A HYBRID DESIGN OPTIMIZATION METHOD USING ENRICHED CRAIG-BAMPTON APPROACH

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A hybrid design optimization method is presented which combines a number of techniques such as Component Mode Synthesis (CMS), Design of Computer Experiments and Neural Networks for surrogate modeling with Genetic Algorithms and Sequential Quadratic Programming for optimization. In the method, the FE analysis is decomposed and reduced by a well-known CMS technique called the Craig-Bampton method. Since the optimization method requires CMS calculations of the updated model at each of its iterations due to the changes in the design variables, one can either reuse the reduction basis of the initial components or compute new reduction basis for the condensation of the system matrices. The first option usually leads to inaccurate results and the last one increases the computation time. In the method, instead of using one of these options, the Enriched Craig-Bampton method, proposed by Masson et al., is employed for efficient optimization. New basis for the modified components are generated by extending the corresponding initial reduction basis with a set of static residual vectors which are calculated using prior knowledge of the initial component designs. Thus, time consuming complete component analyzes are prevented. A theoretical test problem is used for the demonstration of the method.

1. Introduction

Modeling of complex industrial structures requires fine meshed, large size Finite Element (FE) models for certain analysis. On the other hand, dynamic analysis of these structures requires only a few deformation modes which can be calculated with coarse meshed, time efficient FE models. Instead of creating a new coarse mesh for these models, one can keep the fine meshed model but decrease the computation time by employing a suitable reduction method in the analysis. In large projects (e.g. an aircraft project), reducing the complete model is still a cumbersome task. That is why the tendency is to divide the complete structure into several substructures (components) and perform the analysis component wise which is called sub-structuring. The so-called Component Mode Synthesis (CMS) technique is both a sub-structuring and a reduction method.

Reanalysis of a component requires to calculate the reduction basis used for condensation all over again. If an intention is to use an FE model in an optimization process, a number of reanalysis of the components is required in which the total amount of time for the calculation of the reduction bases is more pronounced. For efficient reanalysis of such components several methodologies can be found in the literature [1, 2]. In this research, the method proposed by Masson et al. [3] is employed.

It is still an elaborate task to couple reduced FE models in a global optimization process due to a large number of required reanalysis. Therefore, the ongoing research aims to have a minimum
interaction with FE based models. Nowadays, the common practice is to employ surrogate models that capture the behavior of a parameterized FE model under the extracted data from these parameterized models. These surrogate models take the place of FE models in the optimization problem and are very fast to evaluate.

An efficient hybrid design optimization method has already been proposed in [4] for structures with repetitive component patterns. In this research, this method is extended with an efficient reanalysis technique for the sake of reducing the analysis time even more. The paper is built up as follows: In Section 2, Component Mode Synthesis (CMS), Craig-Bampton (CB) method and Enriched Craig Bampton (ECB) method are explained. In Section 3, the optimization method is introduced briefly which is validated using a theoretical test problem in Section 4. The problem formulation and the comparison of the results with CB and ANSYS full FE model are also presented in this section. Finally, in Section 5 the conclusions and discussion are given.

2. Component Mode Synthesis

Component Mode Synthesis (CMS) involves breaking up a large structure into several substructures, obtaining reduced order system matrices of each component and then assembling these matrices to obtain reduced order system matrices of the entire structure. In CMS, for reduction, nodal displacement vectors of each substructure are replaced by their approximations. The method can be briefly explained as follows:

Let us assume that a FE model of a structure is constructed on a domain $\Omega$ and is divided into $S$ non-overlapping substructures such that each component is defined on the sub-domain $\Omega^c$. Thus, excepting the nodes on the interface boundaries, each node belongs to one and only one component. The linear dynamic behavior of an undamped component, labeled $c$, is governed by the equations,

$$M^c\ddot{u}^c + K^c u^c = f^c + g^c \quad c = 1, 2, \ldots, S \quad (1)$$

where $M^c$, $K^c$ and $u^c$ are the mass matrix, stiffness matrix and vector of local d.o.f of the component, respectively. The vector $f^c$ represents the external loads, and the vector $g^c$ represents the interface forces between the component $c$ and the neighboring components, that ensure dynamic equilibrium at the interfaces. The partitioned form of Eq.(1) can be written as follows:

$$\begin{bmatrix} M^c_{ii} & M^c_{ib} \\ M^c_{bi} & M^c_{bb} \end{bmatrix} \begin{bmatrix} \ddot{u}^c_i \\ \ddot{u}^c_b \end{bmatrix} + \begin{bmatrix} K^c_{ii} & K^c_{ib} \\ K^c_{bi} & K^c_{bb} \end{bmatrix} \begin{bmatrix} u^c_i \\ u^c_b \end{bmatrix} = \begin{bmatrix} f^c_i \\ f^c_b \end{bmatrix} + \begin{bmatrix} g^c_i \\ g^c_b \end{bmatrix} \quad (2)$$

where $i$ and $b$ refer to interior and boundary, respectively.

For reducing the size of the component system matrices $K^c$ and $M^c$, a subspace spanned by the columns of $T^c$ is build in such a way that the solution of Eq. (1) can be written in the form:

$$\{u^c\} = \begin{bmatrix} u^c_i \\ u^c_b \end{bmatrix} \approx T^c \{q^c\} \quad (3)$$

where $q^c$ is a vector of generalized coordinates and $\text{dim}(q^c) \ll \text{dim}(u^c)$. $T^c$ is defined by a reduction basis. After defining the transformation matrix $T^c$, the reduced system matrices of each component are given by: $\bar{K}^c = T^c T^c T^c$, $\bar{M}^c = T^c T^c T^c$, respectively. The external loads and the internal forces are $\bar{f}^c = T^c f^c$, $\bar{g}^c = T^c g^c$, respectively. The next step is the assembly of all these matrices for obtaining a reduced model of the entire structure. It is important to point out that the interface forces $\bar{g}^c$ between components are all cancelled out after the assembly.

2.1 Craig-Bampton Method

In the Craig-Bampton (CB) method [5], the reduction basis are obtained utilizing the fixed interface normal modes and the constrained modes of each component.
The fixed interface normal modes are calculated by restraining all d.o.f. at the interface and solving the eigenvalue problem. The fixed interface normal modes of a component $c$ are:

$$\phi^c = \begin{bmatrix} \{\phi^c_i\}_1 & \{\phi^c_i\}_2 & \cdots & \{\phi^c_i\}_{N_T} \end{bmatrix} = \begin{bmatrix} \phi_j \end{bmatrix}^c, \quad j = 1, 2, \ldots, N_T$$

(4)

where $\{\phi^c_i\}_j$ is the eigenvector of the $j^{th}$ normal mode and $N_T$ is the number of truncated normal modes.

The constraint modes are calculated by statically imposing a unit displacement to the interface d.o.f. one by one while keeping the displacement of other interface d.o.f. zero and the interior d.o.f. of the substructure force free. The constraint mode matrix $\psi$ of component $c$ is defined as:

$$\psi^c = \begin{bmatrix} \psi^c_{ib} \\ I_{bb} \end{bmatrix} = \begin{bmatrix} -K^c_{ii}^{-1}K^c_{ib} \\ I_{bb} \end{bmatrix}.$$ 

(5)

Therefore, the Craig-Bampton transformation matrix $T_{CB}^c$ for component $c$ is

$$T_{CB}^c = \begin{bmatrix} \phi_j & \psi^c_{ib} \\ 0_b & I_{bb} \end{bmatrix}.$$ 

(6)

The assembly is done using the compatibility of the interface degree of freedoms (d.o.f) which is called primal assembly.

2.2 Enriched Craig-Bampton Method

When a design variable of a component $c$ is modified, the transformation matrix $T_{CB}^c$ used for the condensation of the component usually needs to be modified as well. This can be done by recalculation of $T_{CB}^c$ for the modified model all over again which is computationally demanding.

An alternative solution might be using the transformation matrix of the nominal model $T_{CB,N}^c$ with some additional basis vectors which are capable to compensate for the missing information about the modified model. In [3], Masson et al. proposed to use static residual vectors for gathering this additional basis.

In the following, only the modification of one component will be considered. Thus, the superscript "c" will be omitted. The idea can be extended for the rest of the modified components.

The proposed method in [3] assumes that the contribution of static displacements at the interfaces are correctly represented by the constraint modes of the nominal component model for the modified ones. Thus, the missing information about the modified model is estimated by detecting the changes in the nominal fixed interface normal modes $\phi_N$ due to the introduced perturbations on the design variables.

The fixed interface normal modes of a modified substructure are found solving

$$[(K_{ii} + \Delta K_{ii}) - \omega^2(M_{ii} + \Delta M_{ii})]y(\omega) = 0$$

(7)

where $\Delta K_{ii}, \Delta M_{ii}$ stand for the introduced modifications on $K_{ii}$ and $M_{ii}$ respectively, $\omega, y(\omega)$ are the natural frequency and the corresponding response function of the substructure, respectively.

When Eq. (7) is rearranged,

$$[K_{ii} - \omega^2M_{ii}]y(\omega) = f_\Delta(\omega), \quad f_\Delta(\omega) = -[\Delta K_{ii} - \omega^2\Delta M_{ii}]y(\omega)$$

is obtained where $f_\Delta(\omega)$ represents the residual forces acting on the nominal structure due to the structural modifications. In many applications, practical interest is to measure the size of these forces. A reasonable approach for that is to calculate the static response $R_D(\omega)$ of the structure to the residual forces $f_\Delta(\omega)$

$$R_D(\omega) = K_{ii}^{-1}f_\Delta(\omega)$$

(9)
where \( R_D(\omega) \) is the residual displacement function. An error estimation can be done using \( R_D(\omega) \) and it can also be utilized to enrich \( T_{CB, N} \).

Since there is no information on the response \( y(\omega) \) of the modified structure (see Eq. (8)), \( f_\Delta(\omega) \) can not be calculated exactly. Instead, it can be approximated using fixed interface normal modes \( \phi_N \) and the corresponding frequencies \( \omega_N \) of the nominal model such that

\[
y(\omega) \approx \phi_N c(\omega) \tag{10}
\]

where \( c(\omega) \) is a vector function of the generalized d.o.f.

When Eq. (10) is substituted into Eq. (8),

\[
f_\Delta(\omega) \approx -[\Delta K_{ii} - \omega_N^2 \Delta M_{ii}] \phi_N c(\omega) \tag{11}
\]

is obtained where \( R_L = -[\Delta K_{ii} - \omega_{j,N}^2 \Delta M_{ii}] \phi_{j,N}, \ j = 1, 2, \ldots, N_T \) stands for a basis which spans the subspace that contains the residual force vectors. The essential idea in Eq. (11) is if the subspace spanned by \( R_L \) does not contain the exact force vectors with respect to a specific design modification, it may at least contain a reasonable representation of these vectors. The residual displacement function then becomes

\[
R_D(\omega) = K_{ii}^{-1} R_L c(\omega) \tag{12}
\]

where the columns of the residual displacement matrix \( R_D = K_{ii}^{-1} R_L \) stand for the vectors that are going to be employed to enrich the Craig-Bampton transformation matrix of the nominal model. Thus, the transformation matrix of the modified substructure gets the form of

\[
T_{\text{enriched}} = \begin{bmatrix} \phi_{j,N} & \psi_{ib,N} & R_D \\ 0_{bi} & I_{bb} & 0_{br} \end{bmatrix} \tag{13}
\]

where subscript \( N \) stands for nominal.

In order to prevent coupling effects of dependent design variables, \( R_L \) should be calculated for each design variable while keeping the rest of the design variables the same with the nominal values. Additionally, \( R_D \) is rarely of full rank and it needs to be reconditioned before being appended into \( T_{\text{enriched}} \). The details can be found in [1, 3]. The generalized coordinates associated with the static residual vectors can be eliminated for avoiding to get large component system matrices. This is issued in [3] as well which is not employed in this research.

### 3. The Design Optimization Method

The employed design optimization method is illustrated in Fig. 1. In this method, the analysis is decomposed so that each component analysis is carried out separately at the Component Level. The optimization problem, however, is solved as a whole at the Structure Level.

This method combines a number of techniques such as Component Mode Synthesis (CMS), Design of Computer Experiments (DOCE) and Neural Networks (NN) for surrogate modeling with Genetic Algorithms (GA) and Sequential Quadratic Programming (SQP) for optimization. The details of the method will not be addressed here but can be found in [4].

### 4. Demonstration of the Concepts

The structure, resembles a part of a fuselage, concerned is shown in Fig. 2(b) which is composed of 8 identical components. A component consists of a cylinder skin including a floor panel, frames and stiffeners whose model is illustrated in Fig. 2(a).

The reduced system matrices of the entire structure were obtained only by modeling one component. First, the parameterized FE model of a component was generated in the commercial FE
software ANSYS. Its system matrices were calculated for the defined design variables and then they were transferred to MATLAB. For obtaining the reduced system matrices of the component of interest, first of all its transformation matrix was computed and afterwards condensation was performed. This procedure was applied for all the components. Next all these reduced matrices were assembled for gathering the reduced system matrices of the entire structure and, modal and harmonic analysis were performed on the reduced structure model. For the harmonic response analysis, a load of 100 kN in y-direction was applied on the top interface node of component 4 and component 5 as shown in Fig. 2(b). A structural damping with an energy dissipation of 3% was considered which was imposed on the reduced stiffness matrix of the entire structure.

![Figure 1. The design optimization method.](image)

![Figure 2. Theoretical test problem. (a) Component model and its design parameters, (b) Structure model and the corresponding components C_i, i = 1, 2, ..., 8 (c) First bending mode of the initial design.](image)

The skin, floor and frames were modeled with Shell181 elements which is a 4-node element with 6 d.o.f at each node and suitable for analyzing thin to moderately thick shell structures [6]. The stiffeners with I cross-section were modeled with three dimensional Beam44 elements which has 6 d.o.f at each node, allows different cross-sections and permits the end nodes to be offset from the centroidal axes of the beam [6]. The utilized design parameters and the material properties are summarized in Table 1. The Young’s Modulus of the stiffeners (E) in the components were defined as design variables and all the stiffeners that belong to a component were assumed to have the same
design values. Therefore, there exist 8 design variables in total in the overall structure. For the initial design $E_i, i = 1, 2, \ldots, 8$ are set to 300GPa.

Table 1. Summary of the design parameter values and the material properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (R)</td>
<td>1.5m</td>
</tr>
<tr>
<td>Component Length ($L_c$)</td>
<td>2m</td>
</tr>
<tr>
<td>Floor Height (H)</td>
<td>1m</td>
</tr>
<tr>
<td>Frame thickness</td>
<td>0.03m</td>
</tr>
<tr>
<td>Stiffener thickness ($t_s$)</td>
<td>0.004m</td>
</tr>
<tr>
<td>E (Skin)</td>
<td>68.9GPa</td>
</tr>
<tr>
<td>E (Floor-Frame)</td>
<td>6890GPa</td>
</tr>
<tr>
<td>Skin thickness</td>
<td>0.0005m</td>
</tr>
<tr>
<td>Frame width</td>
<td>0.1m</td>
</tr>
<tr>
<td>Stiffener width ($h_s$)</td>
<td>0.035m</td>
</tr>
<tr>
<td>Stiffener height ($h_s$)</td>
<td>0.035m</td>
</tr>
<tr>
<td>ρ (Skin-Frame-Stiffeners)</td>
<td>2710kg/m³</td>
</tr>
<tr>
<td>ρ (Floor)</td>
<td>1355kg/m³</td>
</tr>
</tbody>
</table>

For the optimization problem, the main focus was on the frequency range of 10-30 Hz. This interval involves the first bending frequency of the initial design whose mode shape is illustrated in Fig 2(c). It is desired to decrease the response of the structure due to bending. For that purpose, the sum of the magnitudes of displacements in y-direction at the nodes that lie on the top and the bottom of the components’ interfaces was aimed to be minimized. This objective was targeted by minimizing the total area beneath their response function curve. The interface nodes of a component and its top and bottom nodes are illustrated in Fig. 2(a). The response function curve corresponding to the initial design is shown in Fig. 3, its modal analysis results are summarized in Table 2. In the optimization problem in addition to the main objective, keeping the first bending frequency around 21 Hz was one of the constraints. Additionally, the design variables of components 1-8, 2-7, 3-6, 4-5 were forced to have closer values in order to obtain a symmetric final configuration. Also another constraint was defined for preserving the bending mode of the initial design. The problem formulation is as follows:

$$\min_{E_i, i=1,2,\ldots,8} \text{TotalArea}(E_i)$$

subject to

$$21 - \epsilon \leq \omega \leq 21 + \epsilon$$

$$\text{MAC} \geq 0.9$$

$$\frac{\sqrt{(E_j - E_{(9-j)})^2}}{0.5(E_j + E_{(9-j)} - \sqrt{(E_j - E_{(9-j)})^2})} \leq \frac{\epsilon}{2}, j = 1, \ldots, 4$$

$$6.89 \leq E_i \leq 689, i = 1, \ldots, 8$$

where $\epsilon = 0.02$ and MAC defines a correlation function between the eigenvectors of the initial design and the modified design. For this problem, only the displacements of the interface nodes were taken into account. In Eq.(14), the NN model is trained for three different data sets which are the 80 design samples (input data) and the corresponding total area beneath the response function curve, first bending modes and the MAC values (target data). After the training, the NN models are employed in the problem formulation.

The optimization problem was solved twice. In the first, CB method was utilized during the reanalysis of the components whereas in the second, ECB method was employed for this purpose. In both cases three component libraries were created. The first library is for component 1, the second is for components 2, 3, \ldots, 7 and the third one is for component 8. The reason is that the imposed boundary conditions for gathering the transformation matrices of these components are different. It is important to note that for the same design values, the reduced system matrices of component 8 can be obtained by multiplying the reduced system matrices of component 1 with a rotation matrix. Thus, the same library for both component 1 and component 8 could have been used. These libraries contain information about the already calculated component designs. Therefore, if the data of an already calculated design is required, it is gathered from the corresponding library. Also after the system matrices of a component is computed for a new design, they are stored in the related library. Such a book keeping causes a significant reduction in computation time.

The results of the two demo cases are summarized in Table 2. The response function curves that correspond to the calculated optimum designs for ECB and CB are also shown in Fig. 3. Additionally,
these figures also include the validation results obtained from ANSYS. ANSYS results are computed utilizing the full FE model (ANSYS Full). The response function curves of the calculated optimum designs using different methods are shown in Fig. 4 for comparison.

![Figure 3. Illustration of the initial design and the final design results. (Left) CMS calculations are carried out employing ECB, (Right) CMS calculations are carried out employing CB in the optimization method.](image)

For this application, the benefits of using ECB for reanalysis is apparent. The total required time for optimization was reduced by 40% utilizing ECB and the accuracy of the results are satisfactory. The optimum design values obtained from two cases state that the optimal design should be stiff in the middle and it could be less stiff approaching the sides of the structure in order to fulfill the objective, however the design values do not match each other precisely. One of the reasons might be the difference between the response of ECB and CB for the same design variables. Also, there could be more than one optimum configuration in the defined design space. When a closer look is taken into Fig. 4, one may realize that the response of ECB slightly underestimates the actual result, although the estimation of CB is correct.

![Figure 4. Comparison of the optimization results.](image)
Table 2. Summary of the theoretical test problem results.

<table>
<thead>
<tr>
<th></th>
<th>CB</th>
<th>ECB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Design Variables</td>
<td>[300 300 300 300 300 300 300 300]</td>
<td>[300 300 300 300 300 300 300 300]</td>
</tr>
<tr>
<td>Final Design Variables</td>
<td>[20.4 88.2 320.1 689 689 316.9 89.1 20.6]</td>
<td>[15 58.7 338.6 689 689 335.5 58.2 15]</td>
</tr>
<tr>
<td>Initial Total Area</td>
<td>1.07 m.Hz</td>
<td>-</td>
</tr>
<tr>
<td>Initial Total Area ANSYS</td>
<td>1.06 m.Hz</td>
<td>1.06 m.Hz</td>
</tr>
<tr>
<td>Final Total Area</td>
<td>0.95 m.Hz</td>
<td>0.93 m.Hz</td>
</tr>
<tr>
<td>Final Total Area NN</td>
<td>0.94 m.Hz</td>
<td>0.93 m.Hz</td>
</tr>
<tr>
<td>Validation with ANSYS</td>
<td>0.94 m.Hz</td>
<td>0.94 m.Hz</td>
</tr>
<tr>
<td>Initial Frequency</td>
<td>20.29 Hz</td>
<td>-</td>
</tr>
<tr>
<td>Initial Frequency ANSYS</td>
<td>20.29 Hz</td>
<td>20.29 Hz</td>
</tr>
<tr>
<td>Final Frequency</td>
<td>21.06 Hz</td>
<td>21.01 Hz</td>
</tr>
<tr>
<td>Final Frequency NN</td>
<td>21.02 Hz</td>
<td>21.01 Hz</td>
</tr>
<tr>
<td>Final Frequency ANSYS</td>
<td>21.06 Hz</td>
<td>20.93 Hz</td>
</tr>
<tr>
<td>MAC</td>
<td>0.9947</td>
<td>0.9928</td>
</tr>
<tr>
<td>Total Optimization time</td>
<td>5h8min</td>
<td>3h15min</td>
</tr>
<tr>
<td>Total # of iterations</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Size System Matrices ANSYS</td>
<td>14334</td>
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</tr>
<tr>
<td>Size System Matrices</td>
<td>1698</td>
<td>1842</td>
</tr>
<tr>
<td>Reduction Time (Component)</td>
<td>18sec</td>
<td>7 sec</td>
</tr>
<tr>
<td># fixed interface normal modes</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

5. Conclusions and Discussion

A reanalysis technique proposed by Masson et al. is employed in a hybrid design optimization method. A theoretical test problem is selected which is solved twice using CB and ECB, during the reanalysis part of the method. In the problem, global dynamics of the structure is changed by modifying the design variables of components which are implicitly defined in the reduced model. The results were compared and also they were validated with the ANSYS full FE model. It is observed that employing ECB instead of CB is highly beneficial for the selected problem.

Although the method seems very appealing, attention has to be paid for the selection of an application. The method is based on two very important assumptions: The first one is the correct representation of the constraint modes of the modified design by the nominal ones. The second one is the accurate estimation of the residual force vectors using the fixed interface normal modes of the nominal model. If in the selected application, one of these assumptions are violated, the chance of getting deficient results arises.

REFERENCES