STRAIN PATH DEPENDENT EVOLUTION OF YIELD LOCUS
BASED ON HAH MODEL DESCRIPTION

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ABSTRACT: The plastic behavior of a ferritic stainless steel under non-proportional loading was investigated using a biaxial test setup developed at the University of Twente. With this testing device, a specimen can be subjected to both plane strain tension and simple shear stress states in an arbitrary order. A number of tests are conducted including different types of strain path change and load amplitudes. The results of these tests are discussed in this paper. In order to model the observed material behavior, the recently developed homogeneous anisotropic hardening (HAH) model is used. In general, this distortional model provides the description of many features such as flattening, contraction and rotation of the yield surface. This allows the model to capture complex material phenomena such as the Bauschinger and transient cross-hardening effects. The HAH model is calibrated using forward-reverse simple shear and orthogonal loading tests. The experimental and predicted results are compared and the quality of the HAH approximations under multiple strain path changes is investigated.

KEYWORDS: Non-proportional loading, HAH model, Twente biaxial test setup

1 INTRODUCTION

Non-proportional loading of a material results in an evolution of the hardening behavior, which is different from that given by general stress-strain relations using data obtained, for example, from tension tests. A more complex material behavior under non-proportional loading can be explained by the evolution of the microstructure, for example, the orientation of the grain structure (texture) and the orientation of the dislocation structure in the grains.

The Bauschinger effect is due to the existence of dislocation pile-ups in the vicinity of cell walls accumulating during the deformation. If a load reversal takes place, the dislocations start to move in the opposite direction, which is dislocation free. This motion requires lower stresses than in the forward direction. Therefore the material starts to yield earlier under reverse loading. Another effect, which can be explained by microstructural evolutions, is cross hardening. During a shear deformation, so-called micro-bands develop. A relatively high stress is needed to build the micro-bands and to cut through dislocation walls that formed due to prior deformation in another deformation mode. But once they are established, a lower stress is required because of the relatively dislocation free area within their boundaries. This leads to a stress overshoot occurring at the beginning of orthogonal loading. Such hardening phenomena are investigated by van Riel [4] using experiments with cyclic shear and with continuously changing strain path changes. Subsequently, the models by Teodosiu [5] and Levkovitch [6] are applied to the experiments.

The purpose of the current work is to characterize the material behavior under multiple strain path changes. Tests are done with different orders and amplitudes of plane strain tension and simple shear. For the description of the observed material behavior, the recently developed homogenous anisotropic hardening model by Barlat et al. [1], [2] is used in this study. This model captures a number of phenomena, in particular, Bauschinger effect, hardening stagnation after load reversal as well as cross-hardening and latent hardening after orthogonal loading-path changes. The results of the model calibration and validation regarding the presented experimental data are also discussed.

2 EXPERIMENTS

The material chosen is a 0.3 mm thick stainless steel sheet, AISI 420 (ferritic state). For testing, the Twente biaxial test setup [3] is used. This device

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allows the loading of a specimen in both tension and shear modes. Both loading modes are individually controlled and are used in this work to impose strain path changes to the material. The strains are measured on the surface of the sample with a digital image correlation system based on a dots tracking method and the loads are determined via force-sensors. In the following, the different tests are explained and the observed material effects are discussed.

In the first test setting, the material behavior under reverse loading is investigated. Therefore, forward shear – reverse shear tests are conducted with different amount of shear deformation (from 5% up to 25%). In the second test setting, the behavior under forward-reverse shear with prior deformation is studied. First the sample is loaded under plane strain tension and, then, under forward shear – reverse shear. This test should provide information about the impact of pre-straining on the Bauschinger effect. Finally, tests with two successive orthogonal strain path changes (tension to shear to tension) are performed. The measured stress-strain response can be compared with monotonic simple shear and tension tests to assess whether softening occurs after the first strain path change.

The results of the forward – reverse shear tests after pre-tension are shown in Figure 2. These tests were performed with elastic unloading after the applied tensile deformation. The preloading in plane strain tension is the same for all tests (7%). The shear deformation before reversal was conducted using different amplitudes (2%, 5% and 15%). The results show that, after the first strain path change, the material starts to yield earlier (Bauschinger-like effect). Furthermore, the pre-tension seems to have an impact on the reverse shear behavior. The experiments indicate that the transition to plastic flow after load reversal is quite sharp for small amounts of shear. For higher shear pre-strain, it is getting smoother and similar to the behavior without pre-tension. It seems that the high amplitude of shear (15%) eliminates the pre-tension history. Furthermore, it can be observed that all three tests tend to reach the same stress level for high levels of strains.

The results of the tension – shear – tension tests are shown in Figure 3. Compared to previous case, no reverse shear takes place. Instead the sample is loaded in tension again. Again, the material tends to yield earlier and recovers the stress level of the monotonic hardening curves (Bauschinger-like effect). Compared to the monotonic shear and plane strain tension curves, no softening occurs after both orthogonal strain path changes.

These experiments show that many material effects occur during multiple strain path changes. The
Bauschinger effect, hardening stagnation after reverse loading and Bauschinger-like effect after orthogonal loading are observed. Tests with prior pretension show that these effects are somehow influenced by the loading history. The HAH model was chosen to capture these effects. This model is explained shortly in the following section.

3 MATERIAL MODEL

In this section, the constitutive model, i.e., the yield function and strain hardening approach, is briefly described. In order to capture anisotropic material properties, the Yld2000-2d yield function is selected. For hardening, the recently developed HAH distortional hardening framework is used. The equations described below are a subset of the whole HAH framework. In this chapter only equations for the observed material effects are presented. The complete HAH approach can be found in [1], [2].

The HAH model is a yield function, which consists of a stable component and a fluctuating component. The latter allows a distortion of the yield surface described by the former. Several sets of state variables offer different ways to distort the yield locus. By defining the evolution equations of the state variables in an adequate way, the model is able to describe complex material behavior occurring during non-proportional loading.

It was experimentally observed, that a Bauschinger effect appears during reverse loading. To model this effect, the yield surface flattens at the side opposite to loading. For this purpose the following evolution equation for one of the state variables is used

$$\frac{dg_{1}}{d\varepsilon} = k_{2} \left( \frac{\sigma_{l}}{\sigma_{y}} - g_{1} \right)$$

(1)

This corresponds to an exponential decrease towards a lower bound equal to a fraction $k_{3}$ of the initial yield stress, and controlled by the rate $k_{2}$ [1]. As soon as the load is reversed, the distorted surface near the active stress state tends to recover the stable yield surface component. This is the way the transient hardening stage during reverse loading can be modeled.

Since strain hardening stagnation after load reversal has been experimentally observed, a model based on two dislocation densities (forward $\rho_{f}$ and reversed $\rho_{r}$ dislocation densities) is employed (Rauch et al. [7], [8])

$$\frac{d\rho_{f}}{dy} = \frac{\sqrt{\rho_{f}}}{bK} + \frac{1}{bD} - \kappa \rho_{r}$$

(2)

$$\frac{d\rho_{r}}{dy} = \left( \frac{\sqrt{\rho_{f}}}{bK} + \frac{1}{bD} \right) \frac{\rho_{r}}{\rho_{f}}$$

(3)

This model allows for a total dislocation density that stagnates after reloading, thus leading to the proper flow stress behavior.

It was experimentally observed that for cross-loading conditions, the reloading yield stress was significantly lower than the corresponding stress on

Figure 2: Results of the plane strain tension -- shear -- reverse shear test
the monotonic hardening curve (Bauschinger-like effect). An approach to solve this issue is to contract (shrink) the stable surface in the appropriate direction [2]. For this purpose, the evolution equation of one of the state variables is modified as follows

$$\frac{dg}{d\varepsilon} = [k_s \cos^2 \chi + k_{S}(1 - \cos^2 \chi)] \left[ 1 + (S - 1) \cos^2 \chi - g_s \right]$$

where $S$ (shrink) is a coefficient less than or equal to 1.

4 RESULTS

In this section, the application of the HAH model to the AISI420 under investigation is discussed. The experimental results presented in Section 2 are the basis for the calibration of the homogeneous anisotropic hardening (HAH) law. The model was developed such that isotropic and distortional hardening parameters do not interact. Hence, the material parameters for isotropic hardening are determined first. With these parameters fixed, the parameters of the distortional hardening can be approximated.

For a better understanding of the HAH model, the evolution of the yield surface during all loading cases for plane strain tension followed by forward and reverse simple shear deformation is shown in Figure 4. The thick solid lines correspond to the initial and final yield loci before and after loading while the thin lines illustrate the evolution at several intervals.

Under plane strain tension, the yield locus flattens at the side opposite to loading and contracts in any direction orthogonal to loading. In this manner, the Bauschinger effect under reverse loading and Bauschinger-like effect under orthogonal loading can be described. In the second step, loading is changed from plane train tension to simple shear, which leads to a rotation of the yield surface. During the third loading step, reverse shear is imposed which leads to recovery of the yield surface near the loading direction and flattening in the opposite direction.

In Figure 1, the quality of the model prediction compared with the experimental data for forward – reverse shear can be assessed. A good description of the strain hardening with reverse loading, including flow stress stagnation, is achieved. Small deviations still occur at the end of the second stroke. This is partly due to the relatively poor fit of the monotonic shear curve for higher strains. A better fit at higher strains for the monotonic stress-strain curve can also be obtained but at the expense of the smaller strain levels during reverse loading. However, in general, the model approximation is reasonable and the Bauschinger effect is captured satisfactorily.

In Figure 2 the experimental results for forward – reverse shear under pretension are compared with the model predictions. The so-called Bauschinger-like effect, which occurs due to cross-loading, is captured appropriately by the proposed modifications of the HAH approach. The model is able to describe the gradual yielding after the strain path

![Figure 3: Results of the plane strain tension – shear – plane strain tension test](image-url)
was changed. A more complex aspect to model is the second strain path change. Three different amplitudes of forward shear loading have been chosen to investigate its impact on the behavior under reverse shear loading. The test with the highest amplitude (15%) is well described. For lower amplitudes (2% and 5%) the transition at the beginning of yielding is too sharp.

In Figure 3, the predicted flow curves obtained with two orthogonal strain path changes are shown. Once again, for the first strain path change, the Bauschinger-like effect, which was experimentally observed, is described reasonably well. For the second strain path change, the Bauschinger-like effect is not captured very well. As in the test of Fig. 2, the predicted flow curve exhibits a too sharp transient behavior before recovering the monotonic flow stress level.

All the parameters identified for AISI 420, which lead to the results presented in Fig. 1 to 3, are listed in Table 1.

5 CONCLUSIONS

The mechanical behavior of an AISI 420 stainless steel was investigated under a number of complex deformation paths using a special equipment producing sequences of plane strain tension and simple shear. The corresponding stress-strain curves

![Figure 4: Evolution of the yield surface during loading](image)

**Table 1: Coefficients of the material model for AISI420**

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<th>Yld2000-2d non-quadratic yield function</th>
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*irrelevant because $L = 1$
were captured reasonably well with a slightly modified version of the HAH model. However, some level of inaccuracy still exists, mainly after the second strain path change. In order to overcome these limitations, the model has to be improved further. Hardening effects in metals subject to non-proportional loading histories, particularly for two or more path changes, have to be taken into account. More tests are planned in the coming months to guide and validate further developments of the HAH framework.

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REFERENCES