ABSTRACT

Flexure hinges inherently lose stiffness in supporting directions when deflected. In this paper a method is presented for optimizing the geometry of flexure hinges, while supporting stiffnesses are retained. These hinges are subjected to a load and deflected an angle of up to ±20°. The measure of performance is defined by the first unwanted eigenfrequency, which is closely related to the supporting stiffnesses. During the optimization, constraints are applied to the actuation moment and the maximum occurring stress. Evaluations of three cross flexure hinge types and a butterfly flexure hinge are presented. A flexible multibody modeling approach is used for efficient modeling. Each of these hinge types is described by a parameterized geometric model. The obtained optimal hinge designs are validated with a finite element model and show good agreement. The optimal solution of the butterfly flexure hinge shows the least decrease in the supporting stiffnesses of the evaluated hinges.

Keywords: compliant mechanism, constrained warping, structure optimization, large deflection, butterfly flexure hinge, cross flexure hinge.

INTRODUCTION

In high precision manipulator mechanisms, flexure elements are often utilized for their deterministic static and dynamic behavior [1-3]. Folkersma et al. [4] present a two degree of freedom large stroke elastic mechanism with eleven cross flexure hinges. When the mechanism is in a deflected state, a significant decrease in the first unwanted eigen-frequency is observed. This behavior can be understood by considering a basic flexure element, i.e. a leaf spring flexure [5-7] as is illustrated in Fig. 1. The leaf spring flexure holds high support stiffness in x-, z- and ry-direction, while it has low actuation stiffness in the rz-direction. For small deflections around the undeformed configuration these observations are true and supporting stiffnesses are approximately constant. However, for large deflections significant decrease in supporting stiffnesses are observed. For instance, the stiffness in z-direction in the undeformed configuration is governed by the high in-plane bending stiffness, while in the deformed configuration the low torsional stiffness also plays a role. This will lead to the deteriorating dynamic performance of the mechanism. Hence, the challenge of designing a high performance large stroke compliant mechanism is to retain the supporting stiffnesses of the flexure hinges for large deflections.
Several compliant mechanisms are optimized by Trease [8]. The presented optimization focuses on increasing supporting stiffnesses in an undeflected state. Therefore, geometric non-linearities as a cause of deflection are not included in the calculations. An optimized shape of a hinge flexure is found by Boer [9], where the measure of performance is based on a comparison of stiffnesses to that of a single leaf spring flexure. Both methods do not take a mechanism load into account to which the hinges are subjected.

This paper aims at optimizing the geometry of a number of spatial flexure hinge types with respect to the first unwanted eigenfrequency, subjected to a certain loadcase. This loadcase scales the importance of the supporting stiffness directions by assigning inertia to these directions. These hinges ideally release one rotation and constrain all other directions. Hence, the second eigenfrequency \( f_2 \) will be the first unwanted modeshape of the hinge. In order to prevent the structure from failure, the allowable Von Mises stress will be constrained. Also the actuation moment is constrained, which ensures that the flexure hinge stays compliant in the actuation direction. With these constraints, an optimization routine converges to an optimal geometry, which reduces the loss in supporting stiffnesses to a minimum.

For the optimization an efficient modeling approach is required in order to keep computing time within bounds. It needs to account for the non-linear geometric behavior of the flexure hinges. The SPACAR computer program [10], which is based on a flexible multibody approach with non-linear finite beam elements [11], is well suited to create models for this optimization. The obtained optimal geometries are validated with Finite Element analyses in ANSYS.

Four flexure hinges will be optimized in this paper. First, the conventional cross flexure hinge is considered where the flexures are connected in the pivot point [12]. Secondly, a cross flexure hinge where the flexures are not connected in the pivot is assed [13]. This variant is optimized with three and five crossing flexures. Finally a recently developed compliant mechanism is optimized, the butterfly hinge, which is presented by Henein [14].

### Loadcase
The loadcase is defined by inertia tensor \( \mathbf{J} \) and mass \( m \) in the load coordinate system \( O_{x'y'z'} \) as is illustrated in Fig. 2. This inertia tensor is taken at the pivot of the hinge. Here the projection of the principle axis \( y' \) on the \( x_1'y_1 \) plane is initially aligned with the \( y_1 \) axis. Within this coordinate system the inertia tensor can be rotated an angle \( \phi \) about the \( z_1 \)-axis, which redefines the inertia tensor \( \mathbf{J}' \) to \( \mathbf{J}'(\phi) = \mathbf{RJ}\mathbf{R}^T \).

\[
\mathbf{R}(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{J}'(\phi) = \mathbf{RJ}\mathbf{R}^T. \tag{1}
\]

With this angle \( \phi \), it is possible to alter the orientation of the load during the optimization. This way the load can be optimally oriented with respect to the hinge. In Fig. 3 the hinge is constructed in coordinate system \( O_{xyz} \). The origin of \( O_{xyz} \) will always coincide with the pivot of the hinge. Parameter \( \theta \) describes the angle of deflection of the hinge about the \( z \)-axis, which is established by applying an actuation moment \( M \) on the flexure hinge.

![Figure 2. LOADCASE DEFINED IN THE LOAD COORDINATE SYSTEM \( O_{x'y'z'} \), WITH PRINCIPLE AXIS OF INERTIA \( x'_1'y'_1z' \), WHERE \( \phi \) IS THE ANGLE BETWEEN THE PROJECTION OF \( y'_1 \)-AXIS ON THE \( x_1'y_1 \) PLANE AND \( y_1 \)-AXIS.](image)

![Figure 3. HINGE COORDINATE SYSTEM \( O_{xyz} \), ACTUATION MOMENT \( M \) AND ANGLE OF DEFLECTION \( \theta \) OF THE HINGE.](image)
Optimization

The parameter vector $p$ is hinge dependent and describes the geometry of the flexure hinge and the loadcase orientation. The optimization is governed by a constraint function $C(p)$ and cost function $F(p)$ which are dependent on the parameter vector $p$. Maximizing $f_2$ is achieved by minimizing its inverse, resulting in the following cost function to be minimized by the optimization algorithm,

$$ F(p) = \min_{\theta} f_2(p, \theta)^{-1}, \quad \forall \theta \in [-\theta_{max}, ... \theta_{max}], \quad (2) $$

where $\theta$ is the angle of deflection between $\pm \theta_{max}$. To prevent unbounded growth of the parameters during the optimization and to ensure that the algorithm returns a manufacturable and sustainable flexure hinge, constraints are applied. Constraints on the maximum actuation moment $M(p, \theta) < M_{max}$ and the maximum occurring Von Mises stress $\sigma(p, \theta) < \sigma_{max}$ define the non-linear constraint function,

$$ C(p) = \max_{\theta} \left\{ \frac{\max \sigma(p, \theta) - \sigma_{max}}{M(p, \theta) - M_{max}} \right\}, \quad \forall \theta \in [-\theta_{max}, ... \theta_{max}] \quad (3) $$

The optimal parameter vector $p_{opt}$ that minimizes the cost function, subjected to the non-linear constraints is given by,

$$ p_{opt} = \arg \min_{p} F(p) \text{ such that } C(p) \leq 0. \quad (4) $$

Derivative free optimization algorithms, which can include non-linear constraints, are well suited to find the optimal parameter vector of Eqn. 4. A suitable simplex optimization algorithm is described by Nelder-Mead [15]. A modified version of this algorithm is implemented, such that parameter vectors that violate the constraint function are not admissible.

Constrained warping phenomenon

An increase of torsional stiffness arises when a flexure is clamped at two sides. Due to the clamping, the cross section at the ends is inhibited from warping. Therefore, extra deformation energy is needed to twist the flexure, leading to an increased torsional stiffness. When a flexure is considered with length $L$, height $h$ and thickness $t$, it can be derived from [16] that the aspect ratio,

$$ i = \frac{L}{h}, \quad (5) $$

is a measure for the constrained warping phenomenon. Due to this, the analytical torsional stiffness $K_t$ derived by [17] is increased by a dimensionless stiffening factor $\gamma$,

$$ K_t = \frac{S_t}{L \gamma}, \quad (6) $$

where $S_t$ is the torsional stiffness per unit length. Here $\gamma$ is determined by a numerical Finite Element (FE) method experiment, where the flexure of Fig. 4 is considered. This flexure is at one end clamped in a wall and at the other end attached to a rigid body. Hence, warping is inhibited at both ends. In Fig. 5, $\gamma$ is graphed as a function of the aspect ratio $i$. Here it can be seen that the stiffening factor increases rapidly for short flexures. In the limit for $i$ going to infinity $\gamma$ converges to $\gamma = 1$, indicating that for slender beams the constraint warping phenomenon vanishes.

MODELING

The flexible multibody modeling approach implemented in the SPACAR software [10] is used, which is well-suited to create the models for the optimization of flexure hinges. Leaf spring flexures have a thickness that is at least an order of magnitude smaller than their height and length, and modeling them as a plate seems appropriate. However, in order to keep the models simple with a limited number of degrees of freedom, beam elements are used to model the flexures. Two aspects which are taken into account in the beam elements are transverse shear and torsion–extension coupling. Also, the mass moments of inertia of the beam cross section are considered. Standard beam formulation does not include torsional stiffening due to constraint warping. In the following sections an approach is given for taking this phenomenon into account.
Figure 5. CONSTRAINTED WARping STIFFENING FACTOR $\gamma$ AS FUNCTION OF THE ASPECT RATIO $i$.

To include the constraint warping phenomenon in the flexible multibody model, the flexure is discretized with four beam elements, which is assumed to be sufficient, see Fig. 4. Dimensionless parameters $\tilde{k}_a$ and $\tilde{k}_b$ represent a local increase in torsional rigidity $S_t$. Dimensionless parameters $\tilde{l}_a$ and $\tilde{l}_b$ represent fractions of the total flexure length $L$, which determine the element lengths. Due to connecting these beams in series the equivalent torsional stiffness, obtained at the end of the flexure is,

$$K_t = \frac{S_t}{L} \frac{\tilde{k}_a \tilde{k}_b}{2 \left( \tilde{k}_a \left( \frac{1}{2} - \tilde{l}_a \right) + \tilde{k}_b \tilde{l}_a \right)} \left[ \frac{Nm}{rad} \right].$$  \hspace{1cm} (7)

By comparing Eqn. 7 with Eqn. 6, the stiffening factor $\gamma$ is identified to be,

$$\gamma = \frac{\tilde{k}_a \tilde{k}_b}{2 \left( \tilde{k}_a \left( \frac{1}{2} - \tilde{l}_a \right) + \tilde{k}_b \tilde{l}_a \right)}.$$  \hspace{1cm} (8)

Near the clamped ends the torsional stiffness is very high due to constrained warping, so the assumption $\tilde{k}_a = \infty$ is made. The length over which this rigid behavior occurs is determined by modal analyses, where it is observed that this length is typically less than a quarter of the flexure, hence $\tilde{l}_a = 1/4$ is chosen. Taking the limit of $\tilde{k}_a \rightarrow \infty$ for Eqn. 8 gives for $\tilde{k}_b$,

$$\tilde{k}_b = \gamma (1 - 2 \tilde{l}_a) = \frac{\gamma}{2}.$$  \hspace{1cm} (9)

Parameter $\gamma$ is dependent on the aspect ratio $i$ and is given in Fig. 5. Equation 9 holds for $\gamma \geq 2$. For $1 < \gamma < 2$, $\tilde{k}_b$ will become smaller than one, which implies that the element torsional stiffness becomes less than determined with Saint Venant. Therefore in this region $\tilde{k}_b$ is set equal to 1 and length $\tilde{l}_a$ is obtained from Eqn. (8),

$$\tilde{l}_a = \frac{\gamma - \tilde{k}_b}{2\gamma} = \frac{\gamma - 1}{2\gamma}.$$  \hspace{1cm} (10)

A case dependent flexure model is obtained. The parameter settings are summarized in Tab. 1.

Table 1. FINITE ELEMENT CONSTRAINT WARping DIMENSIONLESS CORRECTION PARAMETERS RELATIONS.

<table>
<thead>
<tr>
<th>Case</th>
<th>$k_a$</th>
<th>$k_b$</th>
<th>$l_a$</th>
<th>$l_b$</th>
</tr>
</thead>
<tbody>
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<td>$\gamma \geq 2$</td>
<td>$\infty$</td>
<td>$\gamma/2$</td>
<td>$1/4$</td>
<td>$1/4$</td>
</tr>
<tr>
<td>$1 &lt; \gamma &lt; 2$</td>
<td>$\infty$</td>
<td>1</td>
<td>$(\gamma - 1)/(2\gamma)$</td>
<td>$-l_a + 1/2$</td>
</tr>
</tbody>
</table>

Finite Element validation

The flexible multibody model used for the optimization algorithm is expected to find the correct optimum. Nevertheless, a validation model of the ultimate solution is made in ANSYS. Here the eigenfrequency and Von Mises stress calculations are verified. An eight-node non-linear thin shell element, Shell-281, is used. This element has bending and membrane capabilities and is well suited for linear, large rotation, and large strain non-linear applications. Stress stiffening and large deflection features are included. With this shell element, the constrained warping and anticlastic curvature phenomena are accounted for. A uniform mesh is made for the flexures in the hinges. The loadcase is modeled using the MASS-21 element, which assigns the principal axes and moments of inertia. For modal analysis of the deflected geometries, pre-stress due to deflections are included.

RESULTS

In the next subsections four flexure hinges are optimized according to the presented method. The solid-, three- and five-flexure cross hinge and the butterfly flexure hinge are considered. From the mechanism presented by Folkersma et al. [4], a suitable loadcase is derived to which the flexure hinges will be subjected. In Tab.2 the entries of loadcase inertia tensor,

$$J = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix},$$  \hspace{1cm} (11)

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and mass \(m\) with the constraints and optimization parameters are given. From inertia tensor \(I\), the principle moments of inertia \(I_{xx}',I_{yy}',I_{zz}'\) and \(I_{xx},I_{yy},I_{zz}\) are calculated. The principle axis of inertia \(x'y'z'\) can also be extracted from the inertia tensor \(I\) and are set in coordinate system \(O_{xyz}'\), see Fig. 2. Principle moments of inertia \(I_{yy}'\) and \(I_{xx}\) will govern the second eigenfrequency. Here \(I_{yy}'\) is of the same order of magnitude as inertia \(I_{zz}\), and the moment of inertia \(I_{xx}\) is an order of magnitude lower than \(I_{yy}'\). The maximum allowable stress is constrained to 600 [MPa] and the actuation moment is constrained to 1.5 [Nm]. The maximum angle of deflection is defined to be \(\pm 20^\circ\). Along the z-axis the hinge height, \(H\), is fixed to 85 [mm]. Young’s and shearing modulus of steel are used, respectively 210 [GPa] and 80 [GPa]. All the flexures are modeled to be ideally clamped at both ends.

### Table 2. OPTIMIZATION SETTINGS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
<th>Unit</th>
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</thead>
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<td>[kg m^2]</td>
</tr>
<tr>
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<td>[kg m^2]</td>
</tr>
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</tr>
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<td>[kg]</td>
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<td>[Nm]</td>
</tr>
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<td>[deg]</td>
</tr>
<tr>
<td>(H)</td>
<td>85</td>
<td>[mm]</td>
</tr>
</tbody>
</table>

### Solid-Flexure Cross Hinge (SFCH)

The solid flexure cross hinge consists of a pair of crossing flexures, where the point of intersection is considered to be the pivot of the hinge [5]. The flexures are joined at this intersection point, see Fig. 6. This geometry is symmetric about the \(x\)-, \(y\)-axes and \(x\)-\(y\) plane. This hinge is parameterized by parameter vector \(p\).

\[
p = \{L \ W \ t \ \phi\},
\]

where \(L\) and \(W\) are respectively the length and width of the hinge. All the flexures will be given equal thickness \(t\).

### Optimal parameter vector.

The optimized SFCH geometry is presented in Fig. 7. Here the top view, \(x\)-\(y\) plane, is given where the loadcase is applied in the pivot point. To interpret the optimal loadcase orientation \(\phi\), see Eqn. 1, the projection of the principle axes of inertia on the \(x\)-\(y\) plane are given in coordinate system \(O_{xyz}\). Principle axis \(z'\) is omitted, since it does not significantly influence the second eigenfrequency. The figure shows that it is optimal to let the principle moment of inertia \(I_{yy}'\) coincide with the \(y\)-axis of coordinate system \(O_{xyz}\). The loadcase orientation outcome is a result of the symmetry of the hinge. The modeshape corresponding to the second eigenfrequency rotates about the \(y\)-axis. In order for the hinge to make this rotation, the flexures are subjected to a twist modeshape. With the optimal parameters \(L\) and \(W\), aspect ratio \(i\) becomes 0.4, see Eqn. 5, which indicates a stiffening factor \(\gamma = 4.0\). The second eigenfrequency is increased significantly by this stiffening effect. Therefore, including the constraint warping phenomenon in the beam model is important. Figure 8 graphs the second eigenfrequency \(f_2\) as a function of the angle of deflection \(\theta\). Initially, at \(\theta = 0^\circ\) the eigenfrequency is 230 [Hz]. This value drops 63 [%] due to loss in supporting stiffnesses in the flexures, to 85.3 [Hz]. This minimum is obtained at \(\theta = \pm \theta_{max}\).

### FE validation.

In Fig. 8, the behavior of the second eigenfrequency is given over the range of deflection calculated by SPACAR and by the FE model. The SPACAR calculations show good agreement with the validation model. Deviations of \(\pm 6\%\) are observed. SPACAR calculations show a maximum occurring stress of 600 [MPa] and an actuation moment of 1.5 [Nm]. FE model analyses show a maximum stress of 673 [MPa] occurring at the pivot where the flexures are connected and an actuation moment of 1.88 [Nm]. These discrepancies can be caused by anticlastic curving of the flexures cross section. This three dimensional behavior is not included in the beam formulation of SPACAR and therefore not included in the optimization. Nevertheless these discrepancies are within acceptable limits. Considering the calculation time, a SPACAR model is typically 20 times faster than a FE model.

---

Figure 6. PARAMETRIZATION OF THE SFCH.
Three-Flexure Cross Hinge (TFCH)

The three flexure cross hinge consists of three crossing flexures, which in contrary to the SFCH are not joined at their intersection point, see Fig. 9. This hinge is parameterized by parameter vector $\mathbf{p}$,

$$\mathbf{p} = \{ L \ W \ h_{o1} \ t \ \phi \},$$

where $L$ represents the length and $W$ the width of hinge, $t$ represents the flexure thickness which is equal for all three flexures. Parameter $h_{o1}$ is the height of both the outer flexures. Since the total height $H$ is taken to be 85 [mm], the inner flexure height $h_i$ is dependent on $h_{o1}$.

**Optimal parameter vector.** The optimized TFCH geometry is presented in Fig. 10. Here the top view, $x$-$y$ plane, is given where the loadcase is applied in the pivot of the hinge. The non-symmetric orientation of the loadcase with respect to coordinate system $O_{xyz}$ is due to the anti-symmetry of the hinge about the $x$- and $y$-axis. The modeshape corresponding to the second eigenfrequency shows to be a rotation of the hinge about the $y'$ axis. This causes the flexure to be loaded in torsion direction and therefore the constrained warping phenomenon is of great importance, see Eqn. 6. For the outer flexures a bending modeshape is observed. In comparison with the SFCH the crossing flexures are not connected at their intersection point anymore, hence the stress concentration here is circumvented. This allows $L$ and $W$ to shorten the flexure lengths, which decreases the aspect ratio $i$, see Eqn. 5. With the optimal parameters $L$ and $W$, aspect ratio $i$ becomes $0.34$ for the inner flexure, which indicates a stiffening factor $\gamma = 4.5$. In Fig. 11 the second eigenfrequency $f_2$ is graphed as a function of the angle of deflection $\theta$. Initially, at $\theta = 0^\circ$ the second eigenfrequency is $304.6$ [Hz]. This value drops $68.8$ [%] due to loss in supporting stiffnesses in the flexures, to $95.0$ [Hz]. Again, this minimum is obtained at $\theta = \pm \theta_{\text{max}}$.

**FE validation.** In Fig. 11 the behavior of the second eigenfrequency is given over the range of deflection calculated by SPACAR and by the FE model. The SPACAR calculations show good agreement with the validation model. Deviations of $\pm 1$ [%] are observed. SPACAR shows a maximum occurring stress of $600$ [MPa] and an actuation moment of $1.5$ [Nm]. FE model analyses show a maximum stress of $674$ [MPa] near the ends of the flexures and an actuation moment of $1.65$ [Nm]. These discrepancies can be caused by anticlastic curving and membrane deformation of the cross sections. Membrane deformation is as well as anticlastic curving not included in the beam formulation of SPACAR and therefore not taken into account in the optimization.
Figure 10. GEOMETRY OF THE OPTIMAL TFCH, WITH PRINCIPLE AXES GIVEN IN THE PIVOT OF THE HINGE.

Figure 11. SECOND EIGENFREQUENCY AS A FUNCTION OF THE ANGLE OF DEFLECTION, FOR THE OPTIMAL SOLUTION OF THE TFCH, DETERMINED BY SPACAR AND FE METHOD.

**Five-Flexure Cross Hinge (FFCH)**

In addition to the hinge considered in the previous subsection, the number of crossing flexures is expanded to five. This hinge is parameterized as is illustrated in Fig. 12. The parameter vector $p$ is,

$$p = \{L \ W \ \ h_{o1} \ h_{o2} \ t \ \ \phi \}.$$  \hfill (14)

In addition to the parameter vector of the TFCH parameter $h_{o2}$ is introduced, which describes the height of the added flexures. These are given the same height in order to maintain symmetry of the hinge.

**Optimal parameter vector.** The optimized FFCH geometry is presented in Fig. 13. Here the top view, $x$-$y$ plane, is given where the loadcase is applied in the pivot of the hinge. As for the TFCH a non-symmetrical orientation of the loadcase with respect to coordinate system $O_{xyz}$ is obtained. The modeshape corresponding with the second eigenfrequency is observed to be a rotation about the $y'$ axis. This causes the flexure to be loaded in torsion direction, see Eqn. 6, while the modeshapes of the remaining outer flexures are primarily a bending shape. With the optimal parameters $L$ and $W$, aspect ratio $i$ of the inner flexure becomes 0.9, which indicates a stiffening factor $\gamma = 2.0$. In Fig. 14 the second eigenfrequency $f_2$ is graphed as function of the angle of deflection $\theta$ for the found optimal configuration and for the FFCH used in the mechanism of [4]. For the optimal configuration, at $\theta = 0^\circ$ the second eigenfrequency is $301.2 \ [Hz]$. This value drops 68.4 [%] due to loss in supporting stiffnesses in the flexures to $95.3 \ [Hz]$. Once again, this minimum is obtained at $\theta = \pm \theta_{max}$. These results are similar to the results found for the TFCH. In comparison to the hinge designed in [4] a significant increase in the second eigenfrequency is observed. The optimization method improved the performance with 113 [%].

**FE validation.** In Fig. 14 the behavior of the second eigenfrequency is given over the range of deflection calculated by SPACAR and by the FE model. Again, the SPACAR calculations show good agreement with the validation model. Deviations of $\pm 1[\%]$ are observed. SPACAR shows a maximum occurring stress of $600 \ [MPa]$ and an actuation moment of $1.5 \ [Nm]$. FE model analyses show a maximum stress of $685 \ [MPa]$ near the ends of the flexures and an actuation moment of $1.59 \ [Nm]$. 

Figure 12. PARAMETRIZATION OF THE FFCH.
Butterfly Flexure Hinge (BFH)

The butterfly flexure hinge geometry is illustrated in Fig. 15. The top view is given, since the cross section is constant along the z-axis. This implies symmetry about the x-y plane. The geometry is also symmetric about the x- and y-axis. The height of the cross section along the z-axis is \( H \), see Tab. 2. Three rigid bodies connect the eight leaf spring flexures in series. Dimensions of rigid bodies 1 and 3 are independent on the optimization parameters. Their total width, in x-direction, is taken to be 6 [mm] and is placed at 3 [mm] from the center line in y-direction. It is assumed that within these dimensions, a sufficiently stiff body can be constructed. Dimensions of rigid body 2 will vary with the optimization parameters. The BFH has a low frequent internal eigenmode due to rigid body 2. In order to suppress this internal eigenmode, the angle of rotation of rigid body 2 should be kinematically coupled with the angle of deflection \( \theta \). Due to symmetry of the hinge, the angle of deflection \( \theta \) is related to the angle of deflection of rigid body 2 \( \theta_{r2} \) by

\[
\theta_{r2} = \theta / 2. \tag{15}
\]

An additional mechanism is needed to constrain the relation of Eqn. 15. Such a mechanism is designed by Henein [14] and shows an increase of the internal eigenfrequency with a factor nine. This is assumed to be sufficient. Therefore the internal mode is ignored in the optimization and these rigid bodies are modeled to be infinite stiff and massless. The BFH is parameterized by the parameter vector \( \mathbf{p} \),

\[
\mathbf{p} = \{ L \ W \ t \ \alpha \ \phi \}, \tag{16}
\]

where \( L \) and \( W \) respectively are the length and the width of the hinge, \( t \) represents the flexure thickness which is equal for all eight flexures. Parameter \( \alpha \) represents the angle between two successive flexures. The latter is constrained to be larger than \( 13^\circ \), in order to prevent collision of the flexures at maximum deflection. Due to symmetry the hinge geometry is fully defined by these parameters. The pivot of this hinge lies at the intersection point of the center lines.

Optimal parameter vector. The optimized BFH geometry is presented in Fig. 16. Here the top view, x-y plane, is given where the loadcase is applied in the pivot of the hinge. Due to the symmetry properties of the BFH, the load orientation angle is identical to the orientation angle found for the SFCH. Hence, the principle axis of inertia \( y' \) coincides with the y-axis of coordinate system \( \mathcal{O}_{xyz} \). The modeshape corresponding to the second eigenfrequency is a rotation about the \( y' \)-axis. Again, the flexures are loaded in torsion direction. The flexure lengths are 28 [mm], which results in a torsional stiffening factor of \( \gamma = 5.3 \). Parameter \( \alpha \) shows to be equal to the minimum value of \( 13^\circ \). Therefore it seems desirable to minimize this angle. In Fig. 17 the second eigenfrequency \( f_2 \) is graphed as a function of the angle of deflection \( \theta \). Initially, at \( \theta = 0^\circ \) the second eigenfrequency is 187.2 [Hz]. This value drops 36.4 [%] due to loss in supporting stiffnesses in the flexures, to 119.6 [Hz]. As with the previous hinges, this minimum is obtained at \( \theta = \pm \theta_{\text{max}} \).
FE validation. In Fig. 17 the behavior of the second eigenfrequency is given over the range of deflection calculated by SPACAR and by the FE model. The SPACAR calculations show good agreement with the validation model. Deviations of ±1 [%] are observed. SPACAR shows a maximum occurring stress of 493 [MPa] and an actuation moment of 1.5 [Nm]. FE model analyses show a maximum stress of 489 [MPa] near the ends of the flexures and an actuation moment of 1.65 [Nm]. Stress results show good agreement and a discrepancy in the actuation moment is observed.

DISCUSSION

In Tab. 3 the optimal parameter vectors of the considered flexure hinges are summarized. The flexure hinge which gives the highest second eigenfrequency over the full angle of deflection is the BFH. Though, the complexity involved with the BFH is significant higher than for the TFCH, since an additional mechanism is needed to constrain the internal eigenfrequency. The difference in second eigenfrequency of the TFCH and FFCH is negligible. Hence, adding two flexures to the cross flexure hinge does not increase the lowest second eigenfrequency. The TFCH shows to outperform the SFCH and is smaller in size.

An assumption made in general, in modeling the various types of hinges, involves the perfect clamping of the flexures. Performances strongly depend on this assumption, since constrained warping contributes significantly due to the high stiffening factors $\gamma$ and this is directly related to clamping. When a hinge is designed, this should explicitly be taken into account. In particular for the BFH, the three rigid bodies have to be designed in such a manner that they provide sufficient clamping.

Moment of inertia $I_{xyy}$ is oriented along the $x$-axis for the TFCH and the FFCH. Both, the BFH and the SFCH prefer the load orientation such that this moment of inertia coincides with the $y$-axis of coordinate system $O_{xyz}$, instead of the $x$-axis. Therefore, the highest supporting stiffness directions strongly differ between the various types of hinges.

The optimization algorithm converges to solutions which run into the non-linear constraints. For various initial simplexes the same optimal parameter vector is found, this implies that a global minimum of Eqn. 4 is obtained.

Table 3. RESULTING OPTIMAL PARAMETER VECTORS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SFCH</th>
<th>TFCH</th>
<th>FFCH</th>
<th>BFH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ [mm]</td>
<td>40.2</td>
<td>26.1</td>
<td>26.2</td>
<td>40.4</td>
</tr>
<tr>
<td>$W$ [mm]</td>
<td>60.1</td>
<td>13.7</td>
<td>12.5</td>
<td>50.2</td>
</tr>
<tr>
<td>$t$ [mm]</td>
<td>0.30</td>
<td>0.44</td>
<td>0.44</td>
<td>0.30</td>
</tr>
<tr>
<td>$\phi$ [deg]</td>
<td>0</td>
<td>101</td>
<td>88</td>
<td>0</td>
</tr>
<tr>
<td>$h_{z1}$ [mm]</td>
<td>-</td>
<td>13.4</td>
<td>6.8</td>
<td>-</td>
</tr>
<tr>
<td>$h_{z2}$ [mm]</td>
<td>-</td>
<td>-</td>
<td>19.9</td>
<td>-</td>
</tr>
<tr>
<td>$h_1$ [mm]</td>
<td>-</td>
<td>58.2</td>
<td>31.6</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha$ [deg]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>13</td>
</tr>
<tr>
<td>$\sigma_{\max}$ [MPa]</td>
<td>600</td>
<td>600</td>
<td>600</td>
<td>493</td>
</tr>
<tr>
<td>$M_{act}$ [Nm]</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>min $f_2$ [Hz]</td>
<td>85.3</td>
<td>95.0</td>
<td>95.3</td>
<td>119.1</td>
</tr>
<tr>
<td>max $\gamma$ [-]</td>
<td>4.0</td>
<td>4.5</td>
<td>2.0</td>
<td>5.3</td>
</tr>
</tbody>
</table>
CONCLUSION

In this paper a method for optimizing flexure hinges is presented. These hinges are parameterized and subjected to a chosen loadcase. Subsequently the hinges are optimized with respect to the first unwanted eigenfrequency. Therefore, by assigning inertia properties to several directions, the loadcase will scale the importance of the supporting stiffnesses in these directions. In the used nonlinear flexible multibody modeling approach, SPACAR, the constrained warping phenomenon is included. Three cross hinge flexures and a butterfly flexure hinge (BFH) are optimized. In the found optima of these hinges, it becomes clear that the constrained warping phenomenon is of great importance, since large stiffening factors are involved. The optima obtained with the flexible multibody models show good agreement with finite element (FE) model validations. The flexible multibody model calculation is typically 20 times faster than a FE model calculation, which results in a considerably faster optimization. The method shows to be able to find optimal geometries for flexure hinges, such that high supporting stiffnesses are obtained in the desired directions.

The BFH showed the least decrease in the supporting stiffnesses. The difference of the lowest second eigenfrequency between the optimal geometry of the three flexure cross hinge (TFCH) and the five flexure cross hinge (FFCH) is negligible. Hence, adding two flexures to the cross flexure hinge does not increase the second eigenfrequency of the hinge for this loadcase. The TFCH showed higher second eigenfrequencies than the solid flexure cross hinge (SFCH). The stress concentration at the pivot of the SFCH is downsized, which appears to be advantageous for the second eigenfrequency of the hinge. This implies that lumped compliance is preferred over distributed compliance.

ACKNOWLEDGEMENTS

This research is financially supported by the Dutch association Point-One, project MOV-ET PNE08006, from the Dutch Ministry of Economic Affairs. The authors acknowledge the contribution from Jaap Meijaard in deriving the constrained warping relations.

REFERENCES