Tailored Work Hardening Descriptions in Simulation of Sheet Metal Forming

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Abstract. In the previous decades much attention has been given on an accurate material description, especially for simulations at the design stage of new models in the automotive industry. Improvements lead to shorter design times and a better tailored use of material. It also contributed to the design and optimization of new materials. The current description of plastic material behaviour in simulation models of sheet metal forming is covered by a hardening curve and a yield surface. In this paper the focus will be on modelling of work hardening for advanced high strength steels considering the requirements of present applications. Nowadays work hardening models need to include the effect of hard phases in a soft matrix and the effect of strain rate and temperature on work hardening. Most material tests to characterize work hardening are only applicable to low strains whereas many practical applications require hardening data at relatively high strains. Therefore, physically based hardening descriptions are needed allowing reliable extensions to high strain values.

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INTRODUCTION

Improvements in the simulation of forming processes and mechanical performance of car bodies lead to shorter design times and a better tailored use of material. A better understanding of the underlying physical principles contributes to the design and optimization of new materials. Robust optimization of forming processes using numerical models can lead to higher efficiency. These developments have one requirement in common: a reliable model to describe the behaviour of new steel grades with complicated multi-phase microstructures at various process conditions, including temperature and strain rate ranges. The common description of plastic material behaviour in simulation models of sheet metal forming is covered by a hardening curve and a yield surface. In this paper, a new hardening model will be presented that describes work hardening, temperature and strain rate influences for advanced high strength steels based on underlying physical principles.

Initially, a more classical approach is used based on the development of dislocation cell structures and its effect on work hardening. Complexity is added to the model in the following steps:
- Initial yielding followed by usual stage III hardening [1]
- Work hardening at high strains indicated as stage IV hardening [2]
- Influence of phase boundaries in multiphase steels [3]
- Influence of temperature and strain rate on work hardening [4]
- Thermo dynamic movement of dislocations over obstacles [5, 6]
- Effect of strain path on work hardening [7, 8, 9, 10, 11]

It is important to recognize that these phenomena can play a role in the material behaviour during forming processes. A hardening model needs to capture these phenomena that have become more important with current steel developments and increased accuracy requirements for manufacturing and performance predictability.

The value of an advanced hardening model relies on the capability to characterize the individual contributions to the hardening behaviour. Developments in measuring techniques, in particular in optical strain measuring systems, have contributed to a more detailed knowledge of mechanical test methods. Combined with better models comes a better interpretation of the test results. Some of the above phenomena require specific tests. It is for instance important to recognize that stage IV work hardening of steel is not captured by uniaxial tensile tests and a reliable extrapolation of hardening to high strain values depends therefore on other test results, e.g. from a stack compression test or a hydraulic bulge test. Advanced hardening models have developed from single curve fitting to more complicated material descriptions requiring input from multiple tests [11, 12].
DESCRIPTION OF WORK HARDENING INCLUDING THE EFFECT OF STRAIN RATE AND TEMPERATURE

Starting point is the well known Taylor relation between the plastic flow stress and the dislocation density [1]:

\[ \sigma_f = \sigma_{f0} + M \cdot \alpha_D \cdot G \cdot b \cdot \sqrt{\rho_i} \]  
(1)

Where:
- \( \sigma_f \) = flow stress (MPa)
- \( \sigma_{f0} \) = initial flow stress (MPa), i.e. combination of back stress and a dynamic term dependent on strain rate and temperature
- \( M \) = Taylor factor depending on crystallographic texture \( \approx 2.7 \)
- \( \alpha_D \) = parameter depending on immobile dislocation distribution with a value \( \approx \frac{1}{3} \)
- \( G \) = shear modulus for iron (steel) \( \approx 81000 \) MPa at Room temperature, decreases with temperature
- \( b \) = Burgers vector i.e. slip distance over one slip position; for iron (steel) \( \approx 20 \) nm = \( 2 \times 10^{-7} \) mm
- \( \rho_i \) = immobile dislocation density

In [13] it is explained that using van Liempt’s extension on the Bergström model [2] of hardening results into the following relation between the dislocation density and equivalent strain

\[ \frac{d\rho_i}{d\varepsilon_t} = B_u' \cdot \rho_i^c \cdot g(\varepsilon_t) - \Omega' \cdot \rho_i \]  
(2)

Where:
- \( \varepsilon_t \) = total equivalent strain (including pre deformation)
- \( B_u' \) = proportional constant for the increase of immobile dislocation in cell walls
- \( c \) = exponent describing the decrease of dislocation cell size (usually a value of 0.5 or 0)
- \( g(\varepsilon_t) = e^{\beta \bar{\varepsilon}_i} \) function describing the effect of change of geometry of dislocation cells on the mean free path, introduced by van Liempt [2]; theoretically related to the strain tensor but for simplicity the scalar equivalent strain is used with the parameter \( \beta \) for this geometry change.
- \( \Omega' \) = constant describing remobilization and annihilation of dislocations dependent on strain rate and temperature.

For convenience, a parameter \( \zeta_i \) is introduced for the dislocation density (and parameter \( B_u' \) is replaced by \( B_u = B_u/(1-c) \)):

\[ \zeta_i = \frac{\rho_i^{1-c}}{(1-c) \cdot B_u} \cdot \frac{B_u}{B_u'} = \frac{\rho_i^{1-c}}{B_u} \]  
(3)

As a consequence, equation (2) can be rewritten (\( \Omega' \) is replaced by \( \Omega = \Omega/(1-c) \))

\[ \frac{d\zeta_i}{d\varepsilon_t} = e^{\beta \bar{\varepsilon}_i} - \Omega \cdot \zeta_i \]  
(4)

Substitution of equation (3) into (1) leads to

\[ \sigma_f = \sigma_{f0} + M \cdot \alpha_D \cdot G \cdot b \cdot (B_u \cdot \zeta_i)^{\frac{1}{2}}(1-c)^{\frac{1}{2}} = \sigma_{f0} + \Delta\sigma_h \cdot \zeta_i^{n'} \]  
(5)

Where: \( n' = \frac{1}{2 \cdot (1-c)} \) and \( \Delta\sigma_h = M \cdot \alpha_D \cdot G \cdot b \cdot B_u' \cdot \zeta_i^{\frac{1}{2}}(1-c)^{\frac{1}{2}} \)
\( \Delta \sigma _{0} \text{ (MPa)} \) is considered as a basic parameter for scaling the work hardening. Equations (4) and (5) are used as the basic equations for the work hardening

Equation (4) is integrated using the variation of constant method between the limits \( \epsilon _{t} = 0 \) and \( \epsilon _{t} + \epsilon _{0} \) with \( \epsilon _{0} \) pre-deformation parameter at constant strain rate and temperature delivers an expression for \( \zeta _{I} \) as a function of \( \epsilon _{0} \).

For avoiding unexpected deflection points in the hardening curve at high strains, only the first order term of Taylor series for \( e^{\beta (\epsilon _{t} + \epsilon _{0})} = 1 + \beta \cdot (\epsilon _{t} + \epsilon _{0}) \) is used. Substitution of this result into equation (5) (with some further simplification) leads to the following expression for the flow stress as a function of equivalent strain [13].

\[
\sigma _{f} = \sigma _{f0} + \Delta \sigma _{\sigma} \cdot \left\{ \beta ^{*} \cdot (\epsilon _{t} + \epsilon _{0}) + \left[ 1 - e^{-\Omega ^{*}(\epsilon _{t} + \epsilon _{0})} \right] ^{\nu} \right\}
\]

(6)

Where: \( \Delta \sigma _{\sigma} = \Delta \sigma _{\sigma} / (\beta + \Omega )^{\nu} \cdot \beta ^{*} = n \cdot \beta \) and \( \Omega ^{*} = \beta + \Omega \).

Equation (6) is the well known Bergström-Van Lierdt strain hardening equation describing work hardening during normal cold forming applications. When \( \epsilon _{0} = 0 \) it is identical to the modified El-Magd equation [6].

In non isothermal forming, the remobilization and annihilation of dislocations (\( \Omega \)) cannot be considered as constant. Equation (4) is integrated using the variation of constant method for strain increment \( j \) between the limits \( \epsilon _{t} = \epsilon _{t,j} \) and \( \epsilon _{t} \) (including first order Taylor series approximation of \( e^{\beta \epsilon _{t}} = 1 + \beta \cdot \epsilon _{t} \)):

\[
\zeta _{t,j} = \zeta _{t,j} + \epsilon _{t} \cdot e^{-\Omega (\epsilon _{t,j} - \epsilon _{t,j-1})} + \frac{1}{\beta + \Omega } \left\{ \beta \left( \epsilon _{t,j} - \epsilon _{t,j-1} \right) + \left( 1 + \beta \cdot \epsilon _{t,j-1} \right) \left[ 1 - e^{-\Omega (\epsilon _{t,j} - \epsilon _{t,j-1})} \right] \right\}
\]

(7)

The initial value of the parameter \( \zeta _{t} \) by using a pre strain value \( \epsilon _{0} \) in equation (7). The above described method differs slightly from the end-result of the temperature independent equation (6) because no further simplification is used in the substitution of equation (7) in equation (5). In equation (7), the parameter \( \Omega \) depends on the strain rate and temperature. The following relation is chosen that differs somewhat from the one Bergström proposes [4].

\[
\Omega = \Omega _{0} - \Delta \Omega \cdot \ln(Z) \quad \text{with} \quad Z = \epsilon _{t} \cdot e^{\frac{Q_{act}}{k_{B} T}}
\]

(8)

Where:
- \( \Omega _{0} \) = Remobilization and annihilation parameter at complete absence of recovery (\( Z \to \infty \))
- \( \Delta \Omega \) = Increase of recovery on remobilization and annihilation parameter
- \( Z \) = Zener Hollomon parameter for dynamic recovery behaviour (s⁻¹)
- \( Q_{act} \) = Activation energy for self diffusion / recovery (eV)
- \( k_{B} \) = Boltzmann constant = 8.617.10⁻⁵ eV/K
- \( T \) = absolute temperature in (K)
- \( \epsilon _{t} \) = Strain rate in (s⁻¹)

Substitution of equation (8) into equation (7) delivers a value for the work hardening term including its strain rate and temperature history effect.

### Additional Work Hardening Terms for Dual Phase Materials

A mixture model has initially been developed for dual phase materials and is recently extended for multi phase materials [3]. In each phase \( k \) (total number of K-phases) terms contribute in the work hardening:

- The hardening caused by statistically stored dislocations (SSD): \( \Delta \sigma _{SSDk} = \Delta \sigma _{\sigma} \cdot \zeta _{k} \) (previously described in equation (5) but now per phase).
- Two different contributions by the presence of geometrically necessary dislocations (GND) in the soft phase \( k \) that bridges the deformation gradient the soft phase and the hard phase \( j \):
  * The dislocation density near phase boundary is higher and causes an additional effect of hardening (short range effect): \( \Delta \sigma _{GNDk} \).
  * A pile up stress is caused by these GND’s (long range effect): \( \Delta \sigma _{pile-upj} \).
The preferred way of summation is by putting SSD and GND as two separated terms in one total term for dislocation densities and by adding the pile up stress contributions directly to the total yield stress per phase k.

\[
\sigma_{k} = \sigma_{f0k} + \sqrt{\left(\Delta \sigma_{SSDj}\right)^2 + \sum_{j=1}^{K} \left(\Delta \sigma_{GNDkj}\right)^2} + \sum_{j=1}^{K} \Delta \sigma_{pile-upkj} \tag{9}
\]

Each phase k has its own strain \( \varepsilon_{k} \); for the SSD-term this can be substituted directly in equation (6) or (7). The GND-term and the pile-up term are determined by the difference between the strains in phase k and j

\[
\Delta \sigma_{GNDkj} = \Delta \sigma_{mk} \cdot \left[Q_{kj} \cdot \left(\varepsilon_{k} - \varepsilon_{j}\right)\right] \quad \text{only if} \quad \varepsilon_{k} > \varepsilon_{j} \tag{10}
\]

and:

\[
\Delta \sigma_{pile-upkj} = \Delta \sigma_{mk} \cdot P_{kj} \cdot \left(\varepsilon_{k} - \varepsilon_{j}\right) \tag{11}
\]

Where:

- \( Q_{kj} = \) Parameter for GND hardening of a softer phase k caused by a harder phase j dependent on the geometry and size of phase j particles and the volume fractions of both phases.
- \( P_{kj} = \) Parameter for the pile-up term is caused by the long range stress field of the harder phase j particle in the softer phase k. This parameter depends on particle geometry of the harder phase j and its volume fraction, and not on its size.

In the original mixture model [3], the different mechanical properties of the different phases are taken into account and are combined with the above described two interaction terms between the different phases. By the nature of the two interaction terms the hard phase is becoming harder by both the GND-term and the pile-up term and the harder phase is apparently softer by the pile-up term. This is the main reason that in the mixture model the flow stress using in the different phases converge very rapidly to the same value. A simplified version of this model for dual-phase steel is proposed here that makes use of this effect. Starting from an assumed relation for the strain difference between the ferrite and martensite with the equivalent strain, it is possible to calculate the ferrite flow stress, which is considered to be representative for the flow stress of the composite. This strain difference function \( f(\varepsilon) \) is defined as follows:

\[
f(\varepsilon) = \varepsilon_{1} - \varepsilon_{2} \tag{12}
\]

Suppose a volume fraction of martensite equal to \( \varepsilon_{M} \), and assuming that the composite strain rate can be calculated by the following linear combination of the ferrite strain rate and the martensite strain rate.

\[
\dot{\varepsilon} = \sum_{k=1}^{2} \left(f_{k} \cdot \dot{\varepsilon}_{k}\right) = f_{1} \cdot \dot{\varepsilon}_{1} + f_{2} \cdot \dot{\varepsilon}_{2} = \left(1 - f_{M}\right) \cdot \dot{\varepsilon}_{1} + f_{M} \cdot \dot{\varepsilon}_{2} \iff \dot{\varepsilon}_{1} - \dot{\varepsilon}_{2} = \frac{\dot{\varepsilon}_{1} - \dot{\varepsilon}}{f_{M}} \tag{13}
\]

Differentiating equation (12) to time and substitution into (13) delivers:

\[
\dot{\varepsilon}_{1} = \dot{\varepsilon} + f_{M} \cdot \frac{df(\varepsilon)}{dt} \tag{14}
\]

Integrating to time delivers a relation between the total strain in the ferrite and the total average strain in the composite

\[
\varepsilon_{1} = \varepsilon + f_{M} \cdot f(\varepsilon) \tag{15}
\]

For the boundary condition of the function it is assumed that at low strains the martensite does not deform:

\[
\varepsilon \to 0 \ . \ f(\varepsilon) \to \varepsilon / (1 - f_{M}) \ .
\]

At high strains it is assumed that the martensite strain increases linearly with the composite strain \( \varepsilon \).

\[
\varepsilon \to \infty \ . \ f(\varepsilon) \to \varepsilon_{dp} + B_{dp} \cdot \left[\varepsilon / (1 - f_{M}) - \varepsilon_{dp}\right].
\]

A transition strain, \( \varepsilon_{dp} \), has been defined for separating the two domains and \( B_{dp} \) is the slope of the linear relation with strain after the transition. A Dirac-function in \( \ln(\varepsilon) \)-space is used to connect these two domains in the following way using the transition strain, \( \varepsilon_{dp} \):
\[ f(\varepsilon) = \frac{\varepsilon}{(1 - f_M)} \left( \frac{\varepsilon / (1 - f_M)}{\varepsilon_{dp} + B_{dp} \cdot \varepsilon / (1 - f_M) - \varepsilon_{dp}} \right)^{-F_{Dirac}(\varepsilon)} \]  
with \[ F_{Dirac}(\varepsilon) = \frac{1}{1 + \left( \frac{\varepsilon}{\varepsilon_{dp} \cdot (1 - f_M)} \right)^{-b_p}} \]  

Where: \( b_p \) is a parameter describing the steepness of the transition at the strain \( \varepsilon_{dp} \).

In principal this description can be combined with the strain rate and temperature dependent work hardening in equation (7). Here only the combination with work hardening independent from strain rate and temperature is considered according to equation (6). The way of working is as follows using the result of strain partitioning in equation (16):

- Substitution of the term \( \varepsilon + f_M \cdot f(\varepsilon) \) in stead of \( \varepsilon \) in equation (6) for the SSD term:
  \[ \Delta \sigma_{SSD} = \Delta \sigma_m \cdot \left\{ \beta^* \cdot (\varepsilon + f_M \cdot f(\varepsilon + \varepsilon_0)) + \left[ \frac{1}{1 - e^{-\beta^* (\varepsilon + f_M \cdot f(\varepsilon + \varepsilon_0))}} \right] \right\} \]

- Substitution of equation (12) in equation (10) for the GND term: \( \Delta \sigma_{GND} = \Delta \sigma_m \cdot \sqrt{Q \cdot f(\varepsilon)} \)

- Substitution of equation (12) in equation (11) for the pile-up term: \( \Delta \sigma_{pile-up} = \Delta \sigma_m \cdot P \cdot f(\varepsilon) \)

- Substitution of these terms in equation (9) for the total behaviour of dual phase materials:
  \[ \sigma = \sigma_0 + \sqrt{(\Delta \sigma_{SSD})^2 + (\Delta \sigma_{GND})^2 + \Delta \sigma_{pile-up}} \]

The model described in the previous section has the ability to describe kinematic hardening via the two GND-terms as shown in [3]. The deformation gradient after the strain reverse is opposite to the new strain direction. In a tension compression term the pile-up stress term \( \Delta \sigma_{pile-up} \) has an opposite sign after the strain reverse. In this reasoning, we have to keep in mind, that this behaviour of the stress tensor is incorporated in the scalar flow stress which is inherently positive. After the reverse, the strain gradient between the two phases will decrease and is becoming zero after the (equivalent) strain is equal to the initial value of the equivalent strain gradient before the strain reverse. This type of dislocation interaction during the kinematic hardening described here is completely based on the plastic strain development and neglects the initial spontaneous rearrangement of dislocations by the accompanying stress reverse. The initial softening [7] after the strain path reverse is much stronger by this stress reversal effect and requires an additional description.

**Dynamic Term**

Besides the influence of temperature and strain rate on work hardening as previously described, a dynamic term as a function of these parameters is included in the initial flow stress:

\[ \sigma_{f0} = \sigma_0 + \sigma^* \]  

Where:
- \( \sigma_0 \) = back stress (MPa), being the initial static yield stress without presence of dislocations
- \( \sigma^* \) = dynamic part of the flow stress (MPa)

For the dynamic term the Krabiell and Dahl relation [5] is used describing the thermal activated movement of dislocations over obstacles dependent on strain rate and temperature:

\[ \sigma^* = \sigma_0^* \left[ 1 + \frac{k \cdot B_0 \cdot T}{\Delta G_0} \cdot \ln \left( \frac{\varepsilon}{\dot{\varepsilon}_0} \right) \right]^{m'} \]  

Where:
- \( \sigma_0^* \) = dynamic stress at zero thermal activation
- \( \Delta G_0 \) = maximum activation enthalpy
- \( m' \) = power for the thermally activated contribution
\( \dot{\varepsilon}_0 \) = limit strain rate for thermally activated movement

When this term is applied in the DP hardening model no strain partitioning is included as described by equations (12) to (13). One term based on the average strain rate is considered for all phases according to equation (18).

**Young’s Modulus as a Function of Temperature**

The shear modulus in equation (1) depends on the temperature. An indirect multiplicative dynamic term is therefore expected that applies to all of the derived contributions to the flow stress. The following simple relation fits well with observations.

\[
\frac{E}{E^0} = \frac{G}{G^0} = 1 - \frac{\Delta \alpha_E}{\varepsilon^T_e} e^{-\frac{\theta_E}{T}} - 1
\]  

(19)

Where:

- \( E^0, G^0 \) = Youngs modulus at zero absolute temperature
- \( \Delta \alpha_E \) = Constant describing decrease of Young’s modulus with temperature, \( 0<\Delta \alpha_E<1 \) (value for steel \( \approx 0.030 \))
- \( \theta_E \) = Transition temperature for E modulus degradation with temperature (value for steel \( \approx 100K \))

The elastic constants for steel do not vary much in the temperature range for cold and warm forming and this is virtually identical for all steel grades [15]. All the three work hardening terms for SSD, GND and pile-up depend on the shear modulus via the quantity \( \Delta \sigma_m \) or \( \Delta \sigma_h \). Furthermore, it is assumed that the back stress \( \sigma_0 \) and the dynamic stress parameter \( \sigma_0' \) are also scaled with the shear modulus. The flow stress, \( \sigma_f \), is corrected for this effect using equation (19). The constants \( \sigma_0, \Delta \sigma_m, \Delta \sigma_h \) and \( \sigma_0' \) in equations (6), (7) and (18) for the flow stress \( \sigma_f \) are in fact the values at zero absolute temperature.

**EXAMPLES OF MODELLING DATA ON EXPERIMENTS**

Datasets of two different materials are used for validation of the proposed model descriptions of the previous chapter:
- Low carbon uncoated IF steel (DC06): For demonstrating the advantage of the model of equation (7) with the influence of recovery on work hardening at temperatures ranging from 20°C-150°C above the one neglecting the recovery effect
- Dual phase steel (DP600) from the Numisheet 2014 Benchmark 1: For demonstrating the advantage of the model of equation (9) with the effect GND’s on work hardening above the one without GND-effect

**Tensile Tests on Low Carbon IF steel (DC06) at Elevated Temperature**

Tensile test were carried out from 20-150°C and at two strain rates (0.004s\(^{-1}\) and 0.04s\(^{-1}\)). The results of these tests are fitted using the thermal model according to equations (5) (7) (8) and (19) in two different ways (parameter values are listed in Table 1, including the remaining standard deviation):
- A version with effect of dynamic recovery \( \Delta \Omega \neq 0 \).
- A version without effect of dynamic recovery \( \Delta \Omega = 0 \).

From the tests, the most extreme values for the Zener-Hollomon parameter, \( Z \), are presented in Figure 1:
- Temperature 20°C and strain rate 0.04 s\(^{-1}\), \( Z = 6.36 \times 10^{15} \) s\(^{-1}\) less recovery (\( \Omega = 10.00 \)).
- Temperature 150°C and strain rate 0.004 s\(^{-1}\), \( Z = 3.29 \times 10^{10} \) s\(^{-1}\) more recovery (\( \Omega = 13.86 \)).

The model results with the parameter \( \Delta \Omega \) larger than zero (with recovery) gives a much better prediction of the measured results. It is accurately describing the work hardening in this temperature range for an IF-steel that is known to be sensitive for dynamic recovery. Using equations (10), (11) and (12) with \( \Delta \Omega = 0 \) is virtually identical to using equation (6).
FIGURE 1. Comparison of results according for the model versions with recovery and without recovery according to Table 1.

TABLE 1. Fitted parameters (grey items) and fixed parameters for modelling of the temperature and strain rate dependent hardening

<table>
<thead>
<tr>
<th>Parameter</th>
<th>With recovery ΔΩ &gt; 0</th>
<th>Without recovery ΔΩ = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>St. dev. (MPa)</td>
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</tr>
<tr>
<td>σ_0 (MPa)</td>
<td>93.4</td>
<td>83.30</td>
</tr>
<tr>
<td>Δσ_m (MPa)</td>
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<td>1471</td>
</tr>
<tr>
<td>Ω_0</td>
<td>19.70</td>
<td>10.91</td>
</tr>
<tr>
<td>ΔΩ</td>
<td>0.266</td>
<td>0 (fixed)</td>
</tr>
<tr>
<td>Q&lt;sub&gt;act&lt;/sub&gt; (eV)</td>
<td>1.0 (fixed)</td>
<td>1.0 (fixed)</td>
</tr>
<tr>
<td>β</td>
<td>1.333 (fixed)</td>
<td>1.333 (fixed)</td>
</tr>
<tr>
<td>n'</td>
<td>0.75 (fixed)</td>
<td>0.75 (fixed)</td>
</tr>
<tr>
<td>ε_0</td>
<td>0.005 (fixed)</td>
<td>0.005 (fixed)</td>
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<tr>
<td>σ_0^* (MPa)</td>
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<td>297.5</td>
</tr>
<tr>
<td>ΔG_0 (eV)</td>
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<td>1.0 (fixed)</td>
</tr>
<tr>
<td>m'</td>
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<td>1.661</td>
</tr>
<tr>
<td>E_0 (s&lt;sup&gt;-1&lt;/sup&gt;)</td>
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<td>10^6 (fixed)</td>
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<tr>
<td>Δα_0</td>
<td>0.03 (fixed)</td>
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<tr>
<td>θ_E (K)</td>
<td>100 (fixed)</td>
<td>100 (fixed)</td>
</tr>
</tbody>
</table>

Mechanical Behaviour of the Numisheet DP600 Steel

FIGURE 2. Comparison of stress strain curves for the model versions according to Table 2 with tensile tests results and bulge test results.

FIGURE 3. Comparison of n-values as a function of strain for model versions according to Table 2 with tensile tests results and bulge test results.
The data of the dual phase steel of the Numisheet 2014 Benchmark 1 is used for demonstrating the performance of the dual phase model described in equations (9) to (13) compared to the basic model (non-thermal model) of equation (6). By using the tensile tests data in all directions and the bulge test data, the work hardening parameters are determined including the ones for the high strain behaviour. Three types of modelled hardening behaviour are presented in Table 2:

- The standard way of fitting without the effect of GND’s by using four parameters of the basic model equation (6).
- A deviation form this standard way of fitting of equation (6) on six parameters resulting into non physical values of the exponent \( n' \) (usually ranging form 0.5 to 1) and the pre-deformation parameter \( \varepsilon_0 \) (usually on a value of 0.005 fixing an average of deformation during temper rolling in steel)
- A fit using the proposed dual phase hardening model described in equations (6) and (9) to (13) on six parameters. Based on the stress strain data it was impossible to fit constants \( P \) and \( Q \) separately in combination with the transient strain parameter \( \bar{e}_{	ext{trans}} \). A fixed ratio is used for the parameters \( P \) and \( Q \). A smaller standard deviation could be achieved at the cost of non realistic values for some of the parameters.

The stress strain curve presented in Figure 2, demonstrate that both the tensile data and the bulge tests data could be described accurately by all models. The results of both tests were aligned by using a fixed stress ratio between tensile test results and bulge tests results. The standard deviation between modelled and measured stress results was in all cases 10 MPa or less. A better view on differences is obtained by plotting modelled n-values as a function of strain in Figure 3. Despite the standard deviation was the highest for the DP-model, it can model the n-value as a function of strain in a better way than the basic model using six parameters. The latter model has a tendency to flatten the n-value as a function of strain. The DP model is very strong in connecting the lower n-values of the tensile test with the higher n-values of the bulge test. These higher measured n-values in the equi-biaxial state are typical for DP-material. The four-parameter version of the basic model exaggerates n-value as a function of strain.

**CONCLUSIONS**

- A set of equations for the flow stress of metals is derived that covers a wide range of applications: reference work hardening, multi-phase work hardening, Bauschinger effect and strain rate and temperature effects.
- Capturing all different influences on work hardening in one unified hardening model is unnecessarily complicating both material characterization and numerical calculations.
- Reference hardening, if necessary in combination with dynamic strain rate and temperature influences, is capable of modelling monotonic stress-strain curves accurately for normal cold forming applications.
- A model that describes the recovery effect can be added for forming applications at enhanced temperatures.
- An advanced description for dual phase metals is developed that includes the effect of kinematic hardening.

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