INTRODUCTION

One of the first analyses of leakage flow in single screw extruders was performed by Rowell and Finlayson, who considered the leakage flow to be a pressure flow only. They approximated the annular space between the flight tip and the barrel as two parallel plates. Their expression for the leakage flow can be written as follows:

$$\dot{V}_l = \frac{\pi D \delta^3 \Delta P \varepsilon_f}{12 \mu_b}$$  \hspace{1cm} (1)

where $\Delta P$ is the pressure drop per turn, $\varepsilon_f$ is an eccentricity factor; when the screw is centered, $\varepsilon_f = 1$, and, when the screw is in contact with the barrel, $\varepsilon_f = 2.5$. The same approach was used later by Carley and Strub. This approach is flawed by the fact that it neglects the drag flow contribution to the leakage flow, as well as the pressure flow due to the cross channel pressure gradient. These two contributions will often be larger than the leakage flow described by Eq. (1).

A much improved analysis of leakage flow was developed by Mohr and Mallouk, including the drag flow contribution and the pressure flow due to the cross channel pressure gradient. Booy carried out a modified analysis following the procedure developed by Mohr and Mallouk. A detailed description of the complete analysis for a Newtonian fluid was given by Tadmor, who writes the leakage flow as

$$\dot{V}_l = \frac{p W \delta \nu_{nr}}{2} + \frac{p w \delta^3 g_z}{12 \mu_f}$$

$$+ \frac{\delta^3 W p H^3 (W + w) g_z}{(\mu \delta^3 W + \mu_f H^3 w) 12}$$

$$+ \left[ \frac{6 \mu \nu_{nr} (H - \delta)}{H^3 g_z} + \frac{1 + w/W}{\tan^2 \phi} \right]$$  \hspace{1cm} (2)

It is well known that the flow behavior of polymer melts in screw extruders is strongly dependent on the pseudoplastic behavior of the melt. Therefore, it is to be expected that the same will be true for the leakage
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FIGURE 1
Coordinate system.

flow. In the following, an analysis of leakage flow will be presented that considers the polymer melt to be a power law fluid to allow an accurate evaluation of the degree of non-Newtonian behavior on the leakage flow characteristics.

FORMULATION OF THE PROBLEM

In order to realistically analyze the leakage flow in a single screw extruder, one should take into account the shear thinning behavior of the polymer melt and the two major flow directions in the flight clearance, the down channel direction z and cross-channel direction x (see Figure 1). The problem, therefore, will be treated as a 2-dimensional flow problem; velocity components in normal direction will be neglected. The flow behavior of the polymer melt will be described by the power law equation, which, for this 2-dimensional problem, can be written as

\[ \tau = m \left[ \left( \frac{dv_x}{dy} \right)^2 + \left( \frac{dv_z}{dy} \right)^2 \right] \left( \frac{dv_x}{dy} \right) \left( \frac{dv_y}{dy} \right) \]  

(3)

It is assumed, at this point, that temperature nonuniformities do not affect the flow behavior. Thus, the polymer melt is described as a temperature-independent power law fluid. The validity of this assumption is questionable because, in the clearance, the polymer melt is typically exposed to very high shear rates. As a result, there will be a significant amount of viscous heat generation, which can quite considerably affect the flow process. In a later section, therefore, this assumption will be relaxed, and the effect of developing temperatures will be analyzed. It is further assumed that the fluid adheres to the wall of the screw and the barrel, i.e., no slippage. If it assumed that inertia forces and body forces are negligible, the momentum balance in down channel direction z can be written as

\[ \frac{\partial P}{\partial z} = \frac{\partial \tau_{zz}}{\partial y} \]  

(4a)

Similarly, the momentum balance in cross channel direction x is

\[ \frac{\partial P}{\partial x} = \frac{\partial \tau_{xx}}{\partial y} \]  

(4b)

Both equations of motion are written in Cartesian coordinates. It is assumed, therefore, that the screw channel can be approximated by a flat rectangular channel with width W and depth H. Channel depth H is the distance between the root of the screw and the barrel; thus, it is the sum of flight height and radial flight clearance. The screw surface is considered to be stationary, while the barrel moves with velocity \( v_b \) relative to the screw.

This flow problem cannot be solved analytically, but requires numerical techniques. There are three unknowns for the screw channel: the cross channel pressure gradient, the shear stress at the root of the screw in z direction \( \tau_{zz} \) and in x direction \( \tau_{xx} \). Initial values of these unknowns can be guessed or calculated from expressions valid for a Newtonian fluid; the Newtonian problem has a rather straightforward analytical solution. The shear stress components \( \tau_x(y) \) and \( \tau_y(y) \) can be written as

\[ \tau_x(y) = \tau_{xx} + g_x y \]  

(5a)

\[ \tau_y(y) = \tau_{yy} + g_y y \]  

(5b)

where \( g_x \) is the down channel pressure gradient and \( g_y \) the cross-channel pressure gradient.

The actual shear stress \( \tau(y) \) is determined from

\[ \tau(y) = \sqrt{\tau_x^2(y) + \tau_y^2(y)} \]  

(6)

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The velocity gradient in $z$ and $x$ direction can be calculated from

$$\frac{dv_z}{dy} = \frac{\tau_{yz} r^2}{m^2}$$

(7a)

$$\frac{dv_x}{dy} = \frac{\tau_{yx} r^2}{m^2}$$

(7b)

The down channel and cross channel velocity profiles can be determined by numerical integration of Eqs. (7a) and (7b). The down channel flow rate $\dot{V}_z$ is determined by integration of the down channel velocity:

$$\dot{V}_z = W \int_0^H v_z(y) \ dy$$

(8a)

Similarly, the cross channel flow rate $\dot{V}_x$ is

$$\dot{V}_x = dz \int_0^H v_x(y) \ dy$$

(8b)

The correctness of the initial guesses for $g_x$, $\tau_{x0}$, and $\tau_{z0}$ can be evaluated by comparing the calculated down channel barrel velocity with the actual barrel velocity $v_{bz}$. Unless the initial guess was perfect, there will be a difference between the calculated and actual value. Thus, the divergence of $v_z(H)$ is

$$\text{div } v_z = \int_0^H \frac{dv_z}{dy} \ dy - v_{bz}$$

(9a)

Similarly, the divergence of $v_x(H)$ is

$$\text{div } v_x = \int_0^H \frac{dv_x}{dy} \ dy - v_{bx}$$

(9b)

If the leakage flow is neglected, the divergence of the cross channel flow rate is

$$\text{div } \dot{V}_x = dz \int_0^H v_x(y) \ dy$$

(10a)

However, if leakage through the flight clearance is taken into account, the divergence of $\dot{V}_x$ becomes

$$\text{div } \dot{V}_x = dz \int_0^H v_x(y) \ dy - \dot{V}_{xL}$$

(10b)

At this point, the cross channel leakage flow $\dot{V}_{xL}$ is not known; it has to be determined later by an iterative process. A first guess can be made by assuming that

the cross channel leakage flow equals the drag flow rate through the flight clearance:

$$\dot{V}_{xL} = \frac{1}{2} \eta \Delta v \ \delta \ dz$$

(11)

Now, in order to obtain more accurate values of $g_x$, $\tau_{x0}$, and $\tau_{z0}$, the value of $\tau_{z0}$ is changed by a small amount of $\delta \tau_{z0}$ and the velocities and flow rate are calculated again. This allows determination of the partial derivatives $\frac{\partial v_z}{\partial \tau_{z0}}$, $\frac{\partial v_x}{\partial \tau_{x0}}$, and $\frac{\partial \dot{V}_{xL}}{\partial \tau_{x0}}$. At this point, $\tau_{x0}$ returns to its original value and $\tau_{z0}$ is changed by a small amount $\delta \tau_{x0}$. Again, the velocities and flow rate are calculated, and the following partial derivatives are determined: $\frac{\partial v_z}{\partial \tau_{x0}}$, $\frac{\partial v_x}{\partial \tau_{z0}}$, and $\frac{\partial \dot{V}_{xL}}{\partial \tau_{z0}}$. The same procedure is repeated with $g_x$ to find $\frac{\partial v_z}{\partial g_x}$, $\frac{\partial v_x}{\partial g_x}$, and $\frac{\partial \dot{V}_{xL}}{\partial g_x}$. New values of $\tau_{x0}$, $\tau_{z0}$, and $g_x$ can be obtained by a Newton–Raphson scheme. The divergence values can be expressed as
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The values of $A_{x}$, $A_{T}$, and $A_{T_{o}}$ can be obtained by solving the three linear equations (12a), (12b), and (12c). At this point, new values of $g_{x}$, $T_{o}$, and $T_{o}$ can be determined, and the procedure is repeated until the divergence values are smaller than a certain tolerance value. The flow diagram of the calculations up to this point is shown in Figure 2. When the calculations of the velocity profiles and flow rates in the channel are complete, a similar analysis has to be performed for the flow in the flight clearance. The cross channel pressure gradient in the flight clearance $g_{x}$ can be calculated from the cross channel pressure gradient in the channel $g_{x}$ and the down channel pressure gradient in the channel $g_{x}$:

$$g_{x} = g_{x} - \frac{\pi(D - 2a)}{p_{w}} (g_{x} \cos \phi + g_{x} \sin \phi) \quad (13)$$

There are two unknowns for the flight clearance, the shear stress at the flight surface in $z$ direction $\tau_{x02}$ and in x direction $\tau_{x02}$. Initial values of these unknowns can be guessed or calculated from expressions valid for a Newtonian fluid. The shear stress components $\tau_{yx2}(y)$ and $\tau_{yx2}(y)$ in the flight clearance can be written as

$$\tau_{yx2}(y) = \tau_{x02} + g_{y}y \quad (14a)$$
$$\tau_{yx2}(y) = \tau_{x02} + g_{x}y \quad (14b)$$

The shear stress, velocity gradients, velocity profiles, and flow rates in the flight clearance can be determined the same way as was done for the screw channel [see Eqs. (6)−8)]. The divergence of $v_{x}(\theta)$ is

$$\text{div } v_{x}(\theta) = \int_{0}^{b} \frac{dv_{x}}{dy} dy - v_{ox} \quad (15a)$$

The divergence of $v_{x}(\theta)$ is

$$\text{div } v_{x}(\theta) = \int_{0}^{b} \frac{dv_{x}}{dy} dy - v_{bx} \quad (15b)$$

New values of $\tau_{x02}$ and $\tau_{x02}$ can be calculated by solving

$$\text{div } v_{x2} = \frac{\partial v_{x}}{\partial \tau_{x02}} \Delta T_{x02} + \frac{\partial v_{x}}{\partial \tau_{x02}} \Delta T_{x02} \quad (16a)$$
$$\text{div } v_{x2} = \frac{\partial v_{x}}{\partial \tau_{x02}} \Delta T_{x02} + \frac{\partial v_{x}}{\partial \tau_{x02}} \Delta T_{x02} \quad (16b)$$

When the divergence values are smaller than a certain tolerance value, the calculated cross channel flow rate in the flight clearance $V_{x2}$ is compared with the cross channel flow rate in the channel $V_{x}$. If the difference is larger than a certain tolerance value, the
program returns to the very beginning and calculates the velocity profiles and flow rates again. This time the leakage flow used to calculate the divergence of \( \mathbf{V} \) [Eq. (10b)] is the cross channel flow in the flight clearance \( V_{x2} \). This whole process is repeated until \( V_x - V_{x2} \) is less than a tolerance value. The flow diagram of the velocity calculations in the flight clearance are shown in Figure 3. The program converges rapidly; usually only three to five iterations are required to obtain the correct leakage flow. The actual flow rate is calculated as follows:

\[
\dot{V} = \dot{V}_x + \dot{V}_{x2} - \dot{V}_x' \pi(D - 2\delta) \cos \phi \tag{17}
\]

where \( \dot{V}_x' \) is the cross channel flow rate per unit distance. The dimensionless cross channel leakage flow rate is defined as

\[
\dot{V}^0 = 2\dot{V}_x'(v_{b0} \delta dz) \tag{18}
\]

The actual leakage flow is

\[
\dot{V}_l = \dot{V}_x' \pi(D - 2\delta) \cos \phi \tag{19}
\]

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\[
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\]

Results will be presented in commonly used dimensionless quantities. The dimensionless throughput is given by

\[
V^0 = 2\dot{V}_x/(WHv_{b0}) \tag{20}
\]

The dimensionless pressure gradient is given by

\[
g^0 = [g,H/6m][H/v_{b0}]n \tag{21}
\]

**FLOW RATE CHARACTERISTICS**

The dimensionless throughput vs. pressure gradient for a zero helix angle and flight clearance is shown in Figure 4 for five values of the power law index. The throughput decreases as the power law index reduces; the effect is very significant for strongly non-Newtonian materials, i.e., with power law index values of one half or less. When the helix angle obtains nonzero values, the throughput is reduced even further. Figure 5 shows the dimensionless throughput vs. pressure gradient for the commonly used square pitch helix angle, 17.67°, with a zero radial clearance for five values of the power law index. Interesting to note here is that the throughput at zero down channel pres-
FIGURE 5
Dimensionless throughput vs. pressure gradient for 17.67 helix angle and zero flight clearance.

FIGURE 6
Comparison numerical results to Eq. (22).

Sure gradient is less than the drag flow rate (dimensionless throughput less than unity) when the power law index is smaller than 1. This results from the fact that the cross channel flow affects the viscosity and thus the down channel flow rate. For this case, the relationship between throughput and pressure gradient can be approximated rather well with the following relationship:

\[ \dot{V} \approx \frac{4 + n}{5} \dot{V}_d - \frac{3}{1 + 2n} \dot{V}_p \]  

(22)

where \( \dot{V}_d \) is the drag flow rate,

\[ \dot{V}_d = \frac{1}{4} WH v_{bz} \]  

(22a)

and \( \dot{V}_p \) is the pressure flow rate,

\[ \dot{V}_p = WH^2 \frac{g_s}{12\mu} \]  

(22b)

with the viscosity evaluated at shear rate \( v_{bz}/H \):

\[ \mu = m(v_{bz}/H)^{n-1} \]  

(22c)

The fit between the numerical results and Eq. (22) is shown in Figure 6, where the solid lines represent the
numerical results and the straight dashed lines the result from Eq. (22). The fit is reasonably good down to a power law index of 0.4; at lower values, the nonlinearity becomes quite pronounced, and, as a result, the fit becomes rather poor.

The throughput vs. pressure gradient curves shown so far were for zero flight clearance. This is a situation that is of little practical use because in real life there must be a nonzero flight clearance. A typical value of the radial flight clearance used in the extruder industry is 0.1% of the screw diameter (δ = 0.001D). This rule is valid for extruders with a diameter of 50 mm (2 in.) or larger; for smaller extruders the radial flight clearance is usually about 0.05 mm (0.002 in.)
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independent of the actual diameter. This is shown in Figure 7.

To illustrate the effect of nonzero flight clearance, one can plot dimensionless throughput vs. pressure gradient for one value of the power law index for a number of values of the flight clearance. This is shown in Figure 8 for a Newtonian fluid for four values of the dimensionless radial clearance $\delta^0$. The dimensionless radial clearance is defined as the actual radial clearance divided by the standard radial clearance:

$$\delta^0 = \delta/0.001D = 1000 \delta/D \quad (23)$$

Each value of $\delta^0$ is represented by a curve. When $\delta^0 = 1$, the throughput is reduced about 2% at low values of the pressure gradient and about 4% at a dimensionless pressure gradient of about 0.5 compared to the case of zero flight clearance.

When $\delta^0$ is greater than 1, the throughput is further reduced. When $\delta^0 = 4$, the throughput at low pressure gradient is about 5% less than with the standard clearance ($\delta^0 = 1$), at medium values of the pressure gradient the throughput will be lowered by about 10%.

When the power law index is less than unity, the relationship between throughput and pressure gradient becomes nonlinear. This is shown in Figure 9 for a power law index of 0.7. The reduction in throughput with increasing clearance becomes larger when the power law index has a lower value. When $\delta^0 = 4$, the throughput at low pressure gradient is reduced by about 6% from the standard clearance. At higher values of the pressure gradient this percentage increases; at $g^p = 0.66$ it actually becomes 100%, i.e., the throughput is reduced to zero.

The same relationship for a power law index of 0.5 is shown in Figure 10. Again, an increasing reduction in throughput occurs when the flight clearance is increased. Figure 11 shows the relationship for a power law index of 0.3. When $\delta^0$ increases from 1 to 4 at zero pressure gradient, the throughput reduces about 7%. The maximum pressure generating capability is reduced by about 20%.

The leakage flow as a function of the pressure gradient for a power law index of 0.7 is shown in Figure 12. The dimensionless pressure gradient $g^p$ is defined by Eq. (21), and the dimensionless leakage flow $V_{xi}$ by Eq. (18). When $V_{xi}$ equals unity, the leakage flow is pure drag flow. When $V_{xi}$ is larger than unity, there is a contribution of the pressure flow aiding the drag component of the leakage flow. When the power law is 0.7, the leakage flow increases almost proportionally with pressure gradient. When the clearance increases, the contribution of pressure flow to the total leakage flow becomes more significant. At $g^p = 0.5$, the pressure flow is about 60% of the drag flow when $\delta^0 = 4$. When the power law index reduces, the contribution of pressure flow increases. This is shown in Figure 13 for a power law index of 0.5. At $g^p = 0.5$, the pressure flow is about 110% of the drag flow when $\delta^0 = 4$. Thus, in this case, the pressure flow contributes more to the leakage flow than the drag flow.

FIGURE 9
Dimensionless throughput vs. pressure gradient for a power law fluid ($n = 0.7$) at various clearance values.
For a power law index of 0.3, the situation is shown in Figure 14. At $g^0 = 0.5$ the pressure flow is about 300% of the drag flow when $\delta^0 = 4$. In this situation, therefore, the leakage flow is primarily determined by the pressure-induced leakage flow. As a result, calculations of leakage flow that assume pure drag flow through the flight clearance will become inaccurate at larger values of the pressure gradient and flight clearance and, particularly, at lower values of the power law index. The strong effect of the power law index is shown in Figure 15, where the leakage flow vs. pressure gradient is shown for three values of the power law index 0.3, 0.5, and 0.7 when $\delta^0 = 4$. It should be noted that the increase in actual leakage flow with $\delta^0$ is much larger than indicated by the dimensionless leakage flow in Figures 12–14. If the dimensionless leakage flow at $\delta^0 = 4$ is twice the value at $\delta^0 = 1$, the actual leakage flow will be eight times higher. This is because the dimensionless leakage flow is normalized with respect to the drag flow through the clearance, which is, of course, dependent on the flight clearance itself.
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FIGURE 12
Leakage flow vs. pressure gradient at $n = 0.7$.

FIGURE 13
Leakage flow vs. pressure gradient at $n = 0.5$. 
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FIGURE 14
Leakage flow vs. pressure gradient at $n = 0.3$.

FIGURE 15
Leakage flow vs. pressure gradient at three power law indices.
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VELOCITY PROFILES

The velocity profiles in the channel and flight clearance are shown in Figure 16. Coordinate $\xi$ is the dimensionless normal coordinate, $v_x^0(\xi)$ the down channel velocity in the channel, $v_x^0(\xi)$ the cross channel velocity in the channel, $v_x^0(\xi)$ the down channel velocity in the flight clearance and $v_x^0(\xi)$ the cross channel velocity in the flight clearance. All velocities are normalized such that the velocity at the barrel surface is always unity. Therefore,

\[
\begin{align*}
  v_x^0 &= \frac{v_x}{v_{bx}}, \\
  v_x^0 &= \frac{v_x}{v_{bx}}, \\
  v_x^0 &= \frac{v_x}{v_{bx}}, \\
  v_x^0 &= \frac{v_x}{v_{bx}}.
\end{align*}
\]

Figure 16 shows the velocity profiles for zero pressure gradient, $g_z^0 = 0$, power law index $n = 0.4$, and dimensionless clearance $\delta^0 = 1$. The $v_x^0$, $v_x^0$, $v_x^0$ profiles are almost linear. However, the $v_x^0$ is positive between $\xi = 1$ and about $\xi = 2/3$; between $\xi = 0$ and $\xi = 2/3$ the cross channel velocity is negative. The extreme value occurs at $\xi = 1/3$. When the pressure gradient is increased to $g_z^0 = 0.4$, the down channel velocity $v_x^0$ becomes distinctly nonlinear, while velocity $v_x^0$ remains almost linear. This is shown in Figure 17. Negative $v_x^0$ velocity components occur between $\xi = 0$ and $\xi = 0.5$. Velocities $v_x^0$ are now clearly increased beyond the drag flow velocity profile. Velocities $v_x^0$ are negative between $\xi = 0$ and $\xi = 0.73$, with the extreme $v_x^0$ values now being considerably smaller than for the case where $g_z^0 = 0$. When the pressure gradient is increased to $g_z^0 = 0.6$, the negative down channel velocity $v_x^0$ values increase even further. This is shown in Figure 18. Negative $v_x^0$ velocity components occur between $\xi = 0$ and $\xi = 0.78$. The cross channel velocities in the flight clearance $v_x^0$ are slightly increased over the case where $g_z^0 = .4$, while velocity profiles $v_x^0$ and $v_x^0$ have changed very little. Thus, with increasing pressure gradient, the most noticeable change occurs in the down channel velocity profiles.

When the clearance is increased to $\delta^0 = 2$, the cross channel velocities in the clearance $v_x^0$ show a substantial increase. This is shown in Figure 19 for the case where the power law index $n = 0.4$ and the dimensionless pressure gradient $g_z^0 = 0.4$. Compared to the case where $\delta^0 = 1$ (see Figure 17), velocity profiles $v_x^0(\xi)$, $v_x^0(\xi)$, and $v_x^0(\xi)$ show little change. When the flight clearance is further increased to $\delta^0 = 3$, velocities $v_x^0$ again show a substantial increase, (see Figure 20), while the other velocity profiles show little change. Thus, with increasing flight clearance, the

![Figure 16: Velocity profiles at $g_z^0 = 0$, $n = 0.4$, and $\delta^0 = 1$.](image)
most pronounced change occurs in the cross channel velocity profile in the flight clearance.

When the power law index is increased to \( n = 0.7 \), velocity profiles \( v_2^o(\xi) \) and \( v_2^a(\xi) \) become more linear. This is shown in Figure 21. Compared to the case where \( n = 0.4 \) (see Figure 17), the negative \( v_2^o \) velocities have increased with a more pronounced peak in the curve. The same trend continues when the power law index is further increased to \( n = 1 \), i.e., a Newtonian fluid. This is shown in Figure 22; in this case, velocity profiles \( v_2^o(\xi) \) and \( v_2^a(\xi) \) are essentially linear.
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FIGURE 19
Velocity profiles at
$g^2 = 0.4$, $n = 0.4$, and $\theta^o = 2$.

FIGURE 20
Velocity profiles at
$g^2 = 0.4$, $n = 0.4$, and $\theta^o = 3$. 
FIGURE 21
Velocity profiles at 
$g_2^* = 0.4$, $n = 0.7$, and $\delta^0 = 1$.

FIGURE 22
Velocity profiles at 
$g_2^* = 0.4$, $n = 1.0$, and $\delta^0 = 1$. 
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ENERGY CONSUMPTION

Once the velocity fields have been obtained, the stress fields can be calculated from the velocity fields. This, in turn, allows one to determine the energy or power consumption, both in the screw channel as well as in the flight clearance. The power consumption in the screw channel is obtained by integrating the down channel and cross channel shear stress over the width of the channel. Thus, the power consumption in the screw channel can be expressed as

\[
Z_{ch} = \frac{L}{\sin \phi} \int_0^W (\tau_{yz} v_{bx} + \tau_{yx} v_{by}) \, dx \tag{25}
\]

where the shear stresses have to be evaluated at the barrel surface. The down channel shear stress \(\tau_{yz}\) and cross channel shear stress \(\tau_{yx}\) can be obtained directly from the velocity profiles by using the constitutive relationship of the fluid. In this case, the power law equation is assumed to apply and the shear stresses can be determined from Eq. (3). For a Newtonian fluid Eq. (25) can be written as

\[
Z = (1 + 3r + 4 \tan^2 \phi) \frac{\mu L W v_{bx}^2}{H \sin \phi} \tag{26}
\]

where the throttle ratio \(r\) is the ratio of pressure flow to drag flow [see Eqs. (22a) and (22b)]:

\[
r = H^2 g / 6 \mu v_{bx} \tag{26'}
\]

The throttle ratio allows a compact expression for the volumetric flow rate of a Newtonian fluid in a screw pump. If the leakage flow is neglected, the flow rate can be written as

\[
\dot{V} = \frac{1}{2} WH v_{bx}(1 - r) \tag{27}
\]

Equations (26) and (27) allow a closed form expression of the specific energy consumption (SEC), this is the amount of energy consumed per unit mass or volume of the fluid. The SEC is important because it determines the temperature rise in the fluid from viscous dissipation, and it is a measure of the amount of mixing that the fluid has been subjected to. The SEC in the screw channel for a Newtonian fluid can be expressed as

\[
\dot{E}_{ch} = \frac{Z_{ch}}{\dot{V}} = \frac{(1 + 3r + 4 \tan^2 \phi) 2 \mu v_{bx} L}{(1 - r) H^2 \sin \phi} \tag{28}
\]

In the analysis of mixing not only the SEC can be important, but also the specific power consumption (SPC); this is the amount of power consumed per unit mass of volume of the fluid. It determines how fast or long it took to achieve a certain level of SEC. Therefore, it describes the intensity of the mixing process; this is important in the analysis of dispersive mixing where not only the total strain, but also the actual stress level plays an important role. The SPC in the screw channel for a Newtonian fluid can be expressed as

\[
\dot{Z}_{ch} = Z_{ch}/\dot{V} = \frac{(1 + 3r + 4 \tan^2 \phi) \mu v_{bx} L}{(1 - r) H^2 \sin \phi} \tag{29}
\]

To determine the energy efficiency of pressure generation, the SEC can be compared to the theoretical energy required to develop pressure. The theoretical power requirement to develop pressure \(\Delta P\) is

\[
Z_{th} = \dot{V} \Delta P \tag{30}
\]

Thus, the specific theoretical energy required to develop \(\Delta P\) is

\[
\dot{E}_{th} = \Delta P \tag{31}
\]

This quantity will be used to assess the energy efficiency of the pumping process.

ENERGY EFFICIENCY

The energy efficiency in the screw channel for pressure development can be expressed as

\[
\varepsilon_{ch} = \frac{\dot{E}_{th}}{\dot{E}_{ch}} \tag{32}
\]

For a Newtonian fluid \(\varepsilon_{ch}\) can be expressed as

\[
\varepsilon_{ch} = \frac{(3r - 3r^2)(1 + 3r + 4 \tan^2 \phi)}{(1 + 3r + 4 \tan^2 \phi)} \tag{33}
\]

For a Newtonian fluid \(\varepsilon_{ch}\) is only a function of the throttle ratio \(r\) and the helix angle \(\phi\). The optimum throttle ratio can be obtained by setting

\[
\frac{d\varepsilon_{ch}}{dr} = 0 \tag{34}
\]

This yields the following expression for the optimum throttle ratio:
The maximum value of $\varepsilon_{ch}$ occurs when the helix angle is zero; in this case $r^* = 1/3$ and $\varepsilon_{ch} = 33.3\%$. At higher values of the helix angle, the optimum throttle ratio increases, and the energy efficiency in the channel decreases. This is shown in Figure 23.

In reality, however, energy is not only consumed in the screw channel, but also in the flight clearance. In fact, the energy consumed in the flight clearance can be a major fraction of the total energy consumption. This is particularly true for fluids that are only weakly shear-thinning. The total energy efficiency for pressure generation can be expressed as follows:

$$r^* = -\frac{1 + 4 \tan^2 \phi}{3} \pm \frac{2}{3} \left( 1 + 5 \tan^2 \phi + 4 \tan^4 \phi \right)^{1/2} \tag{35}$$

The second term in the denominator $\hat{E}_{el}$ is the SEC in the flight clearance. Unless $\hat{E}_{el} = 0$, the energy consumption in the clearance will reduce the total energy efficiency compared to the energy efficiency in the channel. This reduction is quite dramatic, as is shown in Figure 24, comparing $\varepsilon$ and $\varepsilon_{ch}$ for a power
FIGURE 25
Comparison of numerical results to Eq. (40).

FIGURE 26
Power consumption in clearance vs. pressure gradient for \( n = 0.4 \).
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law index of 0.7 as a function of the dimensionless down channel pressure gradient.

The specific energy consumption in the flight clearance is the ratio of the power consumption in the clearance divided by the flow rate. It can written as

\[ \dot{E}_{cl} = \frac{Z_{cl}}{V} \]

(37)

The power consumption can be made dimensionless by dividing it by the Couette power consumption. The dimensionless power consumption in the channel then becomes

\[ Z_{ch}^0 = \frac{Z_{ch}}{Z_c} \]

(38)

where \( Z_c \) is the Couette power consumption in the channel in the case of tangential flow with a linear velocity profile. It can be expressed as

\[ Z_c = m \left( \frac{v_b}{H} \right)^{n+1} \frac{HW \cdot \frac{L}{\sin \phi}}{\sin \phi} \]

(39)

An approximate relationship for \( Z_{ch}^0 \) was proposed by Karian, it can be written as follows:

\[ Z_{ch}^0 = 3n + 1 - 3n \frac{\dot{V}^0 \cos^2 \phi}{\sin \phi} \]

(40)

The results of Eq. (40) are compared to results from numerical calculations in Figure 25 for a helix angle of 20°. The error in using Eq. (40) is quite small, in most cases less than 3%. This is true over a wide range of helix angles, from 0 to at least 60°. With the flow rate relationship expressed by Eq. (22), the dimensionless power consumption in the channel can be written as

\[ Z_{ch}^0 = 3n + 1 - 3n \cos^2 \phi \left( \frac{4 + n}{5} - \frac{g_s H^{n+1}}{(2 + 4n)mv_b} \right) \]

(41)

The actual power consumption in the channel can be expressed as

\[ Z_{ch} = \frac{mv_b^{n+1} WL}{H^n \sin \phi} \left[ 3n + 1 - 3n \cos^2 \phi \left( \frac{4 + n}{5} - \frac{g_s H^{n+1}}{(2 + 4n)mv_b} \right) \right] \]

(42)

Equation (42) allows a relatively accurate determination of the power consumption in the channel without using numerical analyses to obtain results. If it is
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FIGURE 29
Power consumption in clearance vs. clearance at various power law indices.

FIGURE 30
Energy efficiency in the channel vs. pressure gradient at various values of the radial clearance.
assumed that the velocity profile in the clearance is dominated by drag flow, the power consumption in the clearance can be expressed as

$$ Z_{el} = \frac{\mu_{el}wL\nu_0}{\delta \sin \phi} \quad (43) $$

However, it was shown earlier that the velocity profiles in the flight clearance can deviate considerably from the drag flow velocity profile. The power consumption in the clearance can be made dimensionless as follows:

$$ Z_{el0} = Z_{el}Z_{ce} \quad (44) $$

The Couette power consumption in the clearance $Z_{ce}$ is the power consumption in the clearance in case of tangential flow with a linear velocity profile; it can be expressed as

$$ Z_{ce} = m \left( \frac{\nu_0}{\delta} \right)^{n+1} \frac{\delta wL}{\sin \phi} \quad (45) $$

The dimensionless power consumption in the clearance as a function of the dimensionless pressure gradient for a power law index $n = 0.4$ is shown in Figure 26. It can be seen that there is approximately a linear relationship between $Z_{el0}$ and $g^0$. The dimensionless power consumption in the clearance is less than unity for positive pressure gradients. This is due to the pressure flow in the flight clearance, which increases the leakage flow, but at the same time, reduces the power consumption in the clearance. Figure 27 shows the power consumption vs. pressure gradient for a power law index $n = 0.7$. Again, there is an

![Figure 31](image-url)

Maximum energy efficiency in channel vs. radial clearance at various power law indices.
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approximately linear relationship between \(Z^0_0\) and \(g^0_2\). Figure 28 shows the relationship for a Newtonian fluid. The relationship between \(Z^0_0\) and \(g^0_2\) for different values of the power law index can be approximated as follows:

\[
Z^0_0 = 1 - (g^0_2 + g^0_0) a (\delta^0)^b \tag{46}
\]

where

\[
g^0_0 = 0.035 + 0.065n
\]

\[
a = 0.256n^2 - 0.488n + 0.238
\]

\[
b = 1.20 + 0.85n
\]

The actual power consumption in the flight clearance can now be expressed as

\[
Z_{cl} = m \left( \frac{V_b}{\delta} \right)^{n+1} \frac{\delta w L}{\sin \phi} [1 - (g^0_2 + g^0_0) a (\delta^0)^b] \tag{47}
\]

With Eqs. (42) and (47) the total power consumption can be determined directly without using numerical analyses to obtain results. This total power consumption is obtained by simple addition:

\[
Z = Z_{ch} + Z_{cl} \tag{48}
\]

When the clearance increases, the dimensionless power consumption in the clearance reduces. This is shown in Figure 29 for four values of the power law index and at a constant dimensionless pressure gradient \((\delta^0 = 0.3)\). As the power law index becomes smaller, the reduction in power consumption becomes more significant.

The energy efficiency in the channel initially increases with the dimensionless pressure gradient, then reaches a maximum, and then reduces with further increases in pressure gradient. This is shown in Figure 30 for a power law index of 0.4 and 5 values of the flight clearance. It can be seen in Figure 30 that the maximum value of the energy efficiency in the channel

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FIGURE 32
Maximum energy efficiency in channel versus power law index.
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**FIGURE 33**
Total energy efficiency vs. pressure gradient for \( n = 0.3 \).

**FIGURE 34**
Total energy efficiency vs. pressure gradient for \( n = 0.5 \).
FIGURE 35
Maximum total energy efficiency vs. radial clearance for various power law indices.

FIGURE 36
Optimum dimensionless clearance vs. power law index.
reduces with radial clearance. The same is true for other values of the power law index as shown in Figure 31. It is clear from Figure 31 that the maximum energy efficiency in the channel increases strongly with power law index. This is shown in more detail in Figure 32 where the maximum channel energy efficiency is plotted directly against the power law index.

When the total energy efficiency is considered, i.e., when the energy consumption in the clearance is included, a considerably different picture emerges. When the total energy efficiency is plotted vs. the pressure gradient, as shown in Figure 33 for power law index \( n = 0.3 \), the efficiency first increases, then reaches a maximum, and then decreases with the pressure gradient. However, the maximum values of the efficiency do not decrease in a monotone fashion with increasing clearance, as is the case with the channel energy efficiency, but increase at first and, later, decrease with radial clearance. This is shown in Figure 34 for a power law index of 0.5. Figure 35 shows the maximum total energy efficiency \( \eta_{\text{max}} \) vs. the radial clearance for six values of the power law index. It is interesting to note that the clearance at which \( \eta_{\text{max}} \) reaches its largest value increases with the power law index. This optimum dimensionless clearance is shown in Figure 36 as a function of the power law index. It can be seen that the usual value of the clearance, \( \delta^* = 1 \), is appropriate when the power law index is about 0.3; however, the optimum clearance becomes substantially larger when the power law index is larger than 0.3. For a Newtonian fluid, the optimum clearance is about six times the standard clearance. This indicates that, for polymer melts with a relatively low degree of pseudoplasticity, it may be beneficial to employ flight clearance values larger than the standard flight clearance.

It is also interesting to determine how the optimum value of \( \eta_{\text{max}} \) depends on the power law index. This is shown in Figure 37. The optimum value initially increases, then reaches a maximum of about 15.5%, and then reduces with the power law index. The highest value of the total energy efficiency occurs at a power law index of approximately 0.75. This means that for a square pitch screw (helix angle of 17.67°) the best energy efficiency that can be achieved is 15.5%. In general, this efficiency will be substantially lower. Consequently, at least 85% of the energy required for pumping will be dissipated into heat, causing a temperature rise of the polymer melt, provided the heat loss through screw and barrel is less than the viscous heat generation.

To show how much improvement can be obtained by optimizing the radial clearance, Figure 38 shows
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FIGURE 39
Total energy efficiency vs. pressure gradient for five helix angles, 0.4 power law index, and $\delta^0 = 2$.

The optimum energy efficiency as a function of power law index for two cases. The first case, solid line, uses the optimum clearance shown in Figure 36; the second case (dashed line) employs the standard clearance ($\delta^0 = 1$). It is clear that above $n = 0.4-0.5$, a significant difference in energy efficiency exists between the two cases. Thus, optimization of the radial clearance becomes important when the power law index is larger than one half.

The results presented so far are for a constant helix angle of 17.67°. The energy efficiency, however, depends not only on the clearance and channel depth, but also on the helix angle. The total energy efficiency vs. dimensionless down channel pressure gradient is shown in Figure 39 for five different helix angles; the power law index is 0.4 and $\delta^0 = 2.0$. The maximum energy efficiency versus helix angle for the same case is shown in Figure 40. The optimum helix angle occurs at about 16°. Also shown in Figure 40 is the curve for a power law index of 0.7 and $\delta^0 = 2.0$. It is clear that the optimum helix angle increases substantially when the power law index increases. The optimum helix angle is also dependent on the radial clearance. This is shown in Figure 41 where the maximum values of the total energy efficiency are plotted against the helix angle for four values of the clearance. The optimum helix angle reduces when the flight clearance increases. When $\delta^0 = 1$, $\phi_{\text{opt}} = 22.5°$, while when $\delta^0 = 4$, $\phi_{\text{opt}} = 19°$.

The optimum down channel pressure gradient $g_0^*$ is plotted in Figure 42 against the power law index. Two cases are shown, one where the clearance equals 0.001D and one where the optimum clearance is used (see Figure 36). When $\delta^0 = 1$, the relationship can be closely approximated by

$$g_0^* (\delta^0 = 1) = -0.1125n^2 + 0.47875n + 0.08375$$  \hspace{1cm} (49)$$

from which the optimum channel depth can be calculated as follows:

$$H^* (\delta^0 = 1) = \left[ \frac{6mg_{\text{g}}v_{gb}}{g_0} \right]^{1/(n+1)}$$  \hspace{1cm} (50)$$

When $\delta^0 = \delta_{\text{opt}}$, the relationship can be closely approximated by

$$g_0^* (\delta^0 = \delta_{\text{opt}}) = -0.1458n^2 + 0.3787n + 0.1171$$  \hspace{1cm} (51)$$

For this case, the optimum channel depth can be calculated similarly:

$$H^* (\delta^0 = \delta_{\text{opt}}) = \left[ \frac{6mg_{\text{g}}v_{gb}}{g_0} \right]^{1/(n+1)}$$  \hspace{1cm} (52)$$
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FIGURE 40
Maximum energy efficiency vs. helix angle for \( n = 0.4, \ n = 0.7, \) and \( \delta^\circ = 2.0 \).

FIGURE 41
Maximum energy efficiency vs. helix angle at various clearance values.
Above a power law index of about 0.3, \( g^* \) is considerably larger than \( g^* \). Therefore, when the power law index is larger than 0.3, the optimum channel depth will be larger when \( \delta^0 = 1 \), then, when the clearance is taken at its optimum value.

CONCLUSIONS

A numerical technique has been developed to accurately determine the leakage flow of a power law fluid in a screw extruder. The effect of the leakage flow on total throughput becomes rather large for small values of the power law index. The contribution of the pressure flow to the leakage flow is considerable, particularly when the flight clearance is large and the power law index small. The velocity profiles in the channel and clearance are strongly affected by the pressure gradient, flight clearance, and power law index. When the pressure gradient is increased, the down channel velocity profile is most significantly affected. When the flight clearance is increased, the most pronounced change occurs in the cross channel velocity profile in the flight clearance. When the power law index increases, the velocity profile in the flight clearance becomes more linear.

The power consumption in the flight clearance has a large effect on the overall power consumption and energy efficiency, especially for large values of the power law index. The total energy efficiency for pressure development is strongly dependent on the pressure gradient, helix angle, and flight clearance. The maximum energy efficiency that can be obtained is about 15.5% at a power law index of 0.75. The optimum flight clearance for energy efficiency increases with the power law index. When the power law index is larger than 0.3, the optimum flight clearance is larger than the standard flight clearance (0.01 D). For a Newtonian fluid, the optimum flight clearance for energy efficiency is about six times the standard flight clearance. When the standard flight clearance is used, the maximum energy efficiency that can be obtained will be considerably below the value that can be obtained when the optimum flight clearance is used. The optimum helix angle increases with the power law index, but reduces with the flight clearance. Approximate expressions have been developed to calculate the power consumption in the channel and flight clearance and the optimum channel depth.

REFERENCES

1. H. S. Rowell and D. Finlayson, Engineering, 126, 249 (1928).