ANALYSIS OF YARN BENDING BEHAVIOUR

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ABSTRACT

This paper addresses the bending behaviour of individual multi-filament yarns for use in textile forming analyses. Yarn specimens deform nonlinearly under their own weight in a cantilever configuration. A relation between the applied moment and the resulting curvature is established. Numerical simulations are performed to establish the contribution of shear rigidity to the deflection.

Keywords: Composite fabric, Textile modelling; Flexural rigidity; Tow mechanics

INTRODUCTION

Better understanding of the mechanical behaviour of single yarns is needed to improve the predictability of fibre bundle deformation and reorientation during forming processes of composite fabrics. Modelling the draping behaviour of textile fabrics has so far been focused primarily on macro and meso scale [1-4]. Yarn-yarn interactions have been thoroughly investigated, yet the intra-yarn behaviour is a less studied phenomenon. Here, we aim to improve the understanding of the bending characteristics of single, dry multi-filament yarns made of widely used technical fibres such as polyacrylonitrile (PAN) based carbon and E-glass.

Figure 1: Schematic drawing of the cantilever arrangement, in undeformed (a) and deformed state (b)

A preliminary experiment was set up to obtain a relation between the bending moment due to the own weight of a yarn specimen and its curvature. A cantilever setup based on Peirce’s cantilever testing procedure was chosen after consideration of several existing
measurement procedures used in the textile industry [5,6]. The specimen is uniformly loaded by its own weight (Figure 1). The positive Z-direction is upwards, a positive distributed load representing the weight of the yarn acts downwards.

This paper addresses the question whether the deflection of cantilever yarn specimens can be described by beam deflection models. The yarns are assumed to show only elastic deformation. A description of the displacement field of the beam is derived from a linear approach, and extended to a nonlinear beam deflection model, because the experiments showed highly nonlinear bending behaviour. The geometrical nonlinearities occur in the plane of bending, the XZ-plane, in the form of large deformations, and in the cross-sectional shape of the yarn, in the YZ-plane, in the form of flattening. The relations between displacement, strains, and curvature are affected, as well as the relation between the distributed load, the rotation, and the moment distribution.

The deflection of a multi-filament yarn is expected to not only depend on flexural rigidity, but also on shear resistance. One of the discussed beam theories takes the contribution of deformation due to shear forces into account.

An estimation of the contribution of shear to the total deformation is made by means of numerical simulations of a single filament that deflects under its own weight in a cantilever configuration.

**BENDING THEORIES OF BEAMS**

Two beam theories to model the bending behaviour of the yarn specimens are compared: the Euler-Bernoulli Beam Theory (EBT) and the Timoshenko Beam Theory (TBT). The Timoshenko Beam Theory takes the contribution of both shear and flexural rigidity to the deformation of a beam into account, where the Euler-Bernoulli Beam Theory only covers the contribution of flexural rigidity. In the linear case, this contribution is superimposed on the deflection according to the Euler-Bernoulli Beam Theory [7,8].

**Euler-Bernoulli Beam Theory**

The EBT describes the curvature of a beam subject to a bending load as a function of the applied moment $M$ and the flexural rigidity $EI$. The curvature $\kappa$ is equal to the inverse of the radius of curvature $\rho$, and is described as the second derivative of the displacement with respect to the horizontal coordinate $x$, under the assumption of small deflection,

$$\kappa = \frac{1}{\rho} = \frac{M}{EI} = \frac{d^2w}{dx^2}. \quad (1)$$

Eq(1) can be solved for deflection $w(x)$ in terms of a uniformly distributed load $q$ with the following boundary conditions:

$$\frac{dw}{dx}\big|_0 = 0 \quad \text{and} \quad w(0) = 0 \quad (2)$$
This yields the classical deflection equation for small displacements of a cantilever beam with free length (or bending length) $L$ [9]:

$$
\frac{w(x)}{L} = -\frac{qL^3}{24EI} \left( 6 \frac{x^2}{L^2} - 4 \frac{x^3}{L^3} + \frac{x^4}{L^4} \right)
$$

(3)

A downward deflection is negative, as a result of the positive upward $Z$-axis.

**Timoshenko Beam Theory**

The TBT model takes into account the shear resistance $k_s A_{yz} G$ in the beam deflection calculation. This resistance is a product of the shear correction factor $k_s$, the yarn cross-sectional area $A_{yz}$, and isotropic shear modulus $G$. The shear correction factor is introduced to account for the non-uniform shear stress distribution in the beam cross-section, whereas the average shear stress is taken as a reference; $k_s$ depends on geometrical properties [10,11]. In the linear approach the shear contribution to the beam deflection is superimposed on the deflection related to the flexural rigidity $EI$, with the curvature of the beam modelled as the second derivative of the displacement with respect to the horizontal coordinate $x$. Without further derivations, the deflection equation for small displacements of a cantilever beam is [9]:

$$
\frac{w(x)}{L} = -\frac{qL^3}{24EI} \left( 6 \frac{x^2}{L^2} - 4 \frac{x^3}{L^3} + \frac{x^4}{L^4} \right) + 12\Omega \frac{x}{L} \left( 2 - \frac{x}{L} \right)
$$

(4)

with the dimensionless constant

$$
\Omega = \frac{EI}{k_s A_{yz} GL^2}.
$$

(5)

**Nonlinear approach of EBT and TBT deflection**

The large deflection of a multi-filament yarn specimen needs a nonlinear modelling approach, because the differential equation presented in above paragraphs, eq(1), is a linearised solution of the EBT and TBT models. Eq(6) presents the nonlinear differential displacement-curvature equation. The rightmost part of the equation contains the unknown term to describe the deflection as a function of shear and flexural rigidities and an applied load.

$$
\kappa = \frac{1}{\rho} = \frac{d^2w}{dx^2} = f(M, EI, k_s, A_{yz})
$$

(6)

This differential equation cannot be solved analytically, yet a numerical solution of the deflection can be obtained. This is achieved by means of finite element simulations in a standard FEM package; the procedure is described in the next section.

**NUMERICAL SIMULATIONS**

Numerical calculations were performed to estimate the contribution of shear resistance and flexural rigidity to the yarn deflection. Calculations were performed on single filaments to provide an upper bound deflection, that is, a multi-filament yarn will not
deflect further than a single filament of the average diameter of approximately 15 μm. However, a filament diameter of 10 μm is used for the simulations of the upper bound deflection, because this was the smallest filament diameter found in the yarn specimens that were used in the experiments.

A lower bound deflection is formed by the deflection of a homogeneous rod with the yarn dimensions and shape. The deflections of the rod are very small compared to the multi-filament yarn deflection. Heterogeneity of the multi-filament yarn causes a lower shear resistance than that for the homogeneous rod of the same material. Therefore, the lower bound deflection can be shifted closer to the actual yarn deflection by varying the shear modulus $G$. The effect of the modification is simulated for a single filament. The results show the order of magnitude of $G$ required to shift the lower bound deflection. The Young’s modulus is modified to cover the sensitivity of the shear modulus variation for different flexural rigidities.

Table 1 shows the modelling parameters; the values of the shear and Young’s modulus, $G$ and $E$ respectively, are varied in the simulations. The Young’s modulus is multiplied with the factors: $1\times10^{-2}$, $1\times10^{-1}$, 1, 2, and 5. The shear modulus stated in (Table 1) is multiplied by the factors: $1\times10^{-6}$…$1\times10^{0}$ with intervals in powers of 10. The simulations were performed with elastic 2-D beam elements, suitable for large deflections.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Quantity</th>
<th>Unit</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filament length</td>
<td>100</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>Filament diameter</td>
<td>10</td>
<td>μm</td>
<td></td>
</tr>
<tr>
<td>Shear correction factor $k_s$</td>
<td>0.86</td>
<td>-</td>
<td>Circular cross-section</td>
</tr>
<tr>
<td>Shear modulus $G_{12}$</td>
<td>28.7</td>
<td>GPa</td>
<td>Isotropic, $\nu_{12} = 0.22$</td>
</tr>
<tr>
<td>Young’s modulus $E$</td>
<td>70</td>
<td>GPa</td>
<td></td>
</tr>
</tbody>
</table>

**EXPERIMENTS**

An experiment was set up to establish a relation between a known applied load on a multi-filament yarn specimen and the unknown geometrical properties, i.e. the curvature of the yarn in the plane of bending and its cross-sectional shape. A cantilever setup was used as mentioned in the introduction of this paper. Images of the deflected yarn specimen were produced from the plane of bending, the $XZ$-plane. Additional images were produced from above the specimen, the $XY$-plane, to capture the yarn width.

The involved geometrical parameters are: yarn specimen bending length $L$, cross-sectional geometry $A$, which is represented by an ellipse with height $2b$ and width $2a$, tip deflection angle $\theta$, and vertical tip deflection $w$ (see Figure 1). The experiments were conducted with E-glass yarn material of which the specifications are stated in Table 2. Four different sample lengths $L$ were used: 100, 125, 150 and 175 mm (± 0.5 mm). Each sample consisted of ten specimens.
### Table 2: E-glass yarn properties

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Quantity</th>
<th>Unit</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>584</td>
<td>g/1000 m (tex)</td>
<td></td>
</tr>
<tr>
<td>Sizing</td>
<td>1</td>
<td>% mass</td>
<td>Silane-based, aqueous</td>
</tr>
<tr>
<td>Filament diameter</td>
<td>15.3</td>
<td>μm</td>
<td>σ = 1 μm (from SEM data)</td>
</tr>
<tr>
<td>Nr of filaments</td>
<td>1100</td>
<td>-</td>
<td>Calculated approximation</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>73</td>
<td>GPa</td>
<td></td>
</tr>
</tbody>
</table>

### RESULTS AND DISCUSSION

#### Numerical simulation results

In total, 35 simulations were performed from all the combinations of the shear and elasticity moduli values. Deflection plots of the filament simulations are shown as an example in Figure 2 (b) for the different shear modulus values with a constant elastic modulus of 70 and 140 GPa respectively (the latter is purely hypothetical). Figure 2 (c) shows a surface plot of the dimensionless deflection of a typical E-glass filament with constant cross-section/length ratio, and with varying shear modulus $G_{12}$ and Young’s modulus $E$ for all 35 simulations. The grey bullets indicate the tip deflection of the above plotted full deflection curves with constant Young’s moduli. The shear rigidity and flexural rigidity are influenced by varying $G_{12}$ and $E$ respectively. The surface plot of nonlinear calculations shows that the shear rigidity has a significant contribution to the filament deflection starting from an order of magnitude of $10^{-5}$ of the nominal shear modulus ($G_{12} = 28.69$ GPa) and lower.

This rough estimate shows that the influence of shear deformation on the total filament deflection is small compared to the deflection due to bending deformation. It is therefore acceptable to assume that the change in cross-sectional shape is mainly related to the flexural rigidity $EI$, the influence of which is taken into account in the nonlinear EBT model described by eq(6). Further calculations on single filaments can therefore be performed with above mentioned equation without drastic reduction of accuracy. The lower bound deflection represented by the homogeneous rod approach would need a very low shear modulus as well, in order to significantly increase the lower bound deflection. Therefore, the shear modulus is not expected to account for the large difference in deflection between the homogeneous rod and the multi-filament yarn. Note that the assumption is made that the cross section of the filament perpendicular to the neutral axis remains plane by neglecting the shear contribution to the deformation.

#### Experimental results

The curvature of the deflected yarn specimens can be extracted from the image data. Figure 3 shows an example of the deflected specimens. The image data is processed in Optimas, a standard image analysis software package (1995, Meyer Instruments, Inc.). The edges of the deflected yarn specimen were determined with a gradient filter based on a 256 bit grey scale, in a region of interest around the clamped region of the
specimen (Figure 4), measuring 20 mm in width and 30 mm in height. Flattening of the yarn and changes in curvature are largest in this region.

Figure 2: Upper bound deflection simulations (a); simulated filament deformation for various shear moduli (b) and surface plot of dimensionless tip deflections (c). The grey dots in (c) represent the tip deflections of the curves in (b)
A polynomial fit of the form of eq(9) through edge defining points was produced to establish a displacement field $w(x)$. From each image, a bottom and top curve fit were obtained. Table 3 shows the mean fit coefficients and the variance for all four samples.

$$w(x) = C_4 x^4 + C_2 x^2 + C_3$$  \hspace{1cm} (9)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Characteristic</th>
<th>$C_{b1}$</th>
<th>$C_{b2}$</th>
<th>$C_{b3}$</th>
<th>$C_{t1}$</th>
<th>$C_{t2}$</th>
<th>$C_{t3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 mm</td>
<td>Mean</td>
<td>-2.07e-7</td>
<td>-2.60e-2</td>
<td>-1.01</td>
<td>3.85e-7</td>
<td>-2.45e-2</td>
<td>-3.20e-1</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>2.22e-11</td>
<td>4.58e-5</td>
<td>2.11e-2</td>
<td>1.49e-11</td>
<td>3.97e-5</td>
<td>9.10e-3</td>
</tr>
<tr>
<td>125 mm</td>
<td>Mean</td>
<td>-4.09e-6</td>
<td>-3.55e-2</td>
<td>-1.14</td>
<td>-3.98e-6</td>
<td>-3.37e-2</td>
<td>-4.59e-1</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>1.05e-10</td>
<td>4.79e-5</td>
<td>8.64e-2</td>
<td>7.78e-11</td>
<td>4.49e-5</td>
<td>6.78e-2</td>
</tr>
<tr>
<td>150 mm</td>
<td>Mean</td>
<td>-1.50e-5</td>
<td>-3.13e-2</td>
<td>-1.20</td>
<td>-8.51e-6</td>
<td>-3.03e-2</td>
<td>-4.55e-1</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>1.06e-10</td>
<td>1.89e-5</td>
<td>5.37e-2</td>
<td>5.28e-11</td>
<td>1.41e-5</td>
<td>4.50e-2</td>
</tr>
<tr>
<td>175 mm</td>
<td>Mean</td>
<td>-5.46e-6</td>
<td>-3.35e-2</td>
<td>-1.53</td>
<td>-4.24e-5</td>
<td>-3.23e-2</td>
<td>-7.65e-1</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>1.61e-9</td>
<td>7.91e-5</td>
<td>1.11e-1</td>
<td>7.89e-10</td>
<td>6.92e-5</td>
<td>7.60e-2</td>
</tr>
</tbody>
</table>

Subscripts: $b =$ bottom edge curve; $t =$ top edge curve

Figure 5 presents dimensionless deflections for the four sample lengths. The deflections from the experiment are compared to the simulated single filament deflection at a
horizontal distance of 20 mm from the clamping edge, this corresponds to the right edge of the ROI. A filament diameter of 15 μm was used as this is the average filament diameter in the yarn specimens; other properties are equal to those presented in Table 1. The experimental results show no clear deflection trend, whereas the simulated dimensionless single filament deflection increases cubically. This can be explained by the approximately cubic contribution of the length to the beam deflection.

![Figure 5: Comparison of dimensionless deflections at 20 mm horizontal distance from the clamping edge (right edge of ROI)](image)

**Yarn cross-sectional shape**

A variation in the cross-sectional shape of the single deflecting yarn was observed during the experiment. The changes included flattening of the yarn which affects the yarn flexural rigidity $EI$; however, the sample sizes in the conducted experiment were too small to derive a trend. Figure 6 shows two examples of flattened yarn specimens near the clamping region. The metal visible on the left of the images is the edge of the clamping region.

![Figure 6: Flattening of yarn specimens near the clamped region](image)

The influence of flattening on the flexural rigidity of the yarn is limited. Suppose the yarn flattens to half of its original height, the flexural rigidity would then only lower by a factor of eight (for an elliptical cross-section and conservatively assuming the yarn width remains constant). This is not enough to account for the large difference in
deflection between the lower bound deflection and the observed deflection of the multi-filament yarn.

**EBT-model applicability**

The results from preliminary experiments show that the nonlinear Euler-Bernoulli beam equation can be used to adequately describe the large deflection of a single E-glass filament. For multi-filament yarns, however, the experiments show a different stiffness than can be expected from the continuum based Euler-Bernoulli equation; this is due to hitherto uninvestigated mechanisms, which occur between individual filaments, on the micro scale. A possible explanation for the discrepancy between the deflection calculations based on the Euler-Bernoulli beam theory and the observations from experimental work lies in the heterogeneous nature of the yarn. Individual filaments migrate within the yarn and surface-to-surface friction occurs between the filaments. These effects were not taken into account in the EBT model. The results from the performed experiments did not yield enough information to model the change in cross-sectional shape of multi-filament yarns. Further experiments should give more data concerning the deformed yarn geometry.

**Intra-yarn friction**

Friction between individual filaments in a yarn is caused by different mechanisms. The filaments on the outer perimeter of the yarn interact with filaments from other yarns due to undulation in fabrics. Filaments within the yarns are often entangled as a result of fibre migration and length differences due to deformations. A fibre sizing is often applied during production to protect the yarn material and/or improve fibre-matrix bonding. It is another factor that plays a role in the intra- as well as inter-yarn friction behaviour. The effects mentioned above all have influence on the deflection behaviour of yarns based on friction mechanisms. An investigation of these effects is beyond the scope of this article, yet it is useful to consider the matter in future work.

**FEM calculations**

Nonlinear finite element simulations of single filament deflection have been performed and showed that shear effects do not have a significant contribution to the total deflection. The contribution of shear rigidity to the filament deflection was compared to the deflection related to flexural rigidity. The simulations showed that the influence was several orders of magnitude smaller, and therefore the shear term is negligible for a single filament. However, it is useful to assess the shear influence once again for further simulations, including multi-filament yarn deflection calculation. An analysis of the friction mechanism between the individual filaments is important as well. The yarns are expected to show non-elastic behaviour due to friction between individual filaments.

**CONCLUSIONS**

The bending behaviour of multi-filament yarns has been investigated to better describe the effects of both material and geometrical properties. A survey of previously
performed research showed that modelling of fabric behaviour is a mature field of
technology; however, less research has been performed on the meso-micro scale.
Experiments to establish the relation between load and deflection of multi-filament
yarns showed that the bending behaviour is strongly nonlinear. Upper and lower bound
simulations showed that the shear influence on the deflection is negligible. The
discussed beam theories are not suitable to adequately model the deflection of multi-
filament yarns. It can be concluded that more and other mechanisms than those
accounted for in the present model contribute significantly to the deflection.
An investigation of the shear and friction effects in bending yarns by means of further
experiments is recommended to establish the micro-mechanical behaviour. An
extension of the deflection models is subject of further research. Irreversible effects like
plastic deformation and friction mechanisms can be investigated and implemented in the
present purely elastic models.

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