Dynamical Hierarchical Control*

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Introduction
The book deals with optimal control problems of mostly large dimension or of smaller dimension with a complicated (nonlinear) structure. The word 'hierarchical' in the title applies to the optimization techniques discussed rather than to the problem statements themselves. Both open and closed-loop solutions are considered. Because of the complexity of the problems, often only suboptimal solutions can be obtained. Various practical, worked out, examples supplement the optimization methods described.

With respect to the first edition (published by North Holland in 1977) a number of new results have been added increasing the size of the volume by about 25%. There are two main additions: one with respect to open-loop optimization techniques for nonlinear systems and the other with respect to hierarchical estimation and control.

The essence of hierarchical techniques to solve optimal control problems is to reformulate the optimal control problem as a hierarchy of interconnected smaller control problems. Many times such a decomposition into a set of smaller problems suggests itself by the nature of the system model, for instance, because of block diagonal structures or because of weak interactions between several parts of the overall system. The subsystems can be solved independently of each other and, if the appropriate computer facilities are present, parallel. The solutions are coordinated by the top level, which provides the interconnections. Such hierarchical optimization techniques may lead to savings in both computation time and memory storage.

Scope and choice of material
The book almost exclusively deals with hierarchical techniques to solve optimal control problems. Other techniques which can be useful in such problems, such as for instance a discretisation and reformulation to a (non)linear programming problem, or starting with a control law characterized by a finite number of parameters which must be chosen optimally, are not or hardly touched upon. A related topic such as system stability, which may be more important to the engineer than optimality, is not treated or mentioned. Two other related issues, model reduction or aggregation and singular perturbation techniques are briefly dealt with. Some references, though not up to date, are given for further study of these issues. Sensitivity analysis is mentioned, with respect to variations in initial conditions, in one subsection and with respect to a specific optimization procedure. Somewhat surprisingly, numerical stability (how do truncation errors influence the results?), is not even mentioned—apart from one unsupported statement on page 257 of the book—though recently new, promising, results have been obtained, specifically with respect to the numerical solution of Riccati equations (Laub, 1979) and the references cited therein, in spite of the fact that Riccati equations are the subject of many of the presented lower level optimization techniques.

The contents of the book have been based for a considerable part on, and are therefore somewhat biased to, research that has been performed either at Cambridge, England or at Toulouse, France (where the author has worked). Some books which (partly) deal with closely related subjects (e.g. Siljak, 1978, or contributions in Leondes, 1990) are not referred to. In the preface the arguments for writing this book are given: the available literature is somewhat scattered in the scientific journals and is of an abstract nature. The book tries to unify (and succeeds reasonably well in this respect apart from the bias mentioned above) available results for an engineering audience. For the practicing engineer, however, it is required that he has a basic knowledge of standard optimization techniques (dynamic programming, maximum principle), theory of duality, nonlinear programming and in one place, of functional analysis. For easy reference these topics are briefly covered in appendices. Rather than emphasizing convergence properties, which may be difficult to derive for some heuristic methods, many of the methods presented are applied to numerous examples in some of which the dimension of the state space is 20 or higher. The examples are either of an engineering or of an academic nature. Economic applications are not discussed.

Presentation
A rather disappointing feature of the book is its presentation. There are many inaccuracies, in various aspects. Apart from numerous typing errors (the book is camera copied), also in formulas, the following inaccuracies and sloppy features can be distinguished.

One of the more striking negligence is that the numerical data in the optimization process of most examples treated cannot be reproduced or reconstructed. For such examples the computation time is given and on what kind of computer configuration this was realized. Since, however, neither the initial guesses for several items, such as the Lagrange multiplier function \( \lambda(t) \) to be provided by the toplevel, are given, nor the \( e \) in the stopping criterion is given, such a computation time is not informative. Of one particular, 20 dimensional, linear example three figures with simulation results are shown. The coefficients in the system matrix \( A \) however, are not given. For those the reader is referred elsewhere.

There are missing terms in formulas \((25)\) on page 62 and there is a missing formula \((52)\) on page 274. There is a missing reference (on page 278 one is referred to [114], which does not exist), and an incomplete reference (page 323). For a formula indicating the number of elementary multiplication operations in a Runge–Kutta integration procedure (page 171) the reader would have liked a reference. For the discrete minimum principle the reader is referred to a rather old reference (82), which is mathematically incorrect according to Boltynskii (1978). The notation is not always consistent (state variables are indicated by \( x \), on pages 47 and 67 by \( y \)) the formulation of some problem statements is strange (pages 130 and 139), there are undefined quantities (page 67: \( f \), page 225: \( x_k (x + 1/k) \)), and there are nonmatching dimensions (formula (6), page 15). There is no distinction between 1 and I on page 252. There are handwritten, hardly readable additions to figures (pages 63, 76 and 90).

* Dynamical Hierarchical Control, revised edition, by M.G. Singh. Published by North-Holland, Amsterdam (1980). xvi + 324 pp., U.S. $44.00, DH 90.00.
There are wrong or at least debatable conclusions. On page 22 a comparison in terms of the number of multiplications is made between two numerical approaches. However, one iteration of the hierarchical technique is compared to the global solution approach, if I understand the concerning paragraph correctly. Nothing is said about the number of iterations. A similar remark can be made on page 40. On page 254 it is asserted that it is easy to show that for high order systems, the new filter will give substantial savings in computation time. If the formulas on which this assertion is based, are correct, dimensions of system and subsystems can easily be chosen such that this assertion is false. Dynamic programming is used (page 293) to derive, what allegedly is said, the well-known Hamilton-Jacobi equation, which, however, it is not (the min-operation is missing, due to a simplified derivation). On page 102 it is asserted that (88) is the dual problem of (88), which however is only true if the constraints are inequalities, whereas in the text equalities are explicitly admitted.

There are unsupported or hardly supported and noninformative statements. Examples: Page 282: "The second important consideration is that except for a limited class of problems, the optimal control may be nonunique". Page 59: "...there are important cases where it [an assertion previously stated] does apply". Undoubtedly true.

This list of inaccuracies, blemishes in presentation and stylistic imperfections is not exhaustive. Each single item of this list is hardly worth mentioning, but the quantity brings the reader into temptation to lose his/her appetite for the book.

Contents

Chapter I is a (quasi-) philosophical introduction to hierarchies in a tutorial style. It gives a number of characteristics which a hierarchy could or should fulfill. The property "the time horizon of interest increases as one goes up the hierarchy" seems questionable. One of the reasons for the existence of hierarchies is that "Decentralized decision makers need to coordinate their activities to satisfy the overall goal and it is more efficient to have a specialist coordination function and a hierarchy than constant communication between all the decision makers since it increases the burden on each decision maker" (page 5).

Chapter II is the most important chapter of the book since subsequent chapters build on it. The topic is open-loop hierarchical control for linear quadratic problems. A distinction is made between feasible (constraints satisfied at every iteration) and nonfeasible (constraints only satisfied at the optimum) methods. The book deals with nonfeasible methods for the argumentation in the book is referred elsewhere. The problem is to minimize

\[ J = \frac{1}{2} \sum_{i} \left( x_i^T(T)x_i^T + \int_0^T \left( x_i(t) + u_i(t) \right)^T \nu x_i(t) + x_i(t)^T \right) dt \]

where \( x_i \) is the state variables of the \( i \)th subsystem and \( u_i \) is the control of the \( i \)th subsystem. The evolution of the subsystems is governed by

\[ \dot{x}_i = A_i x_i + B_i u_i + C_i x_i, \quad x_i(0) = x_{i0}, \]

where the \( x_i \)s denote the interconnected states. Note that \( i \neq j \) is not excluded in the \( x_i \)-formula and this makes the system description nonunique. The question of what the 'best' \( L_i \) should be in terms of convergence properties say (by adjusting \( A_i \)) is not addressed. For this problem and some variations three hierarchical numerical schemes are discussed: the "Goal coordination method"., or "Interaction balance approach" of Mesarovic, Macko and Takahara (1970), the three level method of Tamura (1975), which can also conveniently handle time delays and lastly the interaction prediction approach of Takahara (1965). The first scheme is designed to define and solve the dual problem statement. At the lower levels standard linear-quadratic problems are solved. At the upper level a gradient technique in function space is used to update the Lagrange multiplier \( \lambda(t) \) which fulfills the role of coordination variable. The "Goal coordination method" is elucidated by a twelfth-order example. The description of computational savings compared to the overall solution method is unclear. The Tamura method is applied to an oversaturated traffic network. The model itself leaves the reader with some questions (external inputs have wrong signs, or are these typing errors?) The interaction prediction approach is applied to river pollution control. A comparison of the methods is made in terms of memory and computer time requirements.

Chapter III deals with a posteriori hierarchical optimization for nonlinear systems. It starts with the Goal coordination method. There are no globally sufficient optimality conditions since a duality gap between primal and dual problem is possible. No remedy for this, other than trial and error, is given. Two examples are presented. Takahara's method is also extended to nonlinear systems. It is shown that for the sufficiently small time horizon the method will converge to the optimal solution (a sufficiently small time horizon essentially suppresses the nonlinearities). Two other methods, of Javdan and Simmons, are briefly discussed.

Chapter IV deals with hierarchical feedback control for linear-quadratic problems. Based on hierarchical methods, the feedback solution depends on both the current state (realized in the subsystems, at the lower levels) and the initial state (caused by the top level). For problems with infinite time horizon the dependence on the initial state disappears. The chapter starts with the synthesis of the interaction prediction approach. Extensions to servomechanisms are discussed. As an example the river pollution model has been taken, both with and without delay elements.

Chapter V extends the results of Chapter IV to the nonlinear case. (Quasi-)linearization is the tool to bring the problem into a form suitable for manipulation. There is a section on sensitivity with respect to initial conditions.

The subject of Chapter VI is suboptimal control for large scale systems. The larger part is devoted to serially connected dynamical systems in which the flow of information is in one direction. The control problem is decentralized towards the subsystems and the subsystem control problems are solved in the downstream order. Time delays between the subsystems can thus easily be handled. A section discusses bounds on the suboptimality. Aggregation, perturbation methods are briefly dealt with near the end.

Chapter VII is an application of the theory described in Chapter VI. The application, dealing with hot steel rolling, is based on a Ph.D. thesis performed at Cambridge. Whereas everything discussed so far took place in a deterministic environment, Chapters VIII and IX deal with stochastic considerations. The first of these chapters treats multi-level state and parameter estimation through hierarchical techniques. Maximum likelihood estimates are obtained by maximizing the a posteriori estimate and thus one deals again with a deterministic problem. Several approaches, to which the names of Pearson, Arafah and Sage and Shah are connected, are discussed. Some of them are suboptimal. Numerical examples intersperse the discussion.

Chapter IX, on optimal stochastic control, starts with a multi-level Kalman filter developed by the author and some of his collaborators. This filter is incorporated in a deterministic control hierarchy to yield optimal stochastic control (separation of estimation and control). The joint problem of identification and optimization is also considered. Towards that end a single new cost function is defined, consisting of two terms; one is the original cost function and the other one a quadratic term which takes care of the identification. An index is not provided, though it would have been useful.

Conclusion

The author succeeded in showing a certain unifying structure in the field of dynamical hierarchical control. In principle, the book could be suitable for a broad class of (engineering) readers. In my view the topics presented are
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slightly unbalanced. More important, however, is that the book is seriously marred by its presentation: it is not accurate in various aspects at various points and it is written in a careless style. For derivations of some results the reader is referred elsewhere. The book, specifically Chapter II, could serve as a first superficial introduction to the subject. The reader might (also) consult other texts such as Findeisen et al. (1980) which came recently to my attention. The subject of Singh's book, however, is interest arousing, especially because of the potentials for real practical applications of the topics presented.

References


About the reviewer

Geert Jan Olsder, born in 1944, has since 1971 been employed at Twente University of Technology, Department of Mathematics. He has spent two years in the U.S.A. (Stanford and Harvard). Currently he is chairman of the Mathematics of Control Committee of IFAC.