FRICIONAL TORQUE NUMBERS FOR BALL CUP AND JOURNAL BEARINGS

D. J. LIGTERINK

Department of Mechanical Engineering, Twente University of Technology, P.O. Box 217, 7500 AE Enschede (The Netherlands)

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Summary

Plastic bearing material wears in ball cup and journal bearings. Contact areas in the ball cup and the journal bearing increase. The frictional torque needed to rotate the ball or journal also increases.

When the coefficient of friction is assumed to be constant during wear-out, the frictional torque increases to a maximum of 1.273 times the frictional torque at zero wear.

1. Introduction

Plastic bearing material is soft compared with metal and offers advantages as a bearing material because it has lower friction in combination with metal than a metal–metal combination. A disadvantage is the high wear rate of plastic. The magnitude of frictional torque during the wear of a soft bearing material is important.

2. Frictional torque in ball cup bearings with a plastic cup

Consider any point P of the contact surface of area A in the plastic cup during the functioning of a ball cup bearing with a normal stress \( \sigma_n \) between ball and cup (ref. 1, Fig. 3). When the ball rotates in the cup about the y axis a shear stress \( \tau \) (N mm\(^{-2}\)) occurs at the surface at the point P:

\[
\tau = f \sigma_n
\]

where \( f \) is the coefficient of friction.

The frictional torque about the y axis is

\[
M = \int_A r_2 \tau \, dA
\]
Substituting eqn. (1) in eqn. (2) gives

\[ M = \int_A r_\phi \frac{\partial n}{\partial A} \]  

(3)

Substitution of eqns. (6), (7) and (9) from ref. 1 in eqn. (3) gives

\[ M = \int_A r (\cos^2 \theta + \sin^2 \theta \cos^2 \phi)^{1/2} fW \frac{\cos \theta}{\omega r (\cos^2 \theta + \sin^2 \theta \cos^2 \phi)^{1/2}} \frac{\Delta d_z}{\Delta t} \times \]

\[ \times r^2 \sin \theta \, d\phi \, d\theta \]

\[ = \int_A fWi^2 \frac{\Delta d_z}{\Delta t} \sin \theta \cos \theta \, d\phi \, d\theta \]

When the friction coefficient \( f \) and the wear modulus \( W \) [1] are supposed to be independent of the coordinates,

\[ M = \int_A fWi^2 \frac{\Delta d_z}{\Delta t} \sin^2 \theta = \int_0^{\theta = \theta_e} \int_0^{\phi = 2\pi} \sin \theta \cos \theta \, d\theta \, d\phi \]

From ref. 2, p. 440,

\[ M(\theta_e) = f \frac{W_i^2 \Delta d_z}{\Delta t} \pi \sin^2 \theta_e \]  

(4)

where \( \theta_e \) is the contact angle (Fig. 1). From ref. 3, eqn. (8), substitution of

\[ W = \frac{F\omega}{2r \Delta d_z / \Delta t} L_{bb}(\theta_e) \]

in eqn. (4) gives

\[ M(\theta_e) = \frac{\pi}{2} fFL_{bb}(\theta_e) r \sin^2 \theta_e \]  

(5)

\[ M_{\theta_e,=0} = frF \]  

(6)

Equation (6) substituted in eqn. (5) gives, if \( M(\theta_e)/M_{\theta_e,=0} = \bar{M} \) is defined as a frictional torque number,

\[ \bar{M}_{bb} = \frac{\pi}{2} L_{bb}(\theta_e) \sin^2 \theta_e \]  

(7a)

Substitution with ref. 3, eqn. (8a), in eqn. (7a) for \( \theta_e \rightarrow 0 \) gives

\[ \bar{M}_{bb, \theta_e \rightarrow 0} = 1 \]  

(7b)
Frictional torque number \( \overline{M} \) for ball cup and journal bearings (with a plastic bearing element) as a function of the contact angle \( \theta_e \): \( \overline{M}_{bb} \), ball cup bearing; \( \overline{M}_{jb} \), journal bearing.

3. Frictional torque in journal bearings with a plastic cylindrical bearing

The frictional torque in a journal bearing is

\[
M = \int r f \sigma_n \, dA
\]  \hspace{1cm} (8)

where \( r \) is the radius of the axis, \( f \) is the coefficient of friction between axis and bearing, \( \sigma_n \) is the normal stress in the contact surface and \( dA \) is an infinitesimal area of the contact surface \( A \).

Substitution of \( \phi = 0 \) in ref. 1, eqn. (7), gives

\[
\sigma_n = \frac{W}{\omega r} \cos \theta \frac{\Delta d_z}{\Delta t}
\]  \hspace{1cm} (9)

and

\[
dA = rb \, d\theta
\]  \hspace{1cm} (10)
where \( b \) is the length of the bearing (see ref. 3, Fig. 1). Substitution of eqns. (9) and (10) in eqn. (8) gives

\[
M = 2 \int_{\theta = 0}^{\theta = \theta_e} W \frac{\Delta d_e}{\Delta t} r f \cos \theta \frac{r b}{r b} d \theta
\]

When the friction coefficient \( f \) and the wear modulus \( W \) are supposed to be independent of \( \theta \),

\[
M = f r^2 b \frac{W}{\omega r} \frac{\Delta d_e}{\Delta t} 2 \int_{\theta = 0}^{\theta = \theta_e} \cos \theta \ d \theta
\]

\[
M(\theta_e) = f r \frac{b W}{\omega} \frac{\Delta d_e}{\Delta t} 2 \sin \theta_e
\]

From ref. 3, eqn. (14),

\[
W = \frac{F \omega}{b \Delta d_e/\Delta t} L_{jb}(\theta_e)
\]

Equation (12) substituted in eqn. (11) gives

\[
M(\theta_e) = f r F 2 L_{jb}(\theta_e) \sin \theta_e
\]

\[
M_{\theta_e = 0} = f r F
\]

Substitution of eqn. (14) in eqn. (13) gives, if \( M(\theta_e)/M_{\theta_e = 0} = \bar{M} \) is defined as a frictional torque number,

\[
\bar{M}_{jb} = 2 L_{jb}(\theta_e) \sin \theta_e
\]

Use of ref. 3, eqn. (14a), in eqn. (15a) for \( \theta_e \to 0 \) gives

\[
\bar{M}_{jb, \theta_e \to 0} = 1
\]

**TABLE 1**

Frictional torque number \( \bar{M}_{bb} \) for ball cup bearings as a function of the contact angle \( \theta_e \) (ball material, metal; cup material, plastic)

<table>
<thead>
<tr>
<th>Contact angle ( \theta_e ) (deg)</th>
<th>Wear number ( L_{bb} ) [3]</th>
<th>Frictional torque number ( \bar{M}_{bb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \infty )</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>9.58</td>
<td>1.008</td>
</tr>
<tr>
<td>30</td>
<td>2.634</td>
<td>1.034</td>
</tr>
<tr>
<td>45</td>
<td>1.375</td>
<td>1.080</td>
</tr>
<tr>
<td>60</td>
<td>0.972</td>
<td>1.145</td>
</tr>
<tr>
<td>75</td>
<td>0.835</td>
<td>1.224</td>
</tr>
<tr>
<td>90</td>
<td>0.811</td>
<td>1.273</td>
</tr>
</tbody>
</table>
TABLE 2
Frictional torque number $M_{jb}$ for journal bearings as a function of the contact angle $\theta_e$ (journal material, metal; cylindrical bearing material, plastic)

<table>
<thead>
<tr>
<th>Contact angle $\theta_e$ (deg)</th>
<th>Wear number $L_{jb}$ [3]</th>
<th>Frictional torque number $M_{jb}$ = $2L_{jb}(\theta) \sin \theta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\infty$</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>1.953</td>
<td>1.011</td>
</tr>
<tr>
<td>30</td>
<td>1.045</td>
<td>1.045</td>
</tr>
<tr>
<td>45</td>
<td>0.778</td>
<td>1.100</td>
</tr>
<tr>
<td>60</td>
<td>0.676</td>
<td>1.171</td>
</tr>
<tr>
<td>75</td>
<td>0.641</td>
<td>1.238</td>
</tr>
<tr>
<td>90</td>
<td>0.637</td>
<td>1.273</td>
</tr>
</tbody>
</table>

The frictional torque numbers for ball cup and for journal bearings are given in Table 1 and Table 2 respectively. The frictional torque numbers are also given graphically in Fig. 1.

Acknowledgments

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Nomenclature

- $dA$: infinitesimal contact area (mm$^2$)
- $b$: length of journal bearing (mm)
- $d_s$: wear depth (mm)
- $\Delta d_s/\Delta t$: wear depth rate (mm s$^{-1}$)
- $f$: friction coefficient (dimensionless)
- $F$: force between two mating bearing elements (N)
- $L_{bb}, L_{jb}$: wear numbers for ball cup and journal bearings respectively (dimensionless)
- $M$: frictional torque (N mm)
- $M_{\theta_e}$: $M(\theta_e)/M_{\theta_e=0}$, frictional torque number (dimensionless)
- $M_{bb}, M_{jb}$: frictional torque numbers for ball cup and journal bearings respectively (dimensionless)
- $P$: any point in the contact surface between bearing elements
- $r$: radius of ball in ball cup bearing or journal in journal bearing (mm)
- $\Delta r$: difference of radii of the unworn mating surfaces (mm)
- $W$: wear modulus (N mm$^{-2}$)
- $\theta_e$: contact angle in bearing
- $\theta, \phi$: spherical coordinates (rad)
- $\sigma_n$: normal stress in contact surface (N mm$^{-2}$)
- $\tau$: shear stress in contact surface (N mm$^{-2}$)
References

