A Bond Graph Algorithm to Determine the Equilibrium State of a System

by P. C. BREEDVELD

Twente University of Technology, Department of Electrical Engineering, P.O. Box 217, 7500 AE Enschede, Netherlands

ABSTRACT: An algorithm is presented which enables one to determine the nature of the equilibrium state of a system with constant inputs by direct inspection of its bond graph representation. The algorithm determines whether or not there exists a unique equilibrium state (within a certain region). If not, the system may have no (or infinitely many) equilibrium states in the linear case.

I. Introduction

The algorithm presented concerns a bond graph procedure, so we assume the reader is familiar with the fundamental bond graph concepts and methods, see (1–3).

When simulation programs like TUTSIM (THTSIM) (4) are used to simulate a system by direct use of the causal bond graph in the form of the so-called "structure table", or even by using the corresponding equations in programs like CSMP (5), the existence of a unique equilibrium state is important. In that case digital simulation can be used to compute by "relaxation" its own equilibrium state, by suppressing the time-variant sources and starting from an arbitrary set of initial conditions.

The algorithm is a formalization and extension of a useful procedure which has emerged from the practice of simulating dynamical systems with the use of bond graphs and TUTSIM (THTSIM) or CSMP (6).

II. Determination of the Equilibrium State of a System

By the equilibrium state of a system we mean that state of a system where all time-derivatives of the state variables are zero in the absence of time-varying disturbances. Note that this definition of equilibrium state corresponds to what is often called steady or stationary state, because it does not exclude stationary processes sustained by constant sources. Another definition of the equilibrium state which is often used in thermodynamics also requires that all momentum-like state variables are zero and thus excludes stationary processes. We shall not be concerned with the latter definition here.

In the linear case the former equilibrium condition can be formulated in terms of the system equations $\dot{x} = Ax + Bu$ by $\dot{x} = 0$ and $\dot{u} = 0$. An (acausal) linear bond graph preprocessor like ENPORT (1) reveals equilibrium information through the eigenvalues. If any are zero, then a degenerate equilibrium condition exists.
In thermodynamic bond graph terms (7), which will be called generalized bond graphs (GBG)† the equilibrium definition means that the flow variable of every bond connected to a storage multiport element has to be zero. In conventional bond graphs the equilibrium condition implies that every bond connected to a $C$-field has a zero-valued flow variable and every bond connected to an $I$-field has a zero-valued effort variable. Hence the problem of finding the equilibrium state of a system can be reformulated (in GBG terminology) as the problem of determining the state which results in zero-valued flows into the storage (multi-)port elements with all sources kept constant. The algorithm, in conventional bond graph terminology, is as follows:

**Preparation step**

1. Decompose, if present, the degenerate multiport capacitors and inertances, i.e. storage multiports with a constitutive ("generalized stiffness or mass") equation which can not be inverted, into a weighted junction structure (WJS) (8) and linear 1-port capacitors or inertances (9), of which the constitutive equation can always be inverted.

   Decomposition of a linear (degenerate) multiport storage element is relatively simple, but nonlinear multiport storage elements generally can not be decomposed in such a way that inversion is possible, because internal modulation would cause algebraic loops (9). Therefore, a nonlinear degenerate multiport capacitor has to be linearized around its (unknown) equilibrium state and can be treated in the same way as a linear multiport capacitor.

**Main steps**

1. Replace all ports of the one- or multiport capacitors and inertances by zero-valued, auxiliary flow- and effort-sources respectively.

2. Assign or re-assign causality and try to eliminate all causal conflicts in the bond graph.

   If this is not possible, the junction structure is indeterminate, i.e. the constitutive equations are dependent. In case the dependent constitutive equations are inconsistent, there is no equilibrium state, but if these equations are consistent, there are infinitely many equilibrium states.

   If one can eliminate the causal conflicts, the remaining system corresponds to a consistent and determinate set of algebraic equations.

3. Solve these equations by finding the inputs (efforts and flows respectively) to the auxiliary flow- and effort-sources. By inversion of the constitutive equations of the storage elements, an equilibrium state is found, which is unique in case of a linear system and unique within a certain region in the nonlinear case.

† Thermodynamic bond graphs (TBG) are easily associated with the study of thermal processes. In contrast, the TBG concept is a generalization of the interdisciplinary framework of thermodynamics. Consequently the adjective "generalized" also applies, and seems to be less confusing.
Bond Graph Algorithm

The formulation of the above algorithm in terms of generalized bond graphs is completely analogous except that the distinction which has been made between capacitors and inertances is not necessary.

The algorithm will be demonstrated by some simple examples.

III. Examples

Figure 1(a) shows a very simple conventional bond graph with linear constitutive equations and the replacement of the storage elements by auxiliary sources. No causal conflicts arise and the equilibrium state can directly be seen to be the zero state. This bond graph is slightly changed in Fig. 1(b) (resistor replaced by a non-zero, constant source) and the equilibrium state is still unique, but the equilibrium state-variable of the \( C \) is non-zero: \( q^0 = Ce^0 \). Figure 2(a) shows a variant of this bond graph which leads to a causal conflict. In this situation this conflict means that there are two flows imposed on the 1-junction by the auxiliary sources (dependent storage elements). These two flows do not conflict, which means that there is only one constraint imposed on the two efforts conjugate to these flows, viz. their sum is zero, such that the (dependent) capacitors have infinitely many equilibrium values \( q_1 = C_1 e^0 \) and \( q_2 = C_2 (-n^{-1} e^0) \) respectively, where \( e^0 \) may have any value. Figure 2(b) is another variant which also contains a causal conflict, but this time the flows imposed on the 1-junction are in conflict, except for the case where the actual flow-source is zero, which has been excluded, hence no equilibrium state exists. Figure 2(c) provides a physical system model of this example.

Figure 3(a) shows an example containing a 2-port capacitor. The application of the algorithm results in the bond graph of Fig. 3(b) if the capacitor is not degenerate. However, if this capacitor is singly degenerate (rank one constitutive or "stiffness"

---

Fig. 1. (a) Example of the replacement of storage elements by auxiliary sources in case of a unique equilibrium state. (b) As (a) but with non-zero equilibrium state.
Fig. 2. (a) Variant of Fig. 1 with causal conflict. (b) Like (a) but without equilibrium state(s). (c) Physical system model corresponding to (b).

matrix), Fig. 3(c) is obtained by decomposition (9). The algorithm then results in Fig. 3(d), using an equivalence rule derived in (10). Note that in both cases there exists an algebraic loop between the two resistors, which corresponds to the set of algebraic equations:

\[ e_{R_1} = R_1 f_{R_2} \]
\[ f_{R_2} = R_2^{-1} (g^{-1} f^0 - e_{R_1}) - R_2^{-1} (g^{-1} f^0 - R_1 f_{R_2}) \]

which can be written:

\[ f_{R_2} = g^{-1} f^0 (R_1 + R_2)^{-1} \]
\[ e_{R_1} = R_1 g^{-1} f^0 (R_1 + R_2)^{-1} . \]
Bond Graph Algorithm

(a) $S_i \rightarrow O \rightarrow \mathbf{C} \rightarrow O \rightarrow R \cdot R_i$
(b) $S_i \rightarrow O \rightarrow S_i \rightarrow O \rightarrow R \cdot R_i$
(c) $\mathbf{C} \rightarrow \mathbf{A} \rightarrow \mathbf{C} \rightarrow \mathbf{A} \rightarrow O \rightarrow R \cdot R_i$
(d) $S_i \rightarrow O \rightarrow \mathbf{C} \rightarrow \mathbf{A} \rightarrow \mathbf{C} \rightarrow \mathbf{A} \rightarrow O \rightarrow R \cdot R_i$

FIG. 3. (a) Example with a 2-port capacitor. (b) Application of the algorithm to the non-degenerate case. (c) Decomposition of (a) in the singly degenerate case. (d) Application of the algorithm to (c).

IV. Conclusion

The algorithm presented is straightforward in regular cases. However, in those cases where it can be seen directly from the bond graph that no (or infinitely many) equilibrium states exist, the algorithm provides a useful tool for gaining an impression of the characteristics of the system before a simulation run is performed. This advantage applies also if the system contains degenerate multiport storage elements.

References

(9) P. C. Breedveld, "Decomposition of multiport elements in a revised multibond graph notation", to appear in the 1984 special issue of J. Franklin Inst. on "Physical structure in modelling".