A STRESS INTEGRATION ALGORITHM FOR PHASE TRANSFORMING STEELS

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Key words: transformation plasticity, homogenization, elastic-plastic composites

Summary. A new stress integration algorithm for the constitutive models of materials that undergo strain-induced phase transformation is presented. The most common materials that fall into this category are metastable austenitic stainless (TRIP) steels. These materials can be classified as metal-matrix composites which comprise a soft and a hard metallic phase. A homogenization algorithm is presented that can estimate the evolution of state variables in each phase for a given strain increment. The elastic-plastic behavior of the phases are calculated individually using large deformation theory and the calculated algorithmic tangent moduli are used in different homogenization schemes. Furthermore, the phase transformation process in austenitic stainless steels involves a volumetric expansion and an inelastic shape change collinear with the deviatoric stress. This transformation plasticity is approximated by a phenomenological model and incorporated in the stress update algorithm.

1 Introduction

Steels with strain-induced martensitic transformations have recently started being used in the industry. There are already a lot of models that describe the constitutive behavior of these materials [1][2]. However, these models tend to be either very expensive micromechanical models or phenomenological ones that rely on experimental results and require reverse fitting to get accurate results.

One of the basic problems that prevent a physically based model to be implemented in macroscale is homogenization. The material, during the deformation process, is a metal matrix composite comprising a soft and ductile austenite phase and a hard martensite phase. Furthermore, during the process the volume fraction of the hard phase covers a range between 0 and 1. Consequently, even if one has a well defined model for predicting the volume fractions of phases accurately, it is still a challenge to model the constitutive behavior based on this knowledge.

There are previous studies that focus on homogenization of elastic plastic composites [3]. But mostly these models are restricted to small strain formulations and are lacking the accuracy of the Self Consistent scheme. In this paper a large deformation homogenization algorithm is presented which utilizes Double Inclusion and Self Consistent schemes. These formulations are used in a 3D, displacement based stress update algorithm.
Additionally, an evolving volume fraction of the hard phase is implemented during which an additional straining occurs due to the transformation plasticity phenomenon. This inelastic deformation involves a volumetric expansion and a shape change collinear with deviatoric stress. Therefore, the algorithm is expanded to tackle this additional phenomenon.

2 Homogenization

2.1 Homogenization models

The homogenization algorithms that will be discussed in this article are based on Eshelby’s solution of the inhomogeneity problem [4]. In this problem the strain concentration due to a single inhomogeneity in an infinite matrix is determined.

\[ \epsilon_1 = \mathcal{H} : \epsilon_0, \quad \mathcal{H} = \left[ S : (C_0^{-1} : C_1 - I) + I \right]^{-1} \]  

where \( S \) is the Eshelby tensor defined by the inclusion theory, \( C \) is the elastic stiffness tensor of the corresponding phases, \( \epsilon_0 \) is the remote strain, \( \epsilon_1 \) is the strain inside the inclusion. \( S \) is a function of \( C_0 \) and the shape of the inclusions.

Different homogenization algorithms mainly differ by the interpretation of the Eshelby theory. However, one common feature is the definition of a microscale imaginary material. Since the strain concentration tensor is defined in microscale there must be a scale transition in all macroscale algorithms. This is usually accomplished by assuming that the averaged microscale quantities are equal to the macroscale quantities.

Furthermore, all the algorithms assume that the mixture rule for phases holds which states that the weighted sum of state variables in each phase with the volume fraction equals the total average value of that variable.

\[ \langle D \rangle = f \langle D_1 \rangle + (1 - f) \langle D_0 \rangle, \quad \langle \overset{\gamma}{\sigma} \rangle = f \langle \overset{\gamma}{\sigma}_1 \rangle + (1 - f) \langle \overset{\gamma}{\sigma}_0 \rangle \]  

where \( f \) is the volume fraction of the inhomogeneities, \( D \) is the deformation rate and \( \overset{\gamma}{\sigma} \) is an objective rate of the Cauchy stress.

Finally, it is assumed that each phase will behave according to its own constitutive behavior, meaning that second order effects such as the boundaries, are neglected.

\[ \langle \overset{\gamma}{\sigma}_1 \rangle = C_1^t : \langle D_1 \rangle, \quad \langle \overset{\gamma}{\sigma}_0 \rangle = C_0^t : \langle D_0 \rangle \]  

where \( C^t \) is the tangent modulus and can be either the continuum tangent or the algorithmic tangent. Different tangent operators have been studied by Doghri et. al. [3] for use in homogenization algorithms.

Equations (2) and (3) are not sufficient to determine the constitutive behavior of the composite, one extra equation is needed. This missing equation is supplied by the Eshelby theory as a strain concentration or a stress concentration equation in the form of

\[ \langle D_1 \rangle = A : \langle D_0 \rangle \quad \text{or} \quad \langle \overset{\gamma}{\sigma}_1 \rangle = B : \langle \overset{\gamma}{\sigma}_0 \rangle. \]  

It is important to realize that the \( A \) tensor is different from the \( \mathcal{H} \) tensor in the sense that the material is no more an infinitely long homogeneous matrix with a single inclusion.
2.1.1 Self Consistent

The Self-Consistent (SC) model was first introduced by Kroner and used by Hill for homogenization of polycrystal behavior. The underlying theory is such that each phase is successively treated as an inhomogeneity inside the composite material. Therefore, the far-field strain needed in Eshelby’s theory is defined in this model as the composite material’s strain, i.e. macro strain.

\[
\langle D_1 \rangle = H_1 : \langle D \rangle, \quad \langle D_0 \rangle = H_0 : \langle D \rangle
\]

The \( \mathbf{A} \) tensor in (4) then becomes,

\[
\mathbf{A} = (1 - f) \mathbf{I} - fH_1^{-1} : H_1
\]

Finally using equations (2), (3), (4) and (6) the composite stiffness results in the following.

\[
C^t = \sum_i f_i C_i^t : H_i
\]

Analyzing the above system of equations it is apparent that the SC algorithm is intrinsically non-linear, since the concentration tensors are functions of the composite stiffness which is to be determined by the concentration tensors as given in (7). On the other hand, due to the nature of the algorithm it is obvious that the behavior of the composite at low and high concentrations will be captured well.

2.1.2 Double Inclusion

In the Double-Inclusion (DI) model Eshelby’s theory is utilized twice, once for each phase as an inhomogeneity in the other. The resulting strain concentration tensors are then interpolated with a function of the volume fraction.

\[
\langle D_1 \rangle = H_1 : \langle D_0 \rangle, \quad \langle D_0 \rangle = H_0 : \langle D_1 \rangle
\]

\[
\mathbf{A} = \left[ (1 - \phi)H_0^{-1} + \phi H_1^{-1} \right]^{-1}
\]

with \( \phi \) being a function of \( f \). It is worth noting that the accuracy of this model relies significantly on this interpolation function used. The DI model is linear as opposed to the SC model hence computationally much more advantageous.

2.2 Transformation plasticity

During the transformation a number of mechanisms in the microstructure occur simultaneously such as, formation of new hard particles within the soft matrix and additional straining due to transformation. The former one results in a dilution effect of the stress in the inhomogeneities since for the conservation of equilibrium in the matrix the newly formed particles must have the average stress in the matrix rather than the one in the inhomogeneities. Hence,

\[
\langle \sigma_1 \rangle = \langle \sigma_1 \rangle + \dot{f} \left( \langle \sigma_0 \rangle - \langle \sigma_1 \rangle \right)
\]

The transformation strain is given as, \( D^{tr} = \dot{f} (Tn + \delta vI/3) \) where, \( \dot{f} \) is the rate of transformation, \( n \) is the deviatoric stress direction, \( \delta v \) is the volume change and \( T \) is the magnitude of the deviatoric strain and given as \( T = 0.15 - 0.1f \).
3 Results

The above formulations are implemented in a displacement driven stress integration algorithm in which a plane-stress deformation path is supplied and the corresponding stresses are calculated. Figure 1a shows the results of the homogenization algorithm on a dual phase steel with constant volume fraction of phases where the phases are chosen to be ferrite and martensite. The results prove that the Double Inclusion model with a good interpolation function is a powerful approximation of the Self Consistent model. Figure 1b illustrates the effects of phase transformation on the material behavior and the distribution of strain and stress to the phases. It is observed that the composite undergoes a softening during the initial stages of transformation due to transformation plasticity.

![Figure 1a: Calculated stress vs. strain for Dual phase steel with 30% martensite.](image1a)

![Figure 1b: Calculated stress vs. strain for TRIP steel undergoing strain-induced transformation.](image1b)

**REFERENCES**


