SIGNIFICANT WAVE HEIGHT RETRIEVAL FROM SYNTHETIC RADAR IMAGES

A. P. WIJAYA & E. VAN GROESEN
Department of Applied Mathematics, University of Twente, Drienerloaan 5
Enschede, 7522 NB, The Netherlands
LabMath-Indonesia, Jl. Dago Giri no 99, Bandung 40391, West Java, Indonesia

In many offshore activities radar imagery is used to observe and predict ocean waves. An important issue in analyzing the radar images is to resolve the significant wave height. Different from 3DFFT methods that use an estimate related to the square root of the signal-to-noise ratio of radar images, in this paper we present a completely different approach. This approach is based on the intuitive observation that the shadowing of waves leads to a visibility that depends on the distance from the radar. We will show that for irregular waves the visibility depends in a characteristic way on the distance normalized by the peak wavelength, and on the ratio of radar height and significant wave height and only slightly on the details of the wave spectrum. By comparing the visibility of a specific sea with results of Monte Carlo simulations of sea states with various significant wave heights, the best LSM fit then determines the significant wave height. In this paper we restrict to long-crested waves and illustrate the method with various cases.

Keywords: significant wave height, visibility, shadowing effect, Monte Carlo simulations.

1. Introduction

Observation of ocean waves with radar imagery can be very useful in many coastal engineering applications. In cases when certain operations can only be performed in periods of limited wave height, such as for helicopter landing on a ship, off-loading operations and turbine installation, knowledge of the significant wave height $H_s$ is very much desired. But since the radar images only provide the radar backscatter which is not related to the elevation, the value of $H_s$ from radar images has to be found in an indirect way.

Several authors estimate the significant wave height using a 3DFFT method applied to the images. The 3DFFT method, introduced in [1] to detect the surface current and wave direction, was extended in [2] and [3] with a method to retrieve the significant wave height. The method used the so-called signal-to-noise ratio in the estimated spectrum of the waves as proposed in [4] and validated by [5] for synthetic aperture radar images. The significant wave height, defined as four times the square root of the spectrum area, was without strong underlying justification estimated to be linearly related to the square root of the signal-to-noise ratio, with two free parameters for which in-situ measurements are used to calibrate the parameters. In [6] an empirical method was proposed to relate the radar intensity to the tilt modulation. Integrating the tilt with some filtering yields the surface wave elevation, hence the wave height is directly determined without in-situ measurement. In this method, the tilt modulation is assumed to be the most dominant in the radar mechanism.

Another approach is based on an analysis of the shadowed areas in radar images. The shadowing effect occurs because waves closer to the radar block radar rays so that parts of the waves further from the radar become invisible. A statistical concept based on the proportion of the visible (‘islands’) and invisible (‘troughs’) part of the waves was presented in [7]. The probability of illumination $P_0$ was
defined and was related to the significant wave height. In [8], it was shown that the estimation of $H_s$ using a constant $P_0$ is only accurate for certain waves condition, for instance when the ratio radar height and the wave height was high. An improved result with varying $P_0$ was introduced in [9]. A method without using any reference measurement presented in [10] estimates the significant wave height from the RMS of the surface slope which is related to the shadowing ratio. The relation can be found by the best fit of the shadowing ratio with a known function, Smith’s function [11]. The results compared to measurement have correlation of 87%.

In this paper we also use the shadowing effect to estimate the significant wave height using a similar idea as in [10] that the shadowing quantity is related to the significant wave height. To measure the shadowing, we define a visibility function as the fraction of the time that the waves at one position are visible by the radar. In this paper, we show that the visibility depends on two dimensionless quantities: the ratio of radar height $H_r$ and significant wave height $H_s$, $h = H_r/H_s$ and the ratio of the distance from the radar $x$ and the peak wave length $\lambda_p$, $\rho = x/\lambda_p$. These quantities help to reduce the number of parameters in the system. The visibility function depends strongly on the dimensionless distance and on the significant wave height, but only slightly on precise properties of the spectrum such as the enhancement factor $\gamma$ in a Jonswap spectrum. This leads us to construct averaged visibility curves for various ratios $h$ by Monte Carlo simulations. Then, having constructed a visibility curve of an actual observed ocean waves, the best fit with the result of Monte Carlo simulations leads to an estimate for the significant wave height of the observed ocean wave. In this process we need to know the values of the radar height and the peak wavelength; the peak wavelength can be calculated at given depth from a peak period which can be found from an estimated normalized spectrum.

This paper is organized as follows. Section 2 will discuss how to design the synthetic sea and the synthetic radar images. In this paper we restrict to linear long-crested waves, and consider only shadowing as the dominant effect in the radar mechanism. The definition of visibility is given in section 3, after which in section 4 the method to approximate the significant wave height of the observed wave is introduced. The result of several test cases is presented in section 5, and the paper finishes with some conclusions and remarks.

2. Synthetic Data

In the first subsection we describe how the synthetic long-crested seas are constructed. In the next subsection the shadowing mechanism is explained and applied to obtain the synthetic radar images.

2.1. Synthetic seas

We consider long-crested seas which will be represented as 1D waves in the $x$-direction. Assuming the waves to be linear, they are described as

$$
\eta(x,t) = \sum_{i=1}^{N} \sqrt{2E(\omega_i)} \Delta \omega \cos(k_i x - \omega_i t + \epsilon_i) \tag{1}
$$

Here, $E(\omega)$ is a spectrum, for which we will take a Jonswap spectrum in the following to simulate wind waves. The wave numbers, $k_i$ are related to the frequencies $\omega_i$ via the dispersion relation, and $\epsilon_i$ are random phases uniformly distributed on $[0,2\pi]$. Since (1) produces periodic waves on a spatial interval of length determined by the smallest wave number, to prevent periodicity in the test cases, we take the length of the spatial domain equal to the group velocity (of the peak wave number) times the duration of the observation time. The snapshots of the observed sea by the radar at time $n \cdot \Delta t$ are denoted by

$$
S_n(x) = \eta(x,n\Delta t), n = 0,1,2,\cdots,M-1 \tag{2}
$$

with $\Delta t$ the radar rotation time and $(M-1)\Delta t$ the length of observation time.
2.2. Synthetic radar images

In this paper we assume that shadowing is the most dominant effect in the radar mechanism. Shadowing is the geometrical effect that part of the waves is invisible by the radar because these are (partly) blocked by waves closer to the radar. Hence, the occurrence of the shadowing at far range is more than at closer range. The shadowing mechanism is well-known to be simulated as follows [12].

Suppose the radar is located at the origin \( x = 0 \) at a height \( H_r \) above the sea level. The straight line from the radar to a point at the sea surface \( S_n(x) \) is given by

\[
l_{\n n}(x') = S_n(x) + \frac{S_n(x) - H_r}{x} (x - x'), \forall x' < x
\]

The point \( S_n(x) \) is visible by the radar if \( l_{\n n}(x') - S_n(x') > 0, \forall x' < x \), otherwise it is invisible. This leads us to define the characteristic function of shadowing, \( \chi \), which has value 1 or 0 if the corresponding point is visible or invisible respectively. This characteristic function can be written as

\[
\chi_n(x) = \frac{1}{2} \left[ 1 + \text{sign} \left( \min_{x' < x} \{ l_{\n n}(x') - S_n(x) \} \right) \right]
\]

Another property of radar imaging is that no information can be sensed in an area close around the radar; in this paper we will take an interval of length 500 m as being invisible by the radar. The synthetic radar images are then given by

\[
I_n(x) = \begin{cases} 
S_n(x) \cdot \chi_n(x), & x > 500 \\
0, & \text{otherwise}
\end{cases}
\]

As an example, Figure 1 shows the wave elevation of a snapshot of a sea \( S_n(x) \) with the corresponding synthetic radar image \( I_n(x) \). In this case the ratio of radar height and significant wave height is \( h = 6 \), and almost all the wave troughs are invisible.

![Figure 1. The wave elevation \( S_n(x) \) (solid-blue) with the corresponding \( I_n(x) \) (dashed-red) for \( h = 6 \).](image)

3. Visibility

To quantify the shadowing effect at a certain position in a sequence of images \( I_n \) we define a visibility function as the fraction of time that the wave is visible by the radar:

\[
\text{vis}(x) = \frac{1}{N} \sum_{n=1}^{N} T_n(x)
\]

Here, \( T_n(x) \) is the visibility of the wave at location \( x \) in image \( n \), which has value 1 or 0 when \( S_n(x) \) is visible or not respectively. It is clear that the visibility function depends not only on the distance from
the radar but also on the height of the waves. In fact, the two length scales in vertical and horizontal direction makes it clear that there are two dimensionless parameters that determine the shadowing process. This leads to define the ratio $h$ of radar height above the sea level divided by a characteristic number of the wave height (for instance the wave height of a monochromatic wave or the significant wave height of an irregular wave). The scaling in the horizontal direction can be done by using the distance divided by a characteristic wave length (such as peak wave length of an irregular wave). Below we show some characteristic cases for irregular wind waves with various values of relevant parameters. Since the visibility is calculated from the synthetic radar images that contain the random phases $\epsilon_i$, Monte Carlo simulations have been performed to obtain the averaged visibility in the plots below. In Figure 2 we show results for different values of 3 parameters: ratio $h$, the peak period $T_p$ which determines the wave length used to make the distance dimensionless, and the peak enhancement factor $\gamma$ in the Jonsswap spectrum. For each choice of these values, we simulate the synthetic sea $S_n$, radar images $I_n$ and visibility $vis$. We repeated the process until the averaged visibility is converged, which was obtained after 10 realizations. Figure 2 shows the averaged visibility for different values $h$ and peak period $T_p$ for fixed value $\gamma = 3$ and depth=50 m.

![Figure 2](image1.png)

**Figure 2.** The averaged visibility of $T_p=9$ (solid-line) and $T_p=12$ (dashed-line). The colours indicate the value of $h$; [blue,green,red,brown,cyan,violet] correspond to $h=2,6,10,12,18$ respectively.

Figure 3 shows that the dependence of the visibility on the peak enhancement factor $\gamma$ is rather small, especially for smaller values of $h$.

![Figure 3](image2.png)

**Figure 3.** The averaged visibility for fixed $T_p=9$, different values $h$ as in Fig.2 and for $\gamma=1,2,3$ (dotted, dashed, solid line respectively).
4. Method

The results of the previous section showed that the visibility depends on the ratio \( h \), and on the peak period \( T_p \), but only slightly on the parameter \( \gamma \). These observations motivates a method to determine \( H_s \) for a sea with unknown \( H_s \) from the dependence of the visibility on (normalized) distance. Required is that the peak period \( T_p \) and the radar height \( H_r \) are known. The method is to determine from a sequence of images the visibility, say \( v(\rho) \), and to find the positioning of this curve in a set of curves as in Figure 2. In more detail as follows. Assume that a database is available of the averaged visibility for various values \( h_i = 2 + 4(i - 1), i = 1,2,3,4,5 \) and \( \gamma = 3 \); we denote the visibility by \( \bar{v}^{h_i}(\rho) \). To estimate the value \( h \) that corresponds to the visibility \( v(\rho) \), we fit the curve \( v \) as good as possible in the data base. That is we use LSM to get an estimate for \( h_{\text{est}} \) by solving

\[
\min_{\alpha_i} ||\alpha \cdot \bar{v}^{h_i} + (1 - \alpha) \cdot \bar{v}^{h_{i+1}} - v(\rho)||^2
\]

where \( ||\cdot||^2 \) is the \( L^2 \) -norm. Then, \( h_{\text{est}} = \alpha \cdot (h^i - h^{i+1}) + h^{i+1} \) and the estimated significant wave height is

\[
H_{s,\text{est}} = \frac{H_r}{h_{\text{est}}}
\]

5. Test Case

In this section we present the result of \( H_s \) retrieval for several cases. We consider waves with peak period \( T_p = 9 \) seconds, significant wave height 1 m above a depth of 50 m. The corresponding group velocity is 7.4 m/s. The length of observation time is 20 minutes with time step (radar rotation time) \( \Delta t = 2 \) seconds. To prevent periodicity in the sea we take the spatial domain [0,9000] m with resolution \( \Delta x = 7.5 \) m; for such waves the peak wave length is 124.8 m, hence one wave length consist of approximately 16 points. After the simulation, the spatial domain is truncated on interval [0,2000] m.

The averaged visibility is collected in a database with parameters \( h^i \) and the fixed value \( \gamma = 3 \) as mentioned in the previous section. For the test cases, we use values of the parameters \( \gamma \) and \( h = H_r \) \( (H_s = 1) \) as given in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \gamma )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

We perform 30 realizations for each case to obtain the averaged result for the sea. The estimated value of \( H_s \) for each case is presented in Figure 4 with 95% confidence interval. The results show that the error is less than 6%, also in the case that the actual sea value of \( \gamma \) differs substantially from that of the data base. Note that the correct value is outside the 95% confidence interval for case 1 whereas it is inside for case 2.
6. Conclusion and remarks

We showed that the shadowing effect in the spatial interval depends for irregular waves with given spectrum just as for monochromatic waves on two dimensionless parameters: the quotient of significant wave height and radar height, and the distance measured in number of wavelengths. The dependence on the spectrum shape is rather robust, slightly depending on the enhancement factor $\gamma$ in the Jonswap spectrum. Using these dimensionless quantities, we showed that by comparing with a constructed data base, the visibility of an observed sea state can be used to estimate the significant wave height directly from the radar images. Required is that the peak period is known in advance, which can be determined by estimates of the (normalized) spectrum determined by known 3DFFT method [1]. For some characteristic cases we showed that the error is less than 6% even in cases that the value of the parameter $\gamma$ is quite different. The method presented here can also be extended to the case of short-crested waves as will be discussed in a separate paper.

Acknowledgments

The motivation for this study was stimulated by some of the challenges in the IOP (Industrial Research Project) entitled “Prediction of wave induced motions and forces in ship, offshore and dredging operations”, funded by Agency NL, a department of the Dutch Ministry of Economic Affairs, Agriculture and Innovation and co-funded by Delft University of Technology, University of Twente, Maritime Research Institute Netherlands, OceanWaves GMBH, Allseas, Heerema Marine Contractors and IHC Merwede.

References


