Model of electrical conductivity of skeletal muscle based on tissue structure

F. L. H. Gielen
Department of Physics & Astronomy, Vanderbilt University, Nashville, Tennessee, USA

H. E. P. Cruts
Research & Innovation Group, Rehabilitation Centre Het Roessingh, Enschede, The Netherlands

B. A. Albers  K. L. Boon  W. Wallinga-de Jonge  H. B. K. Boom
Department of Electrical Engineering, Twente University of Technology, PO Box 217,
7500 AE Enschede, The Netherlands

Abstract—Recent experiments carried out in our laboratory with the four-electrode method showed that the electrical conductivity of skeletal muscle tissue depends on the frequency of the injected current and the distance between the current electrodes. A model is proposed in order to study these effects. The model takes into account the structure of the tissue on the scale of individual fibres. It discerns three main components with respect to electrical properties: (a) extracellular medium with electrical conductivity \( \sigma_{\text{e}} \), (b) intracellular medium with electrical conductivity \( \sigma_{\text{i}} \), (c) muscle fibre membrane with impedance \( Z_m \). The model results show an apparent frequency dependence of the electrical conductivity of skeletal muscle tissue, as well as the way the conductivity is affected by the length the current is conducted.

Keywords—Electrical conductivity, Skeletal muscle, Histological structure, Volume conduction, Model computation


1 Introduction
Electrical conductivities are important parameters in the description and interpretation of electrophysiological phenomena. In a recent paper (GIELN et al., 1984) we presented measurements of the electrical impedance of muscle tissue, carried out in our laboratory using the 'four-electrode method'. In Fig. 1 a characteristic result of our measurements is shown together with values of conductivities of skeletal muscle tissue as reported by other authors (BURGER and VAN DONGEN, 1960; RUSH et al., 1963; EPSTEIN and FOSTER, 1983). Clearly the electrical conductivities as well as the values for the anisotropy \( \sigma_{\text{e}} / \sigma_{\text{i}} \) are frequency dependent in our measurements and in the report of EPSTEIN and FOSTER (1983), whereas the other data from the literature are not, in the frequency range covered by EMG signals (10 Hz–20 kHz).

In our experiments the interelectrode distance (IED) is only 0.05 cm, which is at least ten times smaller than that used by other authors.

At such small IEDs the anatomical structure at the level of individual muscle fibres may become a factor that influences the measured electrical conductivity in a significant way. In order to study this aspect a model has been developed that describes the electrical conductivity of skeletal muscle tissue, taking into account the anatomical structure on the level of individual muscle fibres.

Correspondence should be addressed to Ir. B. A. Albers
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2 Effective electrical conductivity
A usual definition of the impedance \( Z \) of a block of homogeneous medium is

\[
Z = \frac{L}{\sigma S} \tag{1}
\]

where \( L \) is the length of the medium in the direction of the current, \( S \) is the surface area of the medium in the direction perpendicular to the current, \( \sigma \) is the complex electrical conductivity of the medium. If we define \( Z' \) as the impedance of a block of inhomogeneous media, the effective electrical conductivity of a block of inhomogeneous medium can be defined by

\[
\sigma_{\text{eff}} = \frac{L}{Z'S} \tag{2}
\]

with \( Z' = V/I \), where \( V \) is the voltage difference on two opposite sides of a block of inhomogeneous medium with surface area \( S \), and \( I \) the total current in the direction normal to this surface. So with \( \sigma_{\text{eff}} \) the relationship between the voltage difference on two opposite sides of a block of inhomogeneous medium and the current in the direction normal to this surface is given, but not the current distribution and potential field inside this block of inhomogeneous medium.

In the following Sections the effective electrical conductivity of muscle tissue in the directions parallel and transverse to the muscle fibres is derived from a model. We use a simplified representation of the muscle structure.
The influence of connective tissues, blood (vessels) and nerves is neglected. We assume the muscle tissue contains only intra- and extracellular medium and muscle fibre membranes.

3 Effective electrical conductivity transverse to muscle fibres

In the calculation of the effective electrical conductivity transverse to the muscle fibres ($\sigma_{TM}$), the structure of the muscle is simplified so as to consist of parallel hexagonal muscle fibres (see Figs. 2 and 4).

Current is impressed on the extracellular medium only, in a direction perpendicular to the muscle fibres (for instance by means of two large parallel conducting plates).

A unit cell for the computation of the electrical conductivity, transverse to the muscle fibres, is defined as a cell with rectangular cross-section, shown with broken lines in Fig. 2. This unit cell is also shown in Fig. 3, together with a lumped circuit equivalent of the unit cell. In the calculation of the electrical impedance of this unit cell the following parameters are used (see Figs. 2 and 3):

- $A$ = one half diameter of a muscle fibre (this means that the sides of the hexagonal cross section have a length $A$, see Fig. 2)
- $d$ = distance given in Fig. 2, the thickness of the layer of extracellular medium between the fibres is $(3)^{1/2}d$ (we assume $d \ll A$, see Fig. 2)
- $b$ = length of the unit cell parallel to the muscle fibres
- $\sigma_e$ = conductivity of the extracellular medium
- $\sigma_i$ = conductivity of the intracellular medium
- $C_m$ = membrane capacity per unit of surface area
- $G_m$ = membrane conductance per unit of surface area
- $Z_m$ = impedance per unit of surface area
- $p$ = fraction of the total volume of the tissue occupied by intracellular medium
- $R_e$ = electrical resistance of the extracellular medium between $TT'$ and $VV'$, between $VV'$ and $WW'$, and between $WW'$ and $QQ'$ (see Fig. 3).
Current flowing in a direction normal to the muscle fibre membrane meets with the series circuit of three impedances; namely the impedance of the intracellular medium, the membrane impedance and the impedance of the extracellular medium. If we compare the impedances of the intra- and extracellular medium with the much larger value of the membrane impedance for the frequency range concerned here, it is permissible to neglect these intra- and extracellular impedances.

The membrane impedance is approximated by a parallel circuit of \( C_m \) and \( G_m \). According to this approximation, \( 1/Z_m \) equals

\[
\frac{1}{Z_m} = G_m + j\omega C_m
\]

where \( \omega = 2\pi f \) with \( f \) the frequency and \( j = (-1)^{1/2} \).

From eqn. 1 and Fig. 3a the following relation is deduced for \( R_e \):

\[
R_e = \frac{A + d}{(3)^{1/2}\sigma_e db}
\]

Fig. 3b shows a lumped circuit equivalent for the unit cell of Fig. 3a. If, for example, the surfaces PP' TT' and QQ'SS' are two different equipotential surfaces, the modelled effective electrical conductivity \( \sigma_{TM} \) of the unit cell can be computed using eqn. 1 and Fig. 3b. This results in

\[
\sigma_{TM} = \frac{(3)^{1/2}A}{3bR_e} + \frac{(3)^{1/2}A}{2Z_m}
\]

If TT'SS' and PP'QQ' are two different equipotential surfaces, an identical result is obtained for \( \sigma_{TM} \). Using eqn. 4, eqn. 5 can be rewritten as

\[
\sigma_{TM} = \frac{\sigma_e + (3)^{1/2}A}{A + d} \left( \frac{3)^{1/2}}{2Z_m} \right)
\]

From Fig. 3a it follows that the fraction of the total volume of the unit cell occupied by the intracellular medium equals

\[
p = \frac{A^2}{A^2 + 2dA + d^2}
\]

With the assumption \( d \ll A \) this results in

\[
d = \frac{1}{2} \left( \frac{1}{p} - 1 \right)
\]

A combination of eqns. 6 and 8 gives the effective electrical conductivity of the unit cell:

\[
\sigma_{TM} = \frac{1 - p}{1 + p} \sigma_e + \frac{(3)^{1/2}A}{2Z_m}
\]

This formula shows that, due to the presence of \( Z_m \), \( \sigma_{TM} \) is frequency dependent.

4 Effective electrical conductivity parallel to muscle fibres

In the calculation of the electrical conductivity parallel to the muscle fibres (\( \sigma_{TM} \)) the same simplified anatomical structure is used (see Fig. 4). Again the impedances of the intra- and extracellular medium for current flowing in a direction normal to the muscle fibre membrane are neglected. In this case current is impressed parallel to the muscle fibres. Now we take the unit cell as shown in Fig. 4a (broken lines).
\( V_i(y), V_e(y) \) = potentials in the unit cell in the extra- and intracellular medium, respectively, at position \( y \) (we choose \( V_i(0) = V_e(0) = 0 \))

\( i_d(y), i_e(y) \) = current in the unit cell in the extra- and intracellular medium, respectively, at position \( y \)

\( i_m(y) \) = radial component of the current in the unit cell flowing through a segment of muscle fibre membrane per unit length

\( r_e, r_i \) = longitudinal impedance of extra- or intracellular medium per unit length.

The impedance of a membrane segment with length \( dy \) is equal to \( Z_m/Ady \).

According to eqn. 1 the impedances \( r_e dy \) and \( r_i dy \) shown in Fig. 5 can be written as:

\[
\begin{align*}
& r_e dy = \frac{dy}{\sigma_e S_e} = \frac{dy}{\sigma_e(1 - p)Si} \\
& r_i dy = \frac{dy}{\sigma_r S_i} = \frac{dy}{\sigma_r pSi} 
\end{align*}
\]

Consider a part of the unit cell with length \( dy \) at position \( y \) (for the symbols used here, see Fig. 5). From Fig. 5 together with Ohm’s law and the assumptions and definitions given above we derive the following differential equations:

\[
\begin{align*}
\frac{dV_i(y)}{dy} &= -i_d(y) r_e \\
\frac{di_e(y)}{dy} &= -(V_i(y) - V_e(y)) A \\
&= -\left(\frac{V_i(y)(1 + \frac{r_i}{r_e}) + r_i i_d S_i y - V_o}{Z_m}\right) A \\
&= \frac{\text{(total current through } S_i \text{) } \times \text{length } y}{\text{voltage over the unit cell with length } y \times S_i} \\
&= -\frac{i_d}{V_i(y)^2} y
\end{align*}
\]

Substitution of eqn. 19 in eqn. 20 results in the following expression for \( \sigma_{LM} \):

\[
\sigma_{LM} = \frac{\sigma_d(1 - p) + \sigma_i p}{1 + \frac{1 - e^{-\lambda y}}{\lambda y} \frac{\sigma_i p}{\sigma_d(1 - p)}}
\]

where

\[
\lambda = \left(\frac{1}{\sigma_d(1 - p)S_i} + \frac{1}{\sigma_i pS_i}\right) Z_m^{-1/2}
\]

So \( \sigma_{LM} \) depends on frequency as well as on \( \lambda y \), and hence \( \sigma_{LM} \) is different from the electrical conductivity usually encountered. As a consequence the electrical properties of muscle tissue cannot be characterised by means of one longitudinal electrical conductivity, independent of measuring geometry.

The value of \( \lambda y \) characterises three ranges of \( \sigma_{LM}(\lambda y) \) values:

\( a) \quad |\lambda y| < 0.1 \Rightarrow \sigma_{LM}(\lambda y) \approx \sigma_d(1 - p) \)

Eqn. 21 reduces to

\[
\sigma_{LM}(\lambda y) \approx \sigma_d(1 - p)
\]

The current flows almost exclusively in the extracellular medium. In this case the unit cell can be replaced by an identical cell of a homogeneous medium with conductivity \( \sigma_{LLM} = \sigma_d(1 - p) \).

For \( |\lambda y| < 0.1 \)

\[0.95 < \frac{\sigma_{LLM}}{\sigma_{LM}(\lambda y)} < 1.00\]

\( b) \quad |\lambda y| > 100 \Rightarrow \sigma_{LM}(\lambda y) \approx \sigma_d(1 - p) + \sigma_i p \)

Eqn. 21 reduces to

\[
\sigma_{LM}(\lambda y) \approx \sigma_d(1 - p) + \sigma_i p
\]

Now current flows in the intra- as well as in the extracellular medium depending on the conductivity of the intra- and extracellular medium. The unit cell can be replaced by an identical cell of a homogeneous medium with conductivity \( \sigma_{LLM} = \sigma_d(1 - p) + \sigma_i p \).

For \( |\lambda y| > 100 \)

\[1.00 < \frac{\sigma_{LLM}}{\sigma_{LM}(\lambda y)} < 1.05\]

\( c) \quad 0.1 < |\lambda y| < 100 \)

In this case eqn. 21 cannot be simplified, implying that the current distribution depends on the value of \( \lambda y \). Close to the current source (or sink) current flows in the extracellular medium only. Far from the source (or sink) current flows in the intra- as well as in the extracellular medium.

The distribution of current depends on the conductivities of these media and on the impedance of the muscle fibre membrane, and is therefore frequency dependent.

We conclude that according to the model the current distribution depends on the distance between the (electrophysiological) source and the position of the measuring point as well as on the frequency contents of the signal, generated by the source.
5 Numerical results

In Fig. 6 the effect of changing the interelectrode distance from 0.05 cm, used in our experiments (GIELEN et al., 1984), to a value of the same order of magnitude as the interelectrode distance mentioned by RUSH et al. (1963) (> 0.5 cm) is illustrated. The values are calculated for a situation comparable with the four-electrode method, but with infinite large current electrodes. The parameter values used in the computation of \( \sigma_{TM} \) and \( \sigma_{LM} \) are given in the legend to Fig. 6. The results show that the interelectrode distance influences the degree of frequency dependence of the effective electrical conductivities. At frequencies less than 1 kHz for large interelectrode distances the values obtained for the conductivity parallel to the muscle fibres are more than twice the conductivity values obtained with small interelectrode distances.

6 Discussion

The model presented takes into account the anatomical structure of skeletal muscle tissue on the scale of individual muscle fibres. It predicts that the conductivity of electric current by skeletal muscle tissue depends on the frequency contents of the injected current and that the geometrical scale, i.e. the distance between the site of current injection and the location of the voltage measurement, influences the value of the conductivity in the direction parallel to the fibre direction.

Such model predictions have been done earlier in other fibrous structure, NICHOLSON (1965) deduced a formula (p. 398, eqn. 11) that is essentially the same as eqn. 9 in this paper. Nicholson used the theory for diluted suspensions (MAXWELL, 1896) as a starting point for his study. V.

\[
V(y) = \frac{-1}{g_{e\gamma} + g_{i\gamma}} \left( y + \frac{1 - e^{-\gamma y}}{\gamma} \frac{g_{i\gamma}}{g_{e\gamma}} \right) \tag{25}
\]

where

\[
\gamma = \left( B \left( \frac{1}{g_{e\gamma}} + \frac{1}{g_{i\gamma}} \right) \right) Z_m^{1/2}
\]

\[
B = \text{surface/volume ratio of fibres}
\]

\[
Z_m = \frac{1}{G_m + j\omega C_m} \text{ membrane impedance per unit length (see Section 3)}
\]

Eqn. 25 can be compared with our eqn. 19 for the longitudinal case. Both formulas would be identical if and only if the following relations existed between their parameters:

\[
B = \frac{A}{S_t} \tag{26}
\]
\[ g_{x} = \sigma_{x} l \]  
\[ g_{xy} = \sigma_{x}(1 - p) \]  

In the x- and z-direction (transverse directions) the same functional relationship (eqn. 25) exists in the model approach of Plonsey and Barr. However, in our model, no redistribution of current takes place in the transverse direction, making the models differ as to their dependence on the transverse co-ordinates x and z. It may be of interest how far both models correspond if we take distances at which, using the model of Plonsey and Barr, redistribution of current reaches its steady value. From eqn. 25 in the x-direction for \( x' \gg 1 \)

\[ V(x) = \frac{-1x}{\sigma_{TM} + \sigma_{t}} \]  

From our model, a formula for the extracellular potential can be derived, using eqn. 9 for the effective transverse electrical conductivity:

\[ V(x) = \frac{1 - p}{1 + p} \frac{-1x}{\sigma_{e} + \sigma_{t}} \]  

An additional intracellular resistance \( R_{in} \) normal to the muscle fibre membrane is used: \( R_{in} = \frac{(3)^{1/2} \Delta}{(2\sigma_{t})}. \) This parameter is required as we will examine high-frequency behaviour. For high frequencies the membrane impedance will vanish and eqn. 30 becomes

\[ V(x) = \frac{1 - p}{1 + p} \frac{-1x}{\sigma_{e} + \sigma_{t}} \]  

This formula will be identical with eqn. 29 when the following relations exist:

\[ g_{ex} = g_{ey} = \frac{1 - p}{1 + p} \sigma_{e} \]  
\[ g_{zx} = g_{zi} = \sigma_{i} \]  

As mentioned earlier, Plonsey and Barr assumed equal ratios of the intra- and extracellular conductivities in the three principal directions. Eqs. 27 and 28, together with eqns. 32 and 33, show that this assumption, which could be valid in a one-dimensional equivalent of the bidomain model, is in fact in conflict with the geometrical assumptions underlying our model.

An important aspect in the model presented is the redistribution of the current injected into the muscle tissue. This will also occur in the case of current injection caused by an excited muscle fibre membrane. The resulting extracellular action potential will be influenced by this current redistribution. The results presented suggest that volume conduction models aimed at giving a correct description of electrophysiological signals should be adapted in a way that the anatomical structure on the scale of individual muscle fibres is taken into account.

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References


Authors' biographies

Frans L. H. Gielen was born in Emmeloord, The Netherlands, in 1951. He received the B.A. degree in Physics (Cryogenics) from Twente University of Technology in 1976, and the M.Sc. degree in Physics (Biophysics) from the same university in 1979. His Ph.D. research was granted by the Dutch Society for Scientific Research (ZWO-Biofysika) and was performed between 1979 and 1983 in the Biomedical Engineering Group of the Department of Electrical Engineering of Twente University of Technology. His special interests are electrophysiology and electric volume conduction theory with respect to EMG signals. He joined Department of Physics & Astronomy of Vanderbilt University, Nashville, Tennessee, USA in February 1984 where his research is on electric and magnetic measurements and models based on electromagnetic theory, which are used for the interpretation of experimental results.

Kasper L. Boon was born in Amsterdam, The Netherlands, in 1944. He received an M.Sc. degree in Physics in 1967 from the Technical University of Eindhoven and a Ph.D. in Biomedical Engineering from the Twente University of Technology in 1974. He is a senior staff member of the Biomedical Engineering Group, Department of Electrical Engineering of the Twente University of Technology. His research activities are mainly directed towards electromyography.

Hubert E. P. Cruts was born in The Netherlands, in 1958. He received the M.S. degree in electrical engineering from the Twente University of Technology, Enschede, The Netherlands in 1982. He is presently associated with the Research and Innovation Group, Rehabilitation Centre Het Roessingh, Enschede, The Netherlands. His research interests are in biomedical signal processing with application to exercise-ECG analysis.

Bert A. Albers was born in Hengelo, The Netherlands, in 1958. He received his M.Sc. degree in Physics from Twente University of Technology in 1983. In the same year he joined the Biomedical Engineering Division, Department of Electrical Engineering, of the Twente University of Technology. His research interests are electrophysiology and volume conduction in skeletal muscle tissue.
Willemien Wallinga-de Jonge was born in Veendam, The Netherlands, in 1945. She received the M.Sc. degree in Biology from the University of Groningen in 1968 and her Ph.D. in Biomedical Engineering from Twente University of Technology, Enschede, in 1980. She is a senior staff member of the Biomedical Engineering Group, Department of Electrical Engineering of Twente University of Technology. Her research activities there involve the electrical activity of muscle in general and the intracellular and extracellular single fibre activity in particular.

Herman B. K. Boom was trained as a medical physicist at the University of Utrecht, where he received his Ph.D. in 1971. He joined the Departments of Medical Physics and Medical Physiology where he was engaged in research in the field of cardiac mechanics and taught physiology and biophysics. Since 1976 he has occupied the Chair of Medical Electronics in the Electrical Engineering Department of Twente University of Technology. His research interests are cardiovascular system dynamics, bioelectricity and rehabilitation technology.