DAMAGE IN TEXTILE LAMINATES OF VARIOUS INTER-PLY SHIFT

D. S. Ivanov1*, S. V. Lomov1, S. G. Ivanov2, I. Verpoest2

1 Katholieke Universiteit Leuven, Belgium, Department Metallurgy and Materials Engineering (MTM)
Kasteelpark Arenberg 44, 3001 Heverlee – Belgium
2 University of Twente, The Netherlands Faculty of Engineering Technology, PO Box 217, NL-7500AE
Enschede
*Dmitry.Ivanov@mtm.kuleuven.be

Abstract
Deformation mechanisms and failure of textile laminates are strongly affected by inter-layer configurations – a mutual shift of the plies. To model it within a traditional framework, one must construct a representative volume element (RVE), which includes all the plies. This is a time consuming and computationally expensive work. As an alternative, the paper suggests boundary conditions (BC) imitating the interaction with the surrounding non-periodic media. This makes possible analysis on a single unit cell of one ply. The proposed BC respect inter-ply configurations, account for the number of plies, distinguish the ply position, and reproduce the meso stress state with a good accuracy. The BC are constructed through (1) averaging of the known periodic solutions with respect to the ply shifts, (2) separation of the solution to the outer and inner ply cases, (3) energy equilibrium of heterogeneous and effective media. The unit cell finite element (FE) modelling is validated by reference full scale solution on the entire laminate.

1 Introduction
Nowadays, textile multi-ply composites are often employed in important structural applications. The design criteria of the textile laminates are determined in relation to the damage thresholds: intra-yarn crack initiation or delamination onset. A ratio of stress at damage onset to the ultimate strength defines a safety potential of the composite. Hence, the knowledge of the failure stages is an important issue for engineering design.

The damage threshold can be efficiently predicted by means of the meso-scale analysis1 under condition that internal yarn geometry is carefully described. One of the serious impediments to predictive power of the meso modelling of the laminates is that the relative inter-ply shift is not controlled in the production practice. Hence, there is always an arbitrary factor not accounted in deterministic meso models.

There are only a few works, which study the effect of stacking sequence by creating composites with controlled position of the plies. The reason is obvious: complexity of manufacturing. The size of textile unit cell is of an order of centimetres. To handle the out-of-plane pattern, one needs to operate with relatively large textile sheets - large enough to produce the macro testing samples, i.e. tens of centimetres in length. These sheets have to be

1 Model accounting for internal geometry of yarns in textile composite.
placed upon each other with a very fine tuning of the out-of-plane positions: fractions of unit cell dimension. Additionally, the yarns in dry textile can easily move and slip away from the desired position, which may break the controlled pattern over the entire area of a specimen. Hale [1] has manufactured satin composites with various controlled ply shifts. The experiments revealed a clear difference between the surface strains measured by Moire interferometry for various configurations. Carvalho et.al. [2] have produced twill laminates with periodic and an arbitrary stackings and observed a stacking dependent kink band formation and failure mechanisms in the compact compression tests.

The numerical meso modelling allows precise albeit virtual positioning of the layers, and there are clear numerical results showing the effect of the stacking sequence. Ito and Chou [3] considered three stacking patterns: random, symmetric and periodic in an idealised 2D case (cross-section of the composite). The stresses were predicted by the approximate iso-strain and flexural models, which were verified by finite element analysis for the instances of regular patterns. A significant difference in stress distribution for these stackings has been reported. Le Page et.al. [4] in a 2D problem has shown that the energy release rate of cracks in a textile architecture is strongly different for the stacking pattern, the number of plies and the crack position. The issue of BC for infinitely thick composites with a regular periodic ply shift (called “staggered” there) has also been discussed in [4]. Owens et.al. [5] has shown stress distribution in symmetrically and periodically stacked woven composite by full scale FE modelling. The present authors [6] have studied numerically the effect of the ply shift to the strain distribution of 2-ply composite.

The major contribution of this paper is an approach, which allows modelling the laminate efficiently by means of a boundary value problem with unit cell of one ply only, which is done by means of properly adjusted BC. In our previous paper [7], it has been shown that the stress distribution in textile laminate depends on the ply position: in the plies situated on the surface or inside the laminate the stresses are different. It was also demonstrated that the stress depends on the number of plies in the laminate. It was proposed to construct BC of a single ply problem that can accurately reproduce interaction with the other unit cells and hence, the stress distribution at different locations in the laminate. In this paper the methodology will be extended to the case of the laminate with arbitrary ply shifts. In the first section we illustrate the difference between 4 different laminate configurations by estimating the risk of various failure modes. In the second section we describe the methodology of constructing the BC for unit cell of one ply with respect to the unit cell position, the number of plies, and ply configuration. Finally, the reference solution obtained for the full scale finite element analysis is compared with results of the new analysis.

2 Comparison of various lay-ups

1.1 Geometry and properties of components

The numerical exercise is performed for the case of a 6-ply carbon-epoxy unbalanced twill composite. The unit cell of the reinforcement is shown on Figure 1. The unbalanced structure is chosen to present a more complex/general case. The number of fibres is approximately the same in both the fibre directions: the linear density of the warp yarns is twice higher, however yarn spacing is smaller. The architecture exhibits a relatively moderate yarn crimp: the warp yarn, being stretched out from its position in the textile, would become only 1.1% longer. The
dimensions of the unit cell is \(10.2 \times 21.6\) mm in plane of the textile, the thickness is 1 mm. This is the same structure as the one reported in the previous publication [7], where the surface effects have been studied. The reader is referred to this paper for more details on stress distribution in 6-ply and infinite ply laminate with the periodic stacking.

Figure 1. Unit cell of twill woven composites: (a) mesh of the unit cell, (b) mesh of the reinforcement.

Properties of a transversely isotropic material are assigned to the yarn elements according to the local fibre orientation. The stiffness is calculated by the analytical formulae of Chamis [8], with the presumed fibre volume fraction in the yarns of 60\%, the properties of the carbon for the fibres and the properties of the epoxy for the matrix.

The geometrical model of the textile reinforcement is produced in the WiseTex software [9, 10] and mesh of both unit cell and laminates is done in the MeshTex program (Osaka University) [11]. In the finite element model, the full contact of the 0° and 90° direction yarns is avoided by inserting a thin matrix layer between the yarns. The unit cell contains 16,640 8-node brick elements. All the plies/unit cells in the laminate models have a nearly identical mesh pattern.

Figure 2. Four considered stacking sequences (black lines show the configurations of the ply shift)

Several lay-up configurations have been built for the reference solution (Figure 2):
1) “Periodic” where all the plies are in phase;
2) “Symmetric”, where every second ply is shifted to half period in warp direction;
3) “Step”, where all the even plies are in phase, and all the odd plies are shifted. The value of the shift corresponds to the distance between two neighbouring weft yarns, which is unit cell quarter size in the warp direction;
4) “Stairs”, where every next ply is shifted to the same distance relatively to the previous one, in the warp direction. The value of the shift is the same as in the “Step”- configuration.
1.2 Loading

A uniaxial tensile test in the warp direction is simulated – Figure 1. Periodic BC are applied to the edges perpendicular to the laminate plane, which corresponds to the assumption of infinitely wide textile reinforcement. The top and the bottom surfaces are left free allowing for the non-constrained deformation in the thickness (i.e. out-of plane) z-direction. Based on homogenised properties calculated using [12], Poisson’s contraction in the weft y-direction \( \nu_{yy} \) is set as \( \langle \varepsilon_{yy} \rangle = -\nu_{yy} \langle \varepsilon_{xx} \rangle \) [7, 12], hence, the average stresses in other but loading direction are zero.

The result of the elastic solution is the local stress distribution, illustrated on Figure 3, which is used to make rough estimates of various damage related thresholds:

(a) Intra-yarn crack onset is estimated by the Puck criterion [13] with parameters for unidirectional carbon-epoxy fibre bundle [14, 15].
(b) Crack occurrence in the matrix inter-yarn zones, responsible for delamination onset, is estimated by the Mohr-Coulomb pressure dependent criterion with parameters for the epoxy matrix [16].
(c) Fibre failure is estimated simply as the ratio of local stress in the fibre direction to the critical strength of UD composite.

![Stress distribution](image)

**Figure 3.** Stress \( \sigma_x \) in the direction of loading (global coordinate system) in uniaxial tensile test - side view of the laminate deformations, the average deformation \( \langle \varepsilon_{xx} \rangle = 0.1 \% \). The deformed shapes are exaggerated to make the comparison more vivid.

Of course, these thresholds cannot be interpreted as actual predictions of the particular failure as far as the elastic model does not account neither for high plastic deformation of epoxy and UD fibre bundles in shear, nor for the stress redistribution after occurrence of yarn cracking and delaminations. However, these estimates are meaningful to make an immediate comparison of a potential damage development in various configurations. A vast difference for all the damage related aspects is observed – Table 1. Mechanical interpretation of the observed difference is given in the next paragraph.
Table 1. Applied strain at initiation of various damage modes: IYC – intra-yarn cracking, DO – delamination onset, FF – fibre failure. Puck, Mohr-Coulomb – failure criteria.

<table>
<thead>
<tr>
<th>$\varepsilon_{\text{init}}, %$</th>
<th>Periodic</th>
<th>Step</th>
<th>Stairs</th>
<th>Symmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>IYC, Puck, weft</td>
<td>0.15</td>
<td>0.19</td>
<td>0.22</td>
<td>0.26</td>
</tr>
<tr>
<td>IYC, Puck, warp</td>
<td>0.25</td>
<td>0.33</td>
<td>0.37</td>
<td>0.38</td>
</tr>
<tr>
<td>DO, Mohr-Coulomb</td>
<td>0.34</td>
<td>0.48</td>
<td>0.70</td>
<td>0.86</td>
</tr>
<tr>
<td>FF, Max-Stress, warp</td>
<td>1.13</td>
<td>0.95</td>
<td>0.89</td>
<td>0.87</td>
</tr>
</tbody>
</table>

The failure factors show always the same scenario of the damage developments for all the configurations: (1) intra-yarn cracking in 90° yarns, (2) intra-yarn splitting of 0° yarns, (3) delaminations, and eventually (4) fibre failure. This is a well known sequence of the events, see for instance [17, 18]. The 4 considered configurations considered together form a sort of spectre of values for the damage thresholds. Periodic and symmetric configurations turned to be on the opposite side of this spectre for: matrix failure initiates earlier of all 4 configurations in the periodic structure and fibre failure would occur earlier in the symmetric configurations than elsewhere. The conclusion on fibre failure should be treated carefully. In reality, from the moment the plies are completely delaminated we actually deal with a stack of 1-ply composites and the stacking may not play any essential role anymore.

As shown in [7], the effect of lay-up becomes even more explicit as the yarn crimp increases.

1.3 Deformation mechanisms in textile laminates

The internal stress distribution in the textile laminates is strongly governed by the inter-ply configurations. The mechanics of the ply interaction can be schematised and illustrated in a simple way for the periodic and symmetric cases. The simplifications given below are important for understanding the reasons of the observed difference in the stresses and the failure levels: lower strain at inter-yarn crack and delaminations initiations for periodic configuration and lower strain at fibre failure for the symmetric one.

In the uniaxial tensile test, the main load is obviously carried out by the yarns in the loading direction. That is true for any possible configuration. Hence, the major stress factor is the normal stress along the loaded fibres. However, the crimp of the yarns causes an additional important lay-up dependent factor: out-of plane shear stress. The reason of the shear is global equilibrium of the deforming system.

Consider the global coordinate system – Figure 3. The shear is not applied at the macro-level and, thus, it has to be balanced out over the volume of the RVE. The yarns are crimped and the local fibre coordinate system does not coincide with the global one. Consequently, normal tensile stress along the fibres, being projected to the global coordinate system, results in the out-of plane shear stress.

The compensation mechanism of the projected tensile stress is fully determined by the lay-up. In the case of periodic lay-up, the compensation function is delegated to the inter-yarn space: matrix and transverse yarns, while in the loaded yarns the stress state is close to the uniaxial – Figure 4. On the opposite, in the symmetric lay-up, the symmetry imposes the zero shear stress on the ply boundaries and, hence, the shear compensation function is imposed on the
loaded yarns: there will be local shear in the fibre coordinates. Being projected to the global coordinate system, this shear would balance out the action of the projected tensile stress.

![Figure 4](image)

**Figure 4.** Sketch explaining the difference between the deformation mechanisms for the periodic and symmetric configurations

As prescribed by the equilibrium equation, the local shear acting within the yarns of symmetric configuration is responsible for the higher local tensile stress there. Gradient of local intra-yarn shear through the yarn thickness results in higher stress gradient of the stress in the fibre direction. This is why the stress in the fibre direction is higher for the symmetric configuration than for the periodic one, while intra-yarns stress and delamination risks are higher for the periodic structure.

1.4 Issues to consider

There are several important consequences of the lay-up difference:

**Experiments**: It can be expected that the experimental scatter for the textile composites would be larger and more samples need to be tested to collect a representative set of data. Once it is done, the “worst” parameters of all the configurations will determine the practical limits of the composite use.

**Optimisation and perspectives**: There exist various ways to improve toughness of the matrix. The behaviour of polymer matrix is often determined by the type of applied load. For instance, the epoxy exhibits excellent behaviour in pure shear (60% of applied deformation), decent toughness in uniaxial load (4% of applied deformation), and extreme brittleness at 3D stress state (0.6% of applied deformation). Different lay-ups exhibit dramatic difference in deformation mechanisms, in particular location of the shear stress. Hence, the micro or nano optimisation of the structures should be done with respect to the meso-architecture.

**Modelling**: If meso modelling aims at making reasonable predictions of the failure, it has to incorporate analysis of the various composite configurations. It is computationally exhausting to construct the full scale models of laminate with arbitrary/random ply shift. An alternative modelling scheme is proposed in the following paragraph.

2. One unit cell modelling

2.1 Boundary conditions for periodic in-phase laminate

In the previous paper [7], it has been found out that the stress distributions in outer and inner layers are dramatically different even for the case of in-plane loading, i.e. the uniaxial tension.

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2 in comparison with flat UD laminated composites
The local stress and deformation turned out to be dependent on the number of plies. The BC for the one unit cell of the laminate with the periodic lay-up were proposed based on study of the displacement fields obtained in FE analysis.

It was found that the deformed shape, i.e. the deflection of the outer unit cell boundaries (1) is nearly the same for all unit cells in a laminate – it does not depend on the position of the unit cell, and (2) is nearly proportional to the periodic solution. The latter present a laminate with infinite number of periodically stacked plies modelled with known periodic BC. The coefficient of proportionality, named further as a scaling factor, depends on the number of plies.

It was proposed to handle the modelling of the laminate by means of adjusting BC for the unit cell of a single ply. These displacement conditions are applied on the unit cell surfaces parallel to the plane of the laminate. They are obtained on the base of boundary deflection calculated in periodic solution \( u_\infty \). The scaled up function \( \lambda u_\infty \) is set as the nodal displacement for the new boundary value problem. We distinguish two solutions: for the outer plies, where one of two boundaries is left free, and for the inner ply, where both of the surfaces are constrained:

**Upper (free) surface of the outer unit cell:**
\[
t_i = \sigma_{i} z = 0
\]
\[
u_x = u_x^\infty, \quad u_y = u_y^\infty
\]
\[
u_z = \lambda u_z^\infty
\]

**Bottom surface of the outer unit cell:**
\[
u_x = u_x^\infty, \quad u_y = u_y^\infty
\]
\[
u_z = \lambda u_z^\infty
\]

**Upper surface of the inner unit cell:**
\[
u_x = u_x^\infty, \quad u_y = u_y^\infty
\]
\[
u_z = \lambda u_z^\infty - \{e_{zz}\} h
\]

**Bottom surface of the inner unit cell:**
\[
u_x = u_x^\infty, \quad u_y = u_y^\infty
\]
\[
u_z = \lambda u_z^\infty - \{e_{zz}\} h
\]

It is assumed that the stress distribution in all the inner plies is the same. The scaling magnitude depends on the architecture, the number of plies in the laminate, and it is not known in advance. In order to find \( \lambda \), the energies of the effective and the heterogeneous media are equated. The energy equivalence of the heterogeneous and the effective media, known also as the Hill condition, suggests:

\[
\frac{1}{2}\langle e_{ij}\rangle\langle\sigma_{ij}\rangle = \frac{1}{2}\langle e_{ij}\sigma_{ij}\rangle = \frac{1}{2}\langle e_{ij}\sigma_{ij}\rangle
\]

where \( \langle \ldots \rangle \) - is the operator of averaging over the composite volume. To find the value of \( \lambda \), the map of the laminate deformation is synthesised from separately obtained solutions for surface/outer and inner plies. Then, the energy of deformation of the heterogeneous composite is presented as the sum of energies of 2 outer plies and N-2 inner plies.

\[
2\langle\sigma_{ij}\rangle_{outer}\langle e_{ij}\rangle_{outer} + (N - 2)\langle\sigma_{ij}\rangle_{inner}\langle e_{ij}\rangle_{inner} = 2\langle\sigma_{ij}e_{ij}\rangle_{outer} + (N - 2)\langle\sigma_{ij}e_{ij}\rangle_{inner}
\]

where the indexes ‘inner’ and ‘outer’ refer to the corresponding position of a ply in the laminate. The scaling coefficient is iterated until the global equilibrium is achieved.
2.2 The boundary conditions for the shifted laminates

The methodology of BC for the non-periodic laminate with the arbitrary ply configuration inherites from the scheme described above.

In the in-phase periodic structure the plies are the least constrained against the out-of-plane motion than for any other configurations: the plies deform in agreement with each other. Hence, it can be expected that the energy of the single ply deformation would be minimal for this in-phase configuration. On the opposite, the out-of-phase symmetric configuration has the zero deflection – the neighbouring plies cancel the free motion of each other. The staggered arbitrary configuration presents an intermediate case. It is assumed that every ply in the laminate tends to deform as close as possible to the shape of the periodic solution. Therefore, it is proposed to approximate the actual laminate deformed shape by averaging of the periodic displacement profiles. These profiles are considered to be a sort of basis functions. As far as the displacements are continuous every ply needs to adapt its own deformed shape to the shape of the other plies. That brings us to the concept of the average/effective out-of-plane motion of the laminate. For instance, it can be calculated by the simple arithmetic averaging of the basis profiles (Figure 5):

\[ \tilde{u}_z' (\tilde{x}) = \frac{1}{N} \sum_{i=1}^{N} \tilde{u}_z^i (\tilde{x} + \tilde{s}_i) \]  

where \( \tilde{s}_i \) are the shift of every ply from a regular periodic position.

![Figure 5](image.png)

**Figure 5.** Scheme explaining the superposition of the periodic profiles for the step-shift.

The results of the superposition and comparison of it with the actual displacement along the inter-layer boundary are presented in Figure 6. The ply deflections are much less uniform than for a laminate with the periodic ply sequence: the deflection shape is individual for all the plies. However, the average profile is perfectly proportional to the predicted/superimposed one for both the stacking sequences. To match the actual deformed shape, the energy based scaling procedure can be again employed. The proposed methodology is also valid for the symmetric configuration. Deflection given by the symmetry boundary conditions will be reproduced by equation (3).
Hence, the proposed methodology is valid for an arbitrary ply configuration. The boundary conditions for a single unit cell can be indeed adjusted instead of making multiple models of laminates. It should be noticed, that this procedure is expected to be less precise for staggered laminate than for the periodic in-phase one, for there is a high variation of the profiles from ply to ply. However, in average, the profile is predicted correctly. This promises that a good estimation of the internal stress can be obtained.

3. Results

The comparison between the full reference and one unit cell analysis is displayed in Table 2.

<table>
<thead>
<tr>
<th>Layer position</th>
<th>Outer</th>
<th>Outer</th>
<th>Outer</th>
<th>Inner</th>
<th>Inner</th>
<th>Inner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration</td>
<td>Step</td>
<td>Stairs</td>
<td>Periodic</td>
<td>Step</td>
<td>Stairs</td>
<td>Periodic</td>
</tr>
<tr>
<td>$\sigma_x$, warp, [MPa]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FFE</td>
<td>60.3–157.6</td>
<td>84.1–161.5</td>
<td>47.1–132.3</td>
<td>74.0–145.8</td>
<td>103.2–169.4</td>
<td>59.5–123.1</td>
</tr>
<tr>
<td>UCA</td>
<td>65.3–153.9</td>
<td>100.2–167.1</td>
<td>46.8–130.2</td>
<td>78.1–144.4</td>
<td>111.3–170.1</td>
<td>54.8–121.2</td>
</tr>
<tr>
<td>$\sigma_y$, weft, [MPa]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FFE</td>
<td>3.7 – 15.1</td>
<td>3.2 – 12.7</td>
<td>3.2 – 15.1</td>
<td>4.7 – 12.3</td>
<td>4.3 – 10.7</td>
<td>3.9 – 15.7</td>
</tr>
<tr>
<td>UCA</td>
<td>2.6 – 13.2</td>
<td>3.2 – 11.4</td>
<td>3.4 – 14.7</td>
<td>3.1 – 13.9</td>
<td>5.3 – 12.4</td>
<td>3.5 – 15.6</td>
</tr>
<tr>
<td>$\sigma_{yz}$, weft, [MPa]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FFE</td>
<td>-5.3–5.9</td>
<td>-3.5–3.6</td>
<td>-8.6–8.6</td>
<td>-6.0–6.0</td>
<td>-2.4–2.1</td>
<td>-9.5–9.5</td>
</tr>
<tr>
<td>UCA</td>
<td>-6.4–6.4</td>
<td>-2.8–2.8</td>
<td>-8.9–8.9</td>
<td>-6.6–6.6</td>
<td>-2.2–2.2</td>
<td>-9.7–9.7</td>
</tr>
</tbody>
</table>

Table 2. Stress range at 1% applied deformation (minimum–maximum values): x-fibre direction, y-cross-fibre direction, z-thickness direction; FFE – reference full finite element solution, UCA – unit cell analysis performed on 1 unit cell

The table shows the maximum and minimum value of both the outer and inner plies. The inner ply in the reference 6-ply solution is presented by the unit cell at the laminate midline, which means that in the case of the six ply laminate there are two plies separating the considered unit cell from the free surface. It was shown, that in the case of the arbitrary lay-up the individual displacement profiles are not identical for the different plies. However, fitting the average profile of the laminate turned to be sufficient to match the stress distribution and to be able to distinguish two principle cases: outer and inner.

Unlike the ideally periodic stacking sequence, the maximum stress concentration is not necessarily observed in the outer plies. For instance, for the configuration “stairs”, the
maximum tensile stress occurs in the inner plies. It is remarkable that the one unit cell solution feels these differences and provides with the reasonable predictions.

4. Conclusions

The proposed methodology gives a simple and yet efficient tool for modelling the laminate stress distribution in textile laminates. It accounts for (1) the ply shift; (2) the number of plies; (3) the outer and inner ply positions. The demonstrated verification of the single ply solutions insures that the proposed technique can be used for modelling the real stackings, i.e. realisations of the random architecture. Apart from stress predictions in shifted laminates, we foresee the perspectives for:

(1) Modelling of delaminations in textile laminate by means of BC. The displacement constraints can be released at the location where delamination is detected and scaling coefficient can be adjusted with respect to the remaining number of the plies. This concept will be presented further.
(2) Modelling interaction of textile laminates with a different orientation, where plies are oriented at an angle to each other. These structures do not have the representative volume as an element of periodicity. Modelling the interaction of the plies is not well defined problem.

The major challenges of this concept:
(1) There is no yet a closed form solution for finding the scaling coefficient.
(2) To make more precise estimation of the stress one needs to match the individual profiles for every laminate, which requires more conditions than a single condition of the global energy equilibrium.

Acknowledgements

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