DYNAMICS OF A MULTI-BEAM PHOTONIC FREE ELECTRON LASER

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Abstract
A photonic free-electron laser (pFEL) uses free electrons streaming through a photonic crystal (PhC) to generate tunable coherent radiation. Here, we consider a pFEL driven by a set of three low energy (~ 10 keV), low pereance (< 0.1 μP) electron beams. Using a particle-in-cell code, we numerically study the dynamics and calculate the small-signal growth rate and output power of the various modes. We show that for an appropriate design of the PhC and selective placement of the electron beams, single mode operation is possible. We will also present results on the scaling with the number of electron beams.

INTRODUCTION
In a photonic free-electron laser (pFEL) electrons can stream through the many channels that are available in a photonic crystal (PhC) and produce Cerenkov radiation [1]. The function of the PhC is to slow down the phase velocity of the electromagnetic wave to match the velocity of the co-propagating electrons. More precisely, one of the spatial harmonics that is part of a Bloch mode of the PhC will be phase matched with the co-propagating electrons. For a strong interaction, the field amplitude of the phase-matched spatial harmonic should be as high as possible, and therefore interaction with a low spatial harmonic is desirable [2]. If the Bloch mode possesses an electric field component in the propagation direction of the electrons, then the phase-matched spatial harmonic will bunch the electrons and coherent Cerenkov radiation will be emitted. The many channels that are naturally present in a PhC have the advantage that the total current transmitted through the PhC can be increased by adding more electron beams in parallel and extending the transverse extend of the PhC, while maintaining a low electron density in each beamlet. Consequently, beam quality and transport are greatly improved compared to increasing the total current by increasing the current density in a single beamlet [3].

On the other hand, extending the transverse size of the PhC may lead to higher-order transverse Bloch modes that can all couple to the electrons and may introduce mode competition. In order to study the dynamics of a multi-beam pFEL, we use a particle-in-cell (PIC) code [4] to model a 3-beam pFEL operating at microwave frequencies. In the following we will present the PhC used in the pFEL and use PIC simulations to determine the starting current and investigate whether mode competition is present or not and how the mode competition is affected by the design of the PhC or placement of the electron beams.

Figure 1: Schematic view of the unit cell of the photonic crystal used for the 3-beam pFEL. See Table 1 for dimensions. The location of the three electron beams used in the PIC simulations are shown as blue cylinders.

PHOTONIC CRYSTAL
The unit cell of the PhC used in this study is schematically shown in Fig. 1, and its dimensions are summarized in Table 1. It consists of a metal waveguide containing a periodic array of metal posts. The PhC is designed to produce microwave radiation around 15 GHz when low energy (~ 10 keV) electrons are used. The metal waveguide is required to guide/confine the microwave radiation.

The PhC effectively allows for seven electron beams to propagate through the structure. A 3-beam electron gun has been designed for use in a pFEL with the PhC [5]. The choice of emitter in this gun however limits the beam-to-beam spacing to 5 mm, hence the beams shown in Fig. 1 fill every other available slot.

Table 1: Dimensions of the PhC unit cell

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>Width of unit cell / waveguide</td>
<td>20.0 mm</td>
</tr>
<tr>
<td>hw</td>
<td>Height of unit cell / waveguide</td>
<td>7.0 mm</td>
</tr>
<tr>
<td>a</td>
<td>Depth of unit cell &amp; distance between posts</td>
<td>2.5 mm</td>
</tr>
<tr>
<td>wp</td>
<td>Width of posts</td>
<td>1.3 mm</td>
</tr>
<tr>
<td>lp</td>
<td>Depth of posts</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>hp</td>
<td>Height of posts</td>
<td>4.0 mm</td>
</tr>
</tbody>
</table>

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The dispersion diagram, limited to a full Brillouin zone, is shown in Fig. 2 for the lowest 8 Bloch modes of the PhC. Here, $\beta$ is the longitudinal wavenumber of the Bloch mode and $a_x = a$ is the period of the PhC in the $x$-direction, which is taken as the propagation direction of the electrons. Also shown are the dispersion of the slow space-charge wave for an electron beam with radius of 0.5 mm, current of 60 mA, and voltages of 9 and 15 kV. Fig. 2 shows that for such a voltage range, the interaction frequency is expected to be between 15 and 16.2 GHz when only the lowest order Bloch mode would participate in the interaction. The dispersion diagram also shows that the interaction is with the first spatial harmonic and of the backward-wave type, i.e., the phase velocity, $v_p$, and group velocity, $v_g$, are in opposite direction. A priori, interaction is possible with many different Bloch modes, albeit at slightly different frequencies and longitudinal wavenumbers. It is therefore expected that mode competition will be present in this particular device. This will be studied in the next section using a PIC model.

**DYNAMICS OF A 3-BEAM PFEL**

To study the dynamics of a 3-beam pFEL we consider a device with an active gain section consisting of 20 unit cells of the type shown in Fig. 1. A schematic view of the device is shown in Fig. 3. The electron beams are generated by electron sources placed on a metal plate that shorts the waveguide at the upstream side of the device. At the downstream side, the waveguide is continued for some distance before it is terminated in a waveguide port, where the analysis of the generated radiation will take place. The port decomposes the radiation field into eigenmodes of the waveguide port. The waveguide port is positioned sufficiently far from the waveguide section loaded with the PhC to have only propagating waves at the port. The transition from loaded to unloaded waveguide corresponds to a jump in wave impedance, hence, the PhC is placed inside a resonator with the transition acting as partial reflecting mirror. In this paper, all PIC simulation assume metals as perfect electric conductors and electron beams with zero initial energy spread.

**Starting current**

As mentioned above, the active gain section of the pFEL is positioned inside a resonator. The losses in this resonator consist of passive losses (taken here to be zero due to ideal metals) and active losses due to the partly transmitting mirror. It is therefore of interest to find the minimum electron beam current required to overcome the lasing threshold. For this, we considered two situations, one pumps with a single electron beam in the centre of the PhC and one with three beams, each having the same current, as shown in Fig. 1. To determine the starting current, we performed PIC simulation and determined the growth rate of the lowest order waveguide mode at the output port, for various values of the beam current. The growth rate is calculated from the exponential growth of the power versus time (measured at the output port), and is shown in Fig. 4. Note, that the beam current is the current in an individual beamlet. Thus, for equal beam current, the total current in the 3-beam case is three times higher than for the single beam case. The starting currents

![Figure 2: Dispersion of the eight lowest Bloch modes of the PhC.](image)

![Figure 3: PIC model of the 3-beam pFEL. The active region of the laser consists of 20 PhC unit cells. Also shown are the three electron beams. The colour represents the electron energy, for the case shown at a time when the laser is in near saturation.](image)

![Figure 4: Growth rate of the lowest waveguide mode at the output port as a function of the current in each of the three beams (solid blue line) and in a single beam (dashed red line).](image)
are respectively 20 mA and 10 mA for the 1- and 3-beam cases. The starting current is therefore a factor 3(6) lower than the maximum available current from the electron gun when the pFEL is pumped by one(three) beam(s). The total starting current for the 3-beam case is thus 30 mA, which is slightly higher than for the single beam case. This is to be expected as the side beams experience lower local field strength due to the transverse profile of the Bloch mode in the PhC. If, for example, the two sides beams were placed in the channels adjacent to the centre beam, the total starting current would be almost equal. We thus find that by increasing the number of electron beams, the starting current per beamlet is decreased. However, the decrease in starting current is less than proportional to the number of electron beams, which is caused by the dependency of the coupling strength of the beams on the transverse profile of the Bloch mode.

Growth and saturation
As mentioned above, a current of 60 mA is sufficient to drive the pFEL into saturation, even if only a single beam is used. This is illustrated in Fig. 5a, where the power of the two lowest order waveguide modes at the output port is plotted as a function of time for a 60 mA, 9 kV beam of 0.5 mm radius. The growth rate of the TE_{10} mode is 1.3 dB/ns in the exponential growth region. The TE_{10} mode saturates at a level of about 34 W. The second waveguide mode (TE_{20}) is also present, albeit at a much lower level. Initially, it grows with the same growth rates and saturates at several mW. The separation in power level between the modes is almost constant. Despite being overmoded, only a single mode is generated when a single on-axis beam pumps the pFEL. The spectrum obtained from the electric field over the whole time window is shown in Fig. 5b. It shows that the centre frequency is 15.4 GHz with a full width at half maximum (FWHM) of about 10 MHz. The power is stationary for about 80 ns, so the bandwidth is close to being Fourier limited.

When the number of beams is increased to three as displayed in Fig. 3, only two modes are generated as shown in Fig. 6. Initially, the lowest order mode is the dominant mode, however it now grows with 3.2 dB/ns. The frequency and bandwidth remain unchanged. However, after the TE_{10} has saturated at about 100 W, the other waveguide mode, TE_{20}, remains growing to become the dominant mode at power level of about 220 W, while at the same time the power in the TE_{10} mode decreases to about 70 W at the end of 150 ns simulation time. The total power is 290 W, which is far more than three times the total power produced by a single beam. It is interesting to note that with only the two side beams turned on, about 270 W is produced in the TE_{20} mode and less than 10 W in the TE_{10} mode, for the same simulation duration. The latter is to be expected as the first order Bloch mode of the PhC has a node at the axis and the positions of the side beams result in a much lower coupling to the lowest order Bloch mode. Nonetheless, these results show a complicated interaction between the electrons and Bloch modes; depending on the location and number of electron beams, the interaction with the Bloch modes of the PhC either result in a single dominant mode at the output port, which can be either the TE_{10} or TE_{20} mode, or a mixture of the two modes. In the remainder of this paper we investigate methods to obtain single mode operation when the pFEL is pumped with multiple beams.

Single mode operation of a multi-beam pFEL
The transverse extend of the PhC required for multi-beam operation of the pFEL leads to multiple transverse Bloch modes that can be resonant with the electron beams (see Fig. 2). We will discuss several methods to obtain single mode operation and focus on the fundamental TE_{10} mode. We believe that these methods, or a combination thereof, can also be applied to obtain single mode operation at other waveguide modes at the output port, e.g. TE_{20} mode.

From basic considerations [6]-[7], it is clear that the coupling strength of the electrons with a particular Bloch mode is determined by the amplitude of the phase-matched longitudinal component of the spatial harmonic at the position of the electrons. By moving the electron beams closer to each other, the coupling strength with the fundamental Bloch mode is expected to increase while the coupling with the next higher order Bloch mode will decrease as this mode has an antinode, while the fundamental Bloch mode has a node on axis. We therefore place the side beams in the channels immediately adjacent to the on-axis beam, i.e., the beam-to-spacing is 2.5 mm instead of 5 mm. The growth and spectrum of the two lowest modes at the output port...
are shown in Figs. 7a and 7b, respectively. The current and voltage of the beamlets are still 60 mA and 9 kV, respectively. The TE\textsubscript{10} mode experiences a larger growth rate of 4.5 dB/ns (versus 3.2 dB/ns for a beam-to-beam spacing of 5 mm). The power in the TE\textsubscript{10} and TE\textsubscript{20} modes are now close to 100 W and just below 1 W, respectively. Changing the position of the beams changed the coupling to the modes, especially to the first order Bloch mode. Indeed, the fundamental Bloch mode experiences a larger growth, however the saturated power is not changed, while the next higher order Bloch mode is strongly suppressed. The spectrum in Fig. 8b shows that the interaction frequency has not changed but side bands have appeared, which are due to the oscillations visible in the power (Fig. 9a). It is interesting to note, that just before 80 ns the oscillations are out of phase, while shortly after 80 ns they are in phase.

Another way to change the coupling between the electrons and the Bloch modes is by changing the PhC and/or the waveguide. Here we choose to increase the width of the waveguide from 20 to 30 mm. This changes the transverse profile of the Bloch modes, hence the coupling to the electron beamlets. To pump this structure we use the multi-beam configuration where three the 60 mA, 9kV electron beams are spaced 5 mm apart (as in Figs. 1 and 3). The growth and spectrum of the two lowest waveguide modes at the output port is shown in figs. 9a and 9b, respectively. We observe that the output power in that the growth rate for the TE\textsubscript{10} mode is not changed much, however the saturated power is almost a factor 2 higher. Note that the wider waveguide also results in a change in dispersion of the Bloch modes, and hence the operating frequency has shifted to 15.35 GHz. The bandwidth is still about 10 MHz. Again, the field is close to being Fourier limited.

**DISCUSSION AND CONCLUSIONS**

We have demonstrated that multiple low energy and low current electron beams can be used to pump a pFEL. For a particular PhC we showed that a 10 mA current per beamlet is sufficient to overcome the oscillator threshold. Due to the larger transverse size, the pFEL is inherently overmoded, and depending on where the electron beams are placed one or more Bloch modes can grow, thereby showing a complicated lasing dynamics. We have also shown that the PhC waveguide can also be tweaked to improve coupling to a single Bloch mode and may result in both improved mode selectivity and higher saturated power. Still, the dynamics is only investigated for a single beam voltage and further study is required to investigate mode competition and tuning characteristics when the beam voltage is varied. This is further complicated by the presence of longitudinal modes in the resonator. This study is currently underway.

**REFERENCES**


[4] CST Microwave Studio 2011, CST GmBH.

