minimum for the maximum value of $v_c$.

We observed a significant decrease in the friction force in the axial direction if the velocity along the axial direction $v_a$ is combined with the velocity along the circumferential direction $v_c$. From the chosen case study, the ratio $\zeta$ reduces to 0.2. The reduction in the friction force will increase the force fidelity of a flexible guide wire in a curved rigid tube. The backlash—associated with the friction—can be greatly reduced.

**FLEXIBLE MULTIBODY MODELING AND SIMULATION**

We developed a flexible multibody model to study the behavior of a flexible instrument inside a curved rigid tube. The modeling task requires three key components— a finite element model of the instrument, a model of the curved rigid tube, and the contact model to define the interaction between the instrument and the tube.

The flexible instrument is modeled as a series of interconnected flexible beam elements. Each beam element has six deformation modes defined—one mode for the elongation, one mode for the torsion, and four modes for the bending deformations of the element [11, 12]. The curved rigid tube is modeled as a circular tube of uniform cross-section. The shape of the tube is defined by the centerline of the tube. The centerline is defined by a straight line, a circular arc, a Bézier curve, or a combination of these. Different geometric shapes are connected together to define the entire length of the curved tube. The contact model includes the interaction in the normal and tangential directions at the various nodes. Wall stiffness and damping are defined to calculate the interaction in the normal direction. Friction between the instrument and the tube is defined to calculate the interaction in the tangential direction. The details of the developed model can be referred to [8, 9, 13].

A flexible multibody model is set up to validate the analytical results. The effect of the velocity components—along the axial and circumferential directions—on the force transmission is further investigated.

**Description of the Model**

A flexible instrument is defined by a stainless steel wire of circular cross-section of 0.8 mm diameter. The undeformed wire is straight and has no pre-curvature. The entire length of the wire is defined by interconnected spatial beam elements. A curved rigid tube is defined by an arc of radius of curvature 300 mm and a wrap angle $\alpha = 90^\circ$. The inner diameter of the tube is 4.0 mm. Two straight tubes of the same inner diameter are defined at the entry and exit of the curved tube. The wire inside the circular section is defined by 10 equal length spatial beam elements. A section of 130 mm and 200 mm long wire are defined in the straight sections of the tube at the entry and at the exit respectively. Figure 5 shows the configuration of the model.

**Simulation for the Combined Motion**

The model is set up for the simulation. The instrument is assumed straight in the beginning. The initial deformations of the elements are obtained analytically and prescribed in the beginning to place the instrument along the centerline of the defined curved rigid tube. The data used for the model and simulation are given in Table 1. The load at the distal end is applied through a linear spring of stiffness $k_{sp} = 198.3$ N/m and an initial extension of 12.5 mm. A lumped mass and inertia are added at the distal end to account for attachments used for measurement. The mass and inertia properties are given in Table 2.

An input motion profile is defined at the proximal end of the instrument in both directions—translation and rotation—as shown in Fig. 3. In the simulations, the amplitude is $A = 2\pi$ rad, and the frequency is $f = 1$ Hz. The simulation is performed for $t = 6$ s. The rotational motion is triggered at $t = 1.75$ s so that when the instrument reverses the direction at $t = 2$ s, the circumferential velocity $v_c$ due to rotation reaches to the maximum.

The input force $F_{in}$ and the output force $F_{out}$ are calculated during the simulation. The input force $F_{in}$ is the force acting at the proximal end and is equal to the reaction force. The output force $F_{out}$ is the force acting at the distal end and is equal to the spring force. The extension in the spring $x_{extn}$ is calculated from the displacement at the distal end. Therefore, the output force $F_{out} = k_{sp}x_{extn}$. The force ratio $F_{in}/F_{out}$ is calculated for the entire motion range. The ratio $\zeta$ is calculated further using Eqn. (11).

The coefficient of friction $\mu$ between the contacting surfaces is assumed constant. The Coulomb friction force model is used in the simulation.
TABLE 1. Data used for the simulations

<table>
<thead>
<tr>
<th>Description</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$</td>
<td>$7.8 \times 10^3$ kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>Young’s modulus, $E$</td>
<td>$200 \times 10^9$ N/m$^2$</td>
<td></td>
</tr>
<tr>
<td>Shear modulus, $G$</td>
<td>$79 \times 10^9$ N/m$^2$</td>
<td></td>
</tr>
<tr>
<td>Mass/length, $pA$</td>
<td>$3.92 \times 10^{-3}$ kg/m</td>
<td></td>
</tr>
<tr>
<td>Flexural rigidity, $EI$</td>
<td>$4.02 \times 10^{-3}$ N m$^2$</td>
<td></td>
</tr>
<tr>
<td>Torsional rigidity, $GJ$</td>
<td>$3.18 \times 10^{-3}$ N m$^2$</td>
<td></td>
</tr>
<tr>
<td>Size of the transition zone</td>
<td>0.5 mm</td>
<td></td>
</tr>
<tr>
<td>Wall stiffness, $k$</td>
<td>$2.0 \times 10^4$ N/m$^2$</td>
<td></td>
</tr>
<tr>
<td>Wall damping, $c_w$</td>
<td>10.0 Ns/m$^2$</td>
<td></td>
</tr>
<tr>
<td>Velocity coefficient, $v_c$</td>
<td>$2.0 \times 10^3$ (m/s)$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Friction coefficient, $\mu$</td>
<td>0.157</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2. Mass properties of the attachment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational inertia $J_x$</td>
<td>1.36 $10^{-5}$ kg m$^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$J_y$ 17.47 $10^{-5}$ kg m$^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$J_z$ 17.67 $10^{-5}$ kg m$^2$</td>
<td></td>
</tr>
<tr>
<td>Mass $m$</td>
<td>0.100 kg</td>
<td></td>
</tr>
</tbody>
</table>

**Simulation Results**

Simulation results are presented with respect to the given input motion. The input force $F_{in}$ and output force $F_{out}$ are calculated and shown in Fig. 6. We can observe the transients in the beginning. The output force $F_{out}$ varies linearly as it depends on the extension of the spring. Initially, as there is only translational motion along the axial direction, the friction force will be entirely acting along the axial direction. When the direction of motion changes, the direction of friction force also changes. The corresponding jump in the input force $F_{in}$ can be observed at $t = 1.0$ s. The rotational motion is combined with the translational motion after $t = 1.75$ s. The corresponding changes in the input force $F_{in}$ can be observed. As the magnitude of the circumferential velocity $v_c$ increases, the friction force along the axial direction decreases. Therefore, the difference in the input force $F_{in}$ and the output force $F_{out}$ decreases. Inversely, when the magnitude of the circumferential velocity $v_c$ approaches zero, the friction force acts entirely along the axial direction. Therefore, the difference between the forces is the maximum.

**Effect of the velocity ratio on the ratio $\zeta$**

The ratio $\zeta$ is calculated to compare the effect of the combined motion on the force transmission (see Eqn. (11)). The logarithm of the force ratio for the combined motion, $\ln(F_{in}/F_{out})$, is calculated. The magnitude of the logarithm of the force ratio for the axial motion only is $\mu \alpha$. In this case, the value of $\mu$ is 0.157 and $\alpha$ is $\pi/2$ rad. Figure 7 shows the corresponding plot of the ratio $\zeta$. Initially, the value of $\zeta$ is 1.0 as there is no rotational motion combined with the axial motion. However, as the translational motion is combined with the rotational motion, we can observe a decrease in the value of $\zeta$. The analytical value of the ratio $\zeta$ is calculated using Eqn. (14) as previously shown in Fig. 4 and also included in Fig. 7. The simulation result shows a good correlation with the analytical result. As the velocity ratio $v_c/v_a$ increases, the value of the ratio $\zeta$ decreases.
EXPERIMENTS

Various experiments are performed to validate the analytical and simulation results. We used the experimental set-up previously designed to study the motion and force transmission of a flexible instrument inside a curved rigid tube [14]. A brief description of the set-up is given in the following section.

Experimental Set-up

The experimental set-up consists of three basic modules: Actuation module (AM), Force sensing module (FSM), and Tip motion measurement module (T3M). Figure 8 shows the experimental set-up, indicating the three modules together with the key components.

The AM provides the actuation in 2-DOFs, i.e. translation and rotation along the axial axis of the instrument at the proximal end. The FSM enables all 6-DOFs measurement of forces and torques arising from the interaction of the instrument with the curved rigid tube. The T3M measures the translation and rotation of the distal end of the instrument along the axial direction. The details of the design and the evaluation of the set-up can be referred to [14].

The combined motion is effected through the translational and rotational motion of the AM independently. The motions along the two DOFs are measured by the encoders. The proximal end of the wire is attached to the AM at the rotational axis. The motion of the distal end of the wire is measured via the cam through two laser displacement sensors. The forces acting at the ends of the wire are measured from the reaction forces measured by the FSM.

A force is applied to the distal end along the axial direction through a spring. The spring is attached to the base through a string. The configuration allows the loading of the wire in the axial direction. As the string is extremely compliant in bending and torsion, the rotational motion can be combined with the translational motion without loading the wire in torsion. Figure 9 shows the details of the attachment to the distal end.

Configuration of the guide tube and wire

For the guide tube, we used a stainless steel tube of outer diameter of 6 mm and inner diameter of 4 mm. A total length of 800 mm was chosen. The tube was bent to 90° arc of radius of curvature 300 mm. There were straight sections left at both the ends. The input side of the tube was aligned to the AM and the output side of the tube was aligned to the T3M. The tube was rigidly attached to the top plate of the FSM.

We chose a stainless steel wire of diameter 0.8 mm and a total length of 880 mm. The wire has no precurvature. The wire is inserted into the curved rigid tube and the two ends are attached to the AM and the T3M. The axial load is applied through the spring and is attached to the base through a string. The extension of the spring is adjusted so that there is always a tensile load acting on the wire throughout the range of motion in the axial direction.

Experimental Results

Various experiments are performed to see the effect of the combined motion on the value of the ratio $\zeta$. Different velocity ratios $v_c/v_a$ are obtained by combining the constant translation $v_{in}$ with the constant rotational input $\omega_{in}$.

Estimation of the value of $\mu$

An experiment was set up to estimate the value of the coefficient of friction $\mu$ between the stainless steel wire and the tube. The wire is inserted into the curved rigid tube. The distal end is attached to the spring.
A constant velocity, \( v_{\text{in}} = 0.5 \text{ mm/s} \), to-and-fro motion is given to the proximal end. The forces at the two ends of the wire are calculated.

The value of \( \mu \) is calculated from the logarithm of the force ratio. From Eqn. (9),

\[
\ln \left( \frac{F_{\text{in}}}{F_{\text{out}}} \right) = \mu \alpha.
\]

(19)

The mean value, thus, calculated over the two cycles is 0.247. For \( \alpha = \pi / 2 \text{ rad} \), the value of \( \mu \) is 0.157. This is the value of \( \mu \) used in the simulation.

**Constant translation \( v_{\text{in}} \) with constant rotation \( \omega_{\text{in}} \)**

Experiments are performed for different velocity ratios by combining a constant translational input \( v_{\text{in}} \) with constant rotational input \( \omega_{\text{in}} \). The constant velocity input of 0.25, 0.50 and 1.0 mm/s in translation are used for the experiments. The translational motion is combined with the constant angular velocity of 0, \( \pi / 3 \), \( \pi / 2 \) and \( \pi \) rad/s.

**Case 1:** \( v_{\text{in}} = 0.25 \text{ mm/s} \) A constant velocity \( v_{\text{in}} \) of 0.25 mm/s is given to the proximal end of the wire. The motion is reversed after 20 s and the cycle is repeated. Initially, there is no rotational motion combined with the axial motion. The input and output forces are measured and the logarithm of the force ratio, \( \ln(F_{\text{in}}/F_{\text{out}})|_{\text{trans}} \), is calculated. The experiment is repeated with the same axial motion combined with the rotational speed \( \omega_{\text{in}} = \pi / 2 \text{ rad/s} \). The rotational motion is reversed after 10 s and the cycle is repeated. The input and output forces are measured and the logarithm of the force ratio, \( \ln(F_{\text{in}}/F_{\text{out}})|_{\text{comb}} \), is calculated for the combined motion.

The ratio \( \zeta \) is calculated using Eqn. (11) where the denominator is obtained from the mean value of \( \ln(F_{\text{in}}/F_{\text{out}})|_{\text{trans}} \). Figure shows the plot of the ratio \( \zeta \) with respect to the input displacement. Here, we observed that when the translational motion is combined with the rotational motion, the value of the ratio \( \zeta \) is reduced considerably. For a translational speed of \( v_{\text{in}} = 0.25 \text{ mm/s} \) and rotational speed of \( \pi / 2 \text{ rad/s} \), the calculated value of angle \( \phi \) is 68.3\(^\circ\). The value of the ratio \( \zeta \) for the combined motion is also compared with the analytical result, \( \cos \phi = 0.37 \). As the rotational motion changes direction every 10 s, we observe an increase in the value of \( \zeta \) at the end of stroke and also halfway. This is expected as there is no rotation momentarily and, therefore, the value of \( \zeta \) approaches towards 1.0. However, the value of \( \zeta \) reduces as soon as the rotation commences.

The value of \( \zeta \) is calculated for another value of the angular rotation \( \omega_{\text{in}} = \pi / 3 \text{ rad/s} \) combined with the axial motion \( v_{\text{in}} \). The value of \( \zeta \) is also compared with the corresponding value of the analytical value, \( \cos \phi \). The results are summarized in Table 3.

<table>
<thead>
<tr>
<th>( v_{\text{in}} ) [mm/s]</th>
<th>( \omega_{\text{in}} ) [rad/s]</th>
<th>Analytical ( \cos \phi )</th>
<th>Experimental mean</th>
<th>Experimental rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>( \pi / 3 )</td>
<td>0.51</td>
<td>0.71</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>( \pi / 2 )</td>
<td>0.37</td>
<td>0.51</td>
<td>0.24</td>
</tr>
<tr>
<td>0.5</td>
<td>( \pi / 2 )</td>
<td>0.62</td>
<td>0.48</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>( \pi )</td>
<td>0.37</td>
<td>0.36</td>
<td>0.14</td>
</tr>
<tr>
<td>1.0</td>
<td>( \pi / 2 )</td>
<td>0.85</td>
<td>0.95</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>( \pi )</td>
<td>0.62</td>
<td>0.70</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Tab. 3. The reduction in the ratio \( \zeta \) with respect to the velocity ratio is observed in the experiments, although the experimental reduction is somewhat smaller than the analytical values. The rms values of \( \zeta \) are also calculated for different velocity ratios and shown in Tab. 3. The rms values are relatively high due to velocity reversals in the rotational direction. The sudden change in the velocity also excites the higher modes and leads to oscillations, which is reflected in the rms values.

**Case 2:** \( v_{\text{in}} = 0.5 \text{ mm/s} \) In this case, a constant input velocity \( v_{\text{in}} = 0.5 \text{ mm/s} \) is given to the proximal end. The motion is reversed after 10 s and repeated. The translational motion is combined with different rotational speed \( \omega_{\text{in}} = 0, \pi / 2 \) and \( \pi \) rad/s. The rotational motion is reversed after every 5 s. Figure shows the plot of the ratio \( \zeta \) for rotational input of \( \omega_{\text{in}} = \pi \text{ rad/s} \) and translational input of \( v_{\text{in}} = 0.5 \text{ mm/s} \) with respect to the input displacement \( x_{\text{in}} \). The analytical value of the ratio \( \zeta \) is also shown for comparison, which is equal to \( \cos \phi \).

Again, the combined motion results into a reduction in the value of the \( \zeta \). The value of \( \zeta \) reduced to 0.36 as compared to the theoretical value of 0.37 for \( v_{\text{in}} = 0.5 \text{ mm/s} \) and \( \omega_{\text{in}} = \pi \text{ rad/s} \). As the rotational motion corresponding to \( \omega_{\text{in}} = \pi \text{ rad/s} \) reverses after 5 s, there is no rotation momentarily. As a result, the value of the ratio \( \zeta \) approaches towards 1 at the end of stroke and halfway.

The value of the \( \zeta \) is calculated for another value of the angular rotation \( \omega_{\text{in}} = \pi / 2 \text{ rad/s} \) combined with the axial motion \( v_{\text{in}} \). The results are summarized in Table 3. The reduction in the ratio \( \zeta \) with respect to the velocity ratio is observed. The rms values of \( \zeta \) are of the order of 0.15 (see Tab. 3).
Case 3: $v_{in} = 1.0 \text{ mm/s}$ The translational speed $v_{in}$ is changed to 1.0 mm/s. The translational motion is combined with different values of rotational speed $\omega_{in}$—0, $\pi/2$ and $\pi$ rad/s. Both the motions are reversed at the same time after every 5 s. Figure shows the plot of the ratio $\zeta$ for the rotational input of $\omega_{in} = \pi$ rad/s and translational input of $v_{in} = 1.0 \text{ mm/s}$. The plot the ratio $\zeta$ is compared for no rotational motion combined with the translational input. We observe the reduction in the value of $\zeta$ as the translational motion is combined with the rotational motion. The analytical value of the ratio $\zeta$ for the combined motion—a rotational speed $\omega_{in}$ and translational speed $v_{in}/v_{ax}$ is also shown. The value of $\zeta$ reduced to 0.70 as compared to the theoretical value of $\cos\phi = 0.62$ for $v_{in} = 1.0 \text{ mm/s}$ and $\omega_{in} = \pi$ rad/s.

The value of the $\zeta$ is calculated for another value of the angular rotation $\omega_{in} = \pi/2$ rad/s combined with the axial motion $v_{in}$. The results are summarized in Tab. 3. The reduction in the ratio $\zeta$ with respect to the velocity ratio is observed. The rms values of $\zeta$ are of the order of 0.20 (see Tab. 3).

The translational and rotational inputs are abruptly changing the direction of motion. As a result, we see transient vibrations in both the directions. The velocities in translation and rotation along the wire suffer from oscillations, and they are no more constant as the input motion. The ratio $\zeta$ depends on the velocity ratio at the individual contact points and, therefore, we observe oscillations in the experimental results as shown in Fig. 10. This is also reflected in the rms values of $\zeta$ as shown in Tab. 3. Nevertheless, the effect of the combined motion on the ratio $\zeta$ is observed in all the cases as hypothesized. The experimental results show that the value of the ratio $\zeta$ decreases as the velocity ratio $v_c/v_{ax}$ increases as shown in Tab. 3.

**FIGURE 10.** Comparison of the ratio $\zeta$ for the combined motion—a rotational speed $\omega_{in}$ with a translational motion input $v_{in}$. The corresponding analytical values of the ratio $\zeta = \cos\phi$ for the combined motion are also shown.

**DISCUSSION**

Assumptions are made to simplify the analytical model and use it as a tool to estimate the improvement in the force transmission due to the combined motion. We assumed uniform contact and motion transmission along the length of the flexible instrument. However, as the wire is flexible in bending and rotation, the motion at the distal end and all the intermediate contact points will not be same as the input motion at the proximal end. The analytical formulation, however, assumes that the wire is fully compliant in bending and rigid in axial direction and torsion. Nonetheless, the analytical formulation provides the basis of the study and can be used as a guiding principle for further study.

Friction, inertia and the compliance of the instrument leads to deviation from the analytical formulation. The effect of these parameters are observed in the experiments. The flexible multibody can be useful to further investigate the effect of combined motion on various kinematic and dynamic behavior.

The value of $\mu$ is not constant and varies over a range. Therefore, we estimated the value of $\mu$ for each set of experiments. Nonetheless, to determine an overall reduction in $\zeta$ over a cycle, the absolute value of the friction force is not required.

The reduction in $\zeta$ leads to improved force transmission along the axial direction. The application is not only limited to the robotic surgical application, but can be also applied to many mechanical transmission problems using flexible wires through tubes.

**CONCLUSION**

In this paper, we presented how the force transmission of a flexible instrument inside a tube is improved along the axial direction by combining the translational motion input with rotational motion input. An analytical model is derived to
calculate the reduction in friction force. The ratio $\zeta$ is defined to measure the reduction in the friction force along the axial direction due to the combined motion. We set up a flexible multibody model of an instrument inside a curved rigid tube and compared the simulation results with the analytical solution. The simulation result showed a reduction in the value of $\zeta$ in accordance with the analytical result for the given velocity ratio $v_c/v_a$.

We showed the reduction in the ratio $\zeta$ through several experiments combining different constant translational input $v_{\text{in}}$ with constant rotational input. Static and dynamic effects lead to a deviation from the analytical solution. The reduction in the friction force ultimately results into improved force transmission. The developed analytical model offers a simple though effective estimate of the amount of reduction in friction force in axial direction. The knowledge of force transmission with a combination of motions can be used to increase the force fidelity of a flexible instrument in applications like robotic surgery with a flexible instrument. The developed insight can also be useful in designing mechanical transmission using flexible wires, cables and tubes.

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