Transient Behaviour in Highly Dependable Markovian Systems: New Regimes, Multiple Paths

— Extended Abstract RESIM 2010 —

Daniël Reijsbergen Pieter-Tjerk de Boer Werner Scheinhardt reijsbergendp@ewi.utwente.nl ptdeboer@cs.utwente.nl w.r.w.scheinhardt@utwente.nl

In recent years, probabilistic analysis of highly dependable Markovian systems has received considerable attention. Such systems typically consist of several component types, subject to failures, with spare components for replacement while repair is taking place. System failure occurs when all (spare) components of one or several types have failed. In this work we try to estimate the probability of system failure before some fixed time bound $\tau$ via stochastic simulation. Obviously, in a highly dependable system, system failure is a rare event, so we apply importance sampling (IS) techniques, based on knowledge of the behaviour of the system and the way the rare event occurs.

Interestingly, we can discern quite a few different situations to explain why system failure is rare, each with its own typical way of how the rare event is reached, namely: (1) low component failure rates, (2) small value of $\tau$, (3) many spare components and (4) high component repair rates. Each of these can be considered as a limiting regime in which some model parameter tends to 0 or infinity. Classifying this parameter as the ‘rarity parameter’, we can measure the performance of an IS scheme by how well it does in the asymptote involved. We could also combine regimes, which sometimes leads to new cases and sometimes not (e.g. the limit in which both failure and repair rates become small is equivalent to $\tau$ becoming small).

For cases (1) and (2), a combination of balanced failure biasing and forcing was proven to have bounded relative error in [2]. In [1] an alternative estimator was proposed, based on the dominant path to failure, the idea being that when an event is rare, deviations from the most likely path to this event become even more rare. However, in several model checking problems an analysis based on dominant paths fails to identify a well-performing change of measure. The reason is that the contribution of some other paths to the probability of interest is too large to neglect, or, more formally speaking, that the contribution of these paths does not vanish asymptotically.

In our paper, we first prove that in the asymptote of case (3), which is interesting in its own right, the dominant path to failure indeed does determine the entire rare event, as in cases (1) and (2). Then we demonstrate that this is not true for case (4). We propose a state- and time-dependent change of measure for a simple, yet nontrivial, model. Our measure is based on the one in [1] and takes all paths into account that contribute to the probability of interest. Finally, we empirically verify that our estimators have good performance.

References
