THE WHEATSTONE GADGET
A SIMPLE CIRCUIT FOR MEASURING DIFFERENTIAL RESISTANCE VARIATIONS

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ABSTRACT
This paper presents a simple circuit for measuring differential resistance variations. The Wheatstone Gadget, or The Gadget, is able to measure the same parameters as the Wheatstone Bridge in combination with an emitter-coupled pair [1], but with fewer components and fewer supply voltages. The Gadget is a simple circuit with a lot of possibilities, as shown in this paper. It needs only one power supply and is especially designed for small resistor values, which makes it well suitable for micromechanical applications.

INTRODUCTION
In many sensors a physical input quantity causes a variation in resistance, which has to be converted into an electrical output quantity. To increase the sensitivity and decrease undesirable side effects, most of the sensors are constructed in a way that the physical input quantity results in an increase in one resistance and a decrease in another.

An example is the μ-flow [2], an acoustical flow sensor in which an acoustical flow causes a differential resistance variation. As in most micromechanical sensors the applied resistors are metal films because these can be manufactured with small tolerances. Typical values of metal film resistors in micromechanics are below one kΩ.

In this paper the following features of The Gadget will be presented: the transfer function, the common mode rejection ratio (CMRR), the power supply rejection ratio (PSRR), the offset tolerance and the signal to noise ratio (S/N). Furthermore some circuits based on The Basic Gadget will be presented.

THE BASIC GADGET
The Gadget uses only one power supply which can be chosen almost freely. It is basically a Widlar current mirror [1], the output current is a function of the input current and the emitter resistors. If the resistors have the same value the output current will be approximately the same as the input current.

The Basic Gadget is shown in Fig. 1 with at the left the circuit and at the right the block diagram. In this case it is designed with a set of two NPN transistors. The output current of The Gadget is given by:

\[ I_{out} = \frac{I_{in}}{R_2} + \frac{I_{out}}{R_2} \left( \frac{\alpha_{FE1} + 1}{\alpha_{FE2}} \right) \frac{R_1 + R_2}{R_2} \]  

\[ U_T \text{ represents } q/kT=25\text{mV. This transcendental equation cannot be solved analytically. If } R_1=R_2 \text{ and } \alpha_{FE1}<<\alpha_{FE2} \text{ the last term (the DC offset term) can be neglected and eq. (1) simplifies to eq. (2). } \]

\[ I_{out} = \frac{I_{in}}{R_2} \frac{U_T}{R_2} \frac{I_{out}}{1} \]  

(1)

(2)
Under the linearity condition:

\[ I_{R_1} \gg U_2 \ln \left( \frac{I_{\text{out}}}{I} \right) \]  \hspace{1cm} (3)

Finally eq. (1) simplifies to eq. (4). For small differential resistor variations, i.e. \( \Delta R_1 = \Delta R_2 \), and a nominal value \( R_1 = R_2 = R \), the transfer function is eq. (5):

\[ I_{\text{out}} \approx \frac{R_1}{R_2} ; \quad \Delta I_{\text{out}} = 21 \frac{\Delta R}{R} \]  \hspace{1cm} (4,5)

The signal to noise ratio is given by:

\[ S_N = \frac{2 \Delta R I}{R \left( \frac{125 \times 10^{-3}}{R^2} q \frac{q}{e} \frac{e}{R} \frac{1}{k T} + \frac{4k}{R^2} (R_{hB} R_{aB} + 2 T_R R) + \frac{q}{10 R} + 2 q l \right)} \]  \hspace{1cm} (6)

in which \( q \) is the charge of an electron, \( \alpha_{FE} \) the forward current gain, \( R_{hB} \) is the base series resistance and \( T_B \) the temperature of the BJT. \( k \) is Boltzmann’s constant, \( T_R \) the temperature of the sensing resistors and \( BW \) the bandwidth of the signal. Transistor noise models are used as in [1], the influence of 1/f noise is neglected here. In most cases the factor \( 2 q l \) in eq. (6) is dominant in the denominator and the \( S/N \)-ratio will be proportional to the square root of \( I \).

In contrast with the Wheatstone bridge used in combination with an emitter-coupled pair, the effect of a DC differential resistance on The Basic Gadget is not limited. The input voltage of an emitter-coupled pair is limited to approximately 50 mV [1].

\[ \text{Fig. 1;2;3;4: The Basic Gadget; The Gadget; The Gadget Duo Sensation; The Cascade Gadget.} \]

**THE GADGET**

Using the formulas of The Basic Gadget it is easy to find an expression for the transfer function of circuit in Fig. 2, the current source is simply implemented with resistor \( R_3 \) and the output current is converted to a voltage with resistor \( R_4 \).

The DC bias condition of The Gadget is (see also Fig. 1 and 2):

\[ U_{\text{out}} = E - I R_4 > U_{B E 12} + U_{C E, \text{min}} \approx U_B \]  \hspace{1cm} (7)

The bias current is given by eq. (8). Substituting eq. (7) in eq. (8) the DC bias condition yields eq. (9).

\[ I = \frac{E - U_{B E 1}}{R_3 + R} \quad ; \quad R_3 \geq R_4 \]  \hspace{1cm} (8,9)

The transfer function of The Gadget using eq. (5) and eq. (8) is given by eq. (10):

\[ \Delta u_{\text{out}} = -2 I R_4 \frac{\Delta R}{R} = \frac{R_4 (E - U_{B E 1}) \cdot 2}{R_3 + R} \]  \hspace{1cm} (10)

With \( R_3 = R_4 \) an optimal transfer function for the gadget is achieved. The common mode transfer function \( (\Delta R_1 = \Delta R_2) \) is given by eq. (11). The combination of eq. (10) and eq. (11) gives the CMRR:

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\[
\Delta u_{\text{out,common}} = \frac{I \cdot R \cdot R_4 \Delta R}{R_3 R} \quad ; \quad \text{CMRR} = \frac{\Delta u_{\text{out,diff.}}}{\Delta u_{\text{out,common}}} \approx 2 \frac{R_3}{R} \quad (11,12)
\]

If the power supply increases by \( \Delta E \), the current through \( R_3 \) increases with \( \Delta I = \Delta E/(R_3 + R) \), the current through \( R_4 \) increases by \( \Delta I \). The output voltage alters with \( \Delta E \) minus \( \Delta I \cdot R_4 \). This leads to a PSRR of:

\[
\frac{\Delta E}{\Delta U_{\text{out}}} \approx \left( 1 - \frac{R_4}{R_3 + R} \right)^{-1}
\]

(13)

If the resistance \( R_4-R_3 \approx R_3 \) the effect of variations in the power supply will be minimised. In this way it is possible to force a variation of the current by varying \( E \) without a variation of the output voltage eq. (8). Eq. (13) is only valid for \( R_3 = R_2 \).

**THE GADGET DUO SENSATION**

The Gadget Duo Sensation (Fig. 3) consists of two Gadgets. It is capable of measuring two pairs of differentially varying resistors and can be used with voltage or current output. The current output is easy to calculate using eq. (1). When the nominal values of the resistors are equal (\( R_1 = R_2 \) and \( R_3 = R_4 \)) the offset current will be eq. (14). The bias current is given by eq. (15):

\[
I_{\text{out,DC}} \approx \frac{2(\alpha_{\text{FE,NPN}} - \alpha_{\text{FE,PNP}})}{\alpha_{\text{FE,NPN}} \alpha_{\text{FE,PNP}}} I \quad ; \quad I = \frac{E - 2U_{\text{BE}}}{R_1 + R_3 + R_5}
\]

(14,15)

Using eq. (5) the output current due to differential variations of the resistors is:

\[
\Delta I_{\text{out}} = 2I \left( \left[ \frac{\Delta R}{R} \right]_{\text{NPN}} + \left[ \frac{\Delta R}{R} \right]_{\text{PNP}} \right)
\]

(16)

For voltage output the PNP-Gadget is an active load [1] for the NPN-Gadget and vice versa. If \( I_{\text{DC}} \approx 0 \) (\( U_{\text{out}} \approx \frac{1}{2} E \)) the transfer function is:

\[
\Delta U_{\text{out}} = \Delta I_{\text{out}} \left( f_{\alpha,\text{PNP}} \right)_{f_{\alpha,\text{NPN}}} = \frac{U_{A,\text{NPN}} U_{A,\text{PNP}}}{U_{A,\text{NPN}} + U_{A,\text{PNP}}} \frac{\Delta I_{\text{out}}}{I}
\]

(17)

Using \( U_A \) as the Early voltage.

**THE CASCADE GADGET**

Because the output of a Basic Gadget, see eq. (5), has the same dimension and magnitude as the input of The Basic Gadget it is possible to cascade it. In Fig. 4 three Gadgets have been cascaded. The limiting factor for cascading is that the bias current will be reduced, last term of eq. (1). This limitation however can be overcome by injecting currents. If the resistors of each individual Gadget vary little and differentially and assuming \( (\Delta R/R)^2 \approx 0 \), the transfer function of The Cascade Gadget is:

\[
I_{\text{out}} + \Delta I_{\text{out}} = \prod_{i=1}^{N} \left( 1 + 2 \left[ \frac{\Delta R}{R_{-i}} \right] \right) \approx I + \sum_{i=1}^{N} 2 \left[ \frac{\Delta R}{R_{-i}} \right]
\]

(18)

Therefore it can be used as a summation circuit.

**THE AM GADGET**

Supplying The Gadget with an alternating voltage, superimposed on a DC voltage, (see Fig. 2) the output will generate an AM signal with suppressed carrier (called a double-side band signal [3]). The supply voltage of The Gadget is in this case:

\[
E = E_0 [1 + \alpha \cdot \cos(\omega_{\text{HF}} + \phi)]
\]

(19)

using \( \alpha \) (with \( \alpha < 1 \)) as the ratio of the absolute value of the carrier and the DC voltage. The output of the AM Gadget:

\[
\Delta u_{\text{out}} = 2IR_4 \cdot \beta \cos(\omega_{\text{LF}} t) \cdot \text{PSRR}^{-1} \cdot \alpha E \cdot \cos(\omega_{\text{HF}} t) + 2IR_4 \alpha \beta (\cos(\omega_{\text{LF}} t + \omega_{\text{HF}} t) + \cos(\omega_{\text{LF}} t - \omega_{\text{HF}} t))
\]

(20)

using \( \beta \cos(\omega_{\text{LF}} t) \) as \( \Delta R/R \). The PSRR, eq. (13), indicates how much the carrier will be suppressed.
SIMULATION AND MEASUREMENT RESULTS

The Gadget, see Fig 2. Parameters: R3=820Ω, R4=680Ω; ΔR/R=0.01; E=10V.

![Graph](image)

Fig. 5: Output of The Gadget as a function of R (the nominal value of R1 and R2). The line represents the theoretical model (eq. 10), the dots are PSPICE simulations, the open dots measurements

Two notes have to be made, 1, the decrease of the transfer function can be avoided when a current source is used instead of the combination of R3 and a voltage source, 2, because of relative big ohmic losses for small resistor values, the measurements in this range are not as good as predicted.

The Gadget Duo Sensation has conveniently been used in our measurements [2]. One disadvantage of the voltage output is that the exact value of the Early voltage is unknown and thus the transfer function cannot be exactly determined.

The AM Gadget. Measurements have been performed with E=20V, α=0.71, β=8.6%; R3=3300Ω in series with a diode, R4=3513Ω, f_{IF,signal}=200Hz, f_{IF,cARRIER}=10kHz. The model (eq. 20) and PSPICE both predict a factor \frac{1}{2}α=2.8 reduction in the modulated side-band compared to the signal, measurements show a factor 4.9. A PSRR of 45dB (the carrier) and a minimum β of 7.10^{-7}(U_{IF}=10μV_{RMS}) was measured at a side band.

FURTHER RESEARCH

To get a larger S/N ratio other types of transistors, the MOSFET and JFET, are being investigated. More variations of The Gadget are being examined. It is also possible to measure complex impedances like capacitors or inductances.

CONCLUSIONS

The Gadget is a simple circuit measuring differential varying resistances. It is easier to operate and has more tolerances than the Wheatstone bridge. For micromechanical applications it is well suitable, because it works well with low resistor values.

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