Evaluation of flow formulas for submerged vegetation

D.C.M. AUGUSTIJN¹, A.A. GAILEMA¹
& F. HUTHOFF²

ABSTRACT. Much research has been dedicated to the interaction between flow and vegetation, however, this knowledge is only to a limited extend incorporated in models that are being used for river management practices. This may be partly due to the unknown reliability of the developed flow formulas. This contribution evaluates five different flow formulas derived for submerged vegetation: Klopstra et al. (1997); Stone and Shen (2002); Baptist et al. (2007); Huthoff et al. (2007); and Yang and Choi (2010). Each of these models is based on measurable vegetation characteristics to account for flow resistance by the vegetation. The evaluation of the flow formulas is based on the agreement with experimental data from literature, on their behaviour with respect to submergence ratio and on predicted water levels for different vegetation types. All models showed reasonable correlation to experimental data for rigid and flexible vegetation, however, average relative deviations were quite significant in the range of 24 to 43%. Some models showed unexpected behaviour in the ratio of the flow velocities in the surface and vegetation layer and deduced roughness parameters as function of submergence ratio. Predicted water levels for a given velocity varied up to several meters for some vegetation types. This shows that a particular choice for a model may have huge consequences when being used to predict water levels during flood conditions. The flow formulas proposed by Klopstra et al. (1997) and Yang and Choi (2010) show the best fit to experimental data and also show consistent physical behaviour.

KEYWORDS: vegetation resistance, flow formulas, prediction, model comparison.

1. Water Engineering & Management, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands.
2. Water Engineering & Management, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands.
HKV Consultants, P.O. Box 2120, 8203 AC Lelystad, The Netherlands.
1. Introduction

Currently many rivers in Europe are being rehabilitated to improve the ecological status. This provides more room for natural processes and may lead to less predictable developments in vegetation and morphological processes. In densely populated areas safety against flooding needs to be guaranteed. Uncontrolled natural developments may jeopardize safety requiring active management. Therefore, to predict hydraulic responses to vegetation developments in river floodplains computational models are commonly employed that include vegetation obstruction as part of the roughness parameterization. Many of such models have been developed in recent years, but only few are actually used in practice. The objective of this study is to evaluate some of these models and determine their reliability to ease the choice in applying recently developed flow models in current day river management practice.

2. Flow formulas

In this study five different flow formulas for submerged vegetation will be evaluated. Figure 1 shows a typical flow profile for submerged vegetation, including some characteristic parameters. All five models are briefly described below.

2.1. Klopstra et al. (1997)

The formula by Klopstra et al. (1997) is based on the momentum equation for the vegetation layer and a logarithmic velocity profile in the surface layer. The analytical solution is a rather lengthy expression for which is referred to the original paper. The only unknown parameter in the model is the scaling parameter \( \alpha \). Various authors have derived empirical relations for \( \alpha \). Here we will use the relation of Van Velzen et al. (2003) which is also used for Dutch river management practice:

\[
\alpha = 0.0227k^{0.7}
\]

where \( k \) is the height of the vegetation.

2.2. Stone and Shen (2002)

Stone and Shen (2002) derived a flow formula based on the momentum balance and accounting for solidity. The depth-averaged velocity is given by:

\[
U = \sqrt{\frac{2g}{C_D mDk^2}} \left( h - Dk \sqrt{m} \right) i
\]

where \( g \) is the gravitational acceleration, \( C_D \) the drag coefficient, \( m \) the vegetation density, \( D \) the diameter of the plants, \( h \) the water depth and \( i \) the bed slope.

2.3. Baptist et al. (2007)

The flow formula of Baptist et al. (2007) is derived by a genetic algorithm from a large number of simulations of a numerical turbulence model and reads:

\[
U = \frac{2g}{C_D mDk^2} \left( h - \frac{k}{\kappa} \ln \left( \frac{h}{k} \right) \right) \sqrt{h i}
\]

where \( \kappa \) is the Von Karman constant taken as 0.4.

2.4. Huthoff et al. (2007)

Huthoff et al. (2007) derived an analytical expression for bulk flow through and over vegetation using scaling assumptions. The resulting expression for the depth-averaged velocity is:

\[
U = \sqrt{\frac{2g}{C_D mD}} \left( \frac{k}{h} + \frac{h - k}{h} \left( \frac{h - k}{s} \right)^{2} \left( \frac{k}{h} \right)^{3} \right)
\]

where \( s \) is the spacing between the vegetation.
2.5. Yang and Choi (2010)

The flow formula of Yang and Choi (2010) is based on a uniform velocity in the vegetation layer added to the integration of a logarithmic velocity profile in the surface layer:

\[ U = \left( \frac{2g}{C_D m D k} \right)^{1/2} \sqrt{\frac{\ln \left( \frac{h}{k} \right)}{\frac{h - k}{h}}} \sqrt{h} \]  

All parameters are as defined before.

3. Evaluation

The five formulas are evaluated in four different ways: (1) by comparison with experimental data for rigid and flexible vegetation; (2) by comparing the velocity in the vegetation layer and surface layer at different submergence ratios; (3) by comparing the behaviour of predicted roughness parameters; and (4) by comparing predicted water depths for a given velocity.

3.1. Comparison to experimental data

In many studies experimental data have been collected in flumes for flow through and over vegetation. Galema (2010) made a compilation of these data. A difficulty with comparing experimental data from different sources is that not all data are measured in the same way and not all information is given. Some data which gave clear outliers were left out.

In Table 1 the performance of the different flow formulas compared to experimental data is given. On average the difference between measured and computed velocities ranges from 24 to 43%, which is considerable. For predicted water depths the deviations are usually smaller (Augustijn et al., 2008). The flow formulas perform slightly better for rigid vegetation than for flexible vegetation, except for Baptist et al. (2007). The formula of Stone and Shen (2002) performs the least. Figure 2 shows the best fit for the flexible data.

Table 1. Averages of relative deviations between measured and computed velocities for different flow formulas

<table>
<thead>
<tr>
<th></th>
<th>rigid (N=214)</th>
<th>flexible (N=119)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stone and Shen</td>
<td>30.5 %</td>
<td>43.2 %</td>
</tr>
<tr>
<td>Klopstra et al.</td>
<td>23.9 %</td>
<td>24.5 %</td>
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<tr>
<td>Baptist et al.</td>
<td>35.5 %</td>
<td>34.8 %</td>
</tr>
<tr>
<td>Huthoff et al.</td>
<td>21.9 %</td>
<td>34.3 %</td>
</tr>
<tr>
<td>Yang and Choi</td>
<td>24.8 %</td>
<td>27.3 %</td>
</tr>
</tbody>
</table>

Figure 2. Best fit for data on flexible vegetation by flow formula of Klopstra et al. (1997).

3.2. Velocity ratio

Here we evaluate the flow velocities in the vegetation and surface layer as predicted by the different flow formulas. For submerged vegetation, the velocity in the vegetation layer is expected to be lower than the velocity in the surface layer and the difference is likely to increase with growing water depth. Hence, the ratio between the average velocity in the surface layer and vegetation layer, \( U/U_v \), should approach 1 when \( h \) approaches \( k \) and \( U/U_v \) should increase with increasing submergence ratio, \( h/k \). Figure 3 shows \( U/U_v \) as function of \( h/k \) computed by the different flow formulas for natural grassland as defined by Van Velzen et al. (2003) (\( m = 4500 \text{ m}^2; D = 0.003 \text{ m}; k = 0.15 \text{ m}; C_D = 1 \)). The formula of Baptist et al. (2007) is not included in this figure as it does not provide separate expressions for flow velocities in the two layers.

In Figure 3, the formulas by Klopstra et al. (1997) and Yang and Choi (2010) show expected behaviour
as $U/U_v$ is always positive and approaches 1 for low submergence. The flow formula of Stone and Shen (2002) gives a relationship for $U/U_v$ only depending on the submergence ratio $h/k$ and in the limiting condition that $h$ approaches $k$, $U/U_v$ approaches a value of 1.5. This clearly does not agree with the expected physical behaviour. The descriptions for $U_i$ and $U_v$ by Huthoff et al. (2007) show for low submergence ratios values for $U/U_v$ smaller than unity, and this effect becomes more pronounced for sparse vegetation. This means that the formula of Huthoff et al. (2007) gives unrealistic results for low submergence ratios.

### 3.3. Roughness parameters

From the flow formulas roughness parameters can be derived such as Manning’s $n$ or a Nikuradse roughness length, $k_N$. It is generally agreed that for submerged vegetation these parameters decrease in value with increasing submergence ratio. For Manning this behaviour is shown by all five flow formulas. The Nikuradse roughness length decreases with increasing submergence ratios for three out of the five flow formulas (see Figure 4). For the formula of Stone and Shen (2002) the Nikuradse roughness length increases with relative water depth. The flow formula of Baptist et al. (2007) reduces to a constant Nikuradse roughness length which can be expressed as a function of vegetation characteristics (Augustijn et al., 2008). For submergence ratios larger than 5 the flow formulas approach to a constant value for Manning’s $n$. To obtain a constant value for $k_N$ the submergence ratio needs even be larger.

### 3.4. Predicted water depths

Figure 5 shows the predicted water depths for several vegetation types as defined by Van Velzen et al. (2003) at a velocity of 1 m/s. For management purposes the predicted differences are relatively large, varying up to several meters for reed. The formula by Stone and

![Figure 3. Ratio between the velocity in surface layer and vegetation layer ($U_i/U_v$) versus submergence ratio ($h/k$) for natural grassland and different flow formulas.](image)

![Figure 4. Nikuradse roughness length ($k_N$) as function of submergence ratio ($h/k$) for different flow formulas.](image)

![Figure 5. Differences in predicted water depth $h$ by the flow formulas for different vegetation types and a flow velocity of 1 m/s.](image)
Shen (2002) predicts the highest water level for almost all vegetation types.

4. Discussion and conclusions

Based on the evaluation it can be concluded that the flow formula of Stone and Shen (2002) performs the least when compared to experimental data. This formula also shows physical incorrect behaviour as the ratio between the velocity in the surface layer and vegetation layer is the same for all vegetation types and approaches 1.5 for low submergence ratios \((b/k \rightarrow 1)\). Moreover, the Nikuradse roughness length derived from the flow formula of Stone and Shen (2002) increases with increasing submergence ratio which is unexpected.

The flow formula of Huthoff et al. (2007) has the smallest relative deviation for experimental data on rigid vegetation (Table 1), but for low submergence ratios the model predicts larger velocities in the vegetation layer than in the surface layer. This qualitative behaviour of the formula is not in accordance with the general observations. However, the unrealistic behaviour is suppressed when predicting the average flow velocity for the entire flow depth. The formula of Baptist et al. (2007) has the largest average of the relative deviations between measured and computed velocities for rigid vegetation, but for other indicators of the goodness of fit (e.g. linear correlation coefficient or root of the mean squared differences) the formula performs better. The formula of Baptist et al. is equivalent with the White-Colebrook equation with constant Nikuradse roughness length for a given vegetation type, independent of water depth.

The two best performing and physically most correctly behaving flow formulas are those of Klopstra et al. (1997) and Yang and Choi (2010). Both are based on similar principles, i.e. a uniform flow velocity in the vegetation layer based on a force balance and a logarithmic velocity profile in the surface layer. Of these two formulas, the expression by Yang and Choi (2010) is the mathematically more simple one. If the flow formulas by Klopstra et al. (1997) and Yang and Choi (2010) are considered most reliable, they still only predict experimental flow velocities within a band width of approximately 50%. This means that even though the qualitative behaviour of the formulas is as expected, they still do not give accurate predictions. For different plant configurations they predict differences in water levels of up to 2 m for a depth averaged flow velocity of 1 m/s (Figure 5). The existence of this uncertainty should be realized when applying one of these formulas in models used for management applications.

Given the vast amount of research already performed in this area it is questionable whether yet another generally applicable flow formula would perform any better. Research initiatives should be taken to monitor the resistance in the field where non-ideal conditions may introduce aspects which are unaccounted for in flow formulas evaluated here.

References


