ON THE CALCULATION OF THE RESPONSE OF (PLANAR) HALL-EFFECT DEVICES TO INHOMOGENEOUS MAGNETIC FIELDS

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Abstract

The calculation of Hall potentials in a rectangular Hall plate is treated for the case in which the device is subject to a magnetic field B that is inhomogeneous in the y-direction perpendicular to the direction of initial current flow. The potentials are presented in the form \( \phi_H(\vec{r}') = \text{const} \int_{\text{width}} B(y) G(y, \vec{r}') \, dy \) for the normal Hall effect (\( \vec{r}' \) is the position vector).

Analytical expressions are given for the weight function G (which depends on the form of the device), together with graphs for a number of typical examples. An analogous expression is derived for the planar Hall effect in ferromagnetic thin films.

1 Introduction

In many applications of the Hall effect the magnetic fields to be sensed are inhomogeneous. For instance, when magnetically coded information must be read (magnetic recorder, magnetic ruler or encoder, magnetic card reader, etc.) the fields to be sensed decrease rapidly with the distance to the medium which carries the information. In order to design the proper layout for a sensor in such an application, one must have an insight into the response of Hall sensors to inhomogeneous fields. Responses of Hall transducers of any layout can be computed rigorously by means of numerical routines with the help of a computer. However, a designer's interest is served better by a more explicit expedient such as a set of graphs or analytical expressions in closed form.

Hlášník and Kokavec [1] presented an elegant procedure to compute Hall responses to inhomogeneous fields, leading to tables of numerical data, computed once and for all and generally applicable. Unfortunately, their data concern only voltage differences between the Hall contacts of a traditional device, while for a general approach one needs values of the Hall potentials at individual points of the device (an example of this need is given...
in Section 5) The Hall potential can be defined as the difference between the potentials with a magnetic field on and off, and under the assumption that an intrinsic magnetoresistivity is absent. The latter restriction must be made because in the case of an intrinsic magnetoresistivity, it may be necessary to discriminate between a Hall and a magnetoresistive component of a potential, especially in the case of inhomogeneous applied fields.

In this treatment we consider an intrinsic magnetoresistivity either to be absent or we will pay special attention to the definition of the potential. Our first aim will be to derive formulae and graphs which can be used for the calculation of Hall potentials. We have restricted ourselves to the case of magnetic fields which are inhomogeneous in the \( y \)-direction (see Fig. 1). The results are applicable not only for the normal Hall effect (caused by the Lorentz force on the charge carriers) but also for the so-called planar Hall effect in ferromagnetic films, our second purpose will be to give a detailed treatment of this latter case so that results for the normal and planar Hall effect can be compared. Finally, we will treat some simple applications as illustrations of the use of the described procedures.

2 Formulation of the theory

Hlásník and Kokavec [1] have considered a normal Hall transducer with the simple rectangular geometry shown in Fig. 1(a) and governed by the equation

\[ \vec{E} = \rho \vec{\sigma} + R_H \vec{B} \times \vec{\sigma} \]  

with \( \vec{\sigma} \) the current density and \( R_H \) the Hall constant. \( \rho \) is considered to be independent of \( \vec{B} \), so an intrinsic magnetoresistivity is absent. Next, they have transposed the boundary problem into a problem for an infinite Hall plate by periodic mirror-imaging of the applied magnetic field which also changes sign on passing a boundary (see Fig. 1(b)). They state that the boundary conditions of the part ABCD in both configurations are equal, while the

![Fig 1](image-url)
latter problem can be treated in a general way leading to the expression (see Appendix 1)

$$\phi_H(\vec{r}') = \frac{1}{2\pi} \int_S \vec{E}_B \cdot \nabla \ln |\vec{r} - \vec{r}'| dS$$

where $\phi_H(\vec{r}')$ is the Hall potential at point $\vec{r}'$, defined as the difference in potential with the field $\vec{B}$ on and off (so that $\phi_H(\vec{r}') = 0$ for $\vec{r}'$ at the current contacts). The integral in eqn (2) is over the (infinite) surface $S$ and $\vec{E}_B = R_H(\vec{B} \times \vec{\sigma})$, the Hall field. Under the condition that the Hall angle is small, $\vec{\sigma}$ is approximately equal to $\vec{\sigma}_0$, the current density at $\vec{B} = 0$. In that case eqn (2) can be approximated by

$$\phi_H(\vec{r}') = \frac{1}{2\pi} \int_S R_H(\vec{B} \times \vec{\sigma}_0) \cdot \nabla \ln |\vec{r} - \vec{r}'| dS$$

$\vec{E}_B = R_H(\vec{B} \times \vec{\sigma}_0)$ is the first-order approximation for the Hall field, while $\phi_H(\vec{r}')$ can be considered as the second-order approximation of the Hall potential. For large Hall angles Hlášník and Kokavec devised an iterative procedure, but in our treatment we confine ourselves to the second-order approximation. With some mathematics eqn (3) can be transformed into

$$\phi_H(\vec{r}') = \frac{R_HI}{2\pi wt} \int_S B(\vec{r}') \frac{y - y'}{(x - x')^2 + (y - y')^2} dS$$

with $I$ the total current through the Hall plate, $w$ its width and $t$ the thickness. Note that $B(\vec{r}')$, which is directed perpendicular to the $xy$-plane, can be any function of $\vec{r}'$. For the sake of clarity, however, we will confine ourselves to the case that $B$ is constant in the $x$-direction. This is the situation which marks the applications mentioned in the introduction. Hlášník and Kokavec have treated the general problem and, if necessary, the following theory can be generalized in the same way. However, this will be at the cost of the simplicity of the resulting expressions. A second reason to restrict to field inhomogeneities in the $y$-direction will become clear in Section 4.

With $B$ constant in the $x$-direction, eqn (4) transforms into

$$\phi_H(\vec{r}') = \frac{R_HI}{2\pi wt} \int_{-w/2}^{+w/2} B(y) G(y, x', y', w, l) dy$$

Restricting ourselves to the case that $x' = 0$ (positions along the median $EF$ of the device) we find for $G(= G(y, 0, y', w, l))$
\[
G = \sum_{n = -\infty}^{+\infty} (-1)^n \left[ \text{artg} \frac{\pi(2n + 1)l}{4w} - \text{artg} \frac{\pi(y - y')}{2w} \right] + \text{artg} \frac{\pi(2n + 1)l}{4w} - \text{artg} \frac{\pi(y - y' + w)}{2w} \\
+ \text{artg} \frac{\pi(2n - 1)l}{4w} - \text{artg} \frac{\pi(-y - y' + w)}{2w} \right]
\]

or, alternatively

\[
G = 2 \sum_{n = -\infty}^{+\infty} \left[ \text{artg} \sinh \frac{\pi(-y - y' + (2m + 1)w)}{l} - \text{artg} \sinh \frac{\pi(y - y' + 2mw)}{l} \right] + \pi/2 \left[ \text{sign}(y - y') - 1 \right]
\]

We have omitted the mathematical steps which lead from eqn (4) to eqns (5), (6) and (7). For completeness, however, these are given in Appendix 2.

We have computed a large number of \(G\)-values leading to the graphs of Fig 2 and we have experienced that very fast convergence of the series is obtained. If series (6) is used in the regime \(l/w > 1/2\) and series (7) for \(l/w < 1/2\), it is sufficient to take the terms \(m = 0\) and \(n = \pm 1\) to reach an accuracy of 0.1% of \(2\pi\) in the worst case (around \(l/w = 1/2\)). Including the terms with \(m = 0\) and \(n = \pm 2\) will give very accurate results for all practical purposes.

As can be seen from Fig 2, the \(G\)-functions, especially those with \(l/w\) small and \(y'\) not close to \(\pm w/2\) (points along the interior of the median), are antisymmetric in nature. Since they operate as weight functions in eqn (5), the antisymmetry generally leads to a small value of the potential. It can be shown that in the limit \(l/w \ll 1\) and \(y'\) not close to the edges, the series (7) degenerates into a few simple terms giving

\[
-4 \text{artg} \exp\{+\pi(y - y')/l\} \quad \text{for} \quad -w/2 < y < y'
\]

\[
G(y, 0, y', w, l) = 4 \text{artg} \exp\{-\pi(y - y')/l\} \quad \text{for} \quad y' < y < w/2
\]

Note that in this limit the parameter \(w\) has disappeared. The width of the specimen no longer influences the determination of the potentials. Physically this is explained by the presence of the short-circuiting current boundaries, as a consequence of which the free edges are no longer "seen" from a point in the interior.

At the edges (\(y' = \pm w/2\)) and still under the condition \(l/w \ll 1\) we find comparable expressions.
Fig 2. $G$-functions at the median $x' = 0$ and $y'$ from $-w/2$ to $+w/2$ in steps of $w/10$ respectively. For every value of $y'$, $G$ as a function of $y$ has a negative part for $y < y'$ and a positive part for $y > y'$. The jump at $y = y'$ always has the value $2\pi$. The extreme value for $G$ with $y = y' = -w/2$ is $2\pi$ and with $y = y' = +w/2$ is $-2\pi$. $l/w$ values are different for the four plots: (a) $l/w = 1/4$, (b) $l/w = 1/2$, (c) $l/w = 1$, (d) $l/w \geq 4$. 
\[ G(y, 0, w/2, w, l) = -8 \operatorname{artg}\{e^{\pi y/l - \pi w/2l}\} \quad (l/w \ll 1) \] (9)
\[ G(y, 0, -w/2, w, l) = 8 \operatorname{artg}\{e^{-\pi y/l - \pi w/2l}\} \quad (l/w \ll 1) \]

As we have seen, the collapse of the \( G \)-functions as \( l/w \to 0 \) is a consequence of the short-circuiting contacts. In fact the \( G \)-functions can be correlated with the form effects as computed by Lippmann and Kuhrt [2]. For that purpose one has to compute

\[ \frac{\{\phi_H(0, w/2) - \phi_H(0, -w/2)\}}{\{\phi_H(0, w/2) - \phi_H(0, -w/2)\}_{l/w > 1}} = \frac{V_{\text{Hall}}}{V_{\text{Hall}(l/w > 1)}} \] (10)

for homogeneous fields, leading to

\[ \frac{V_{\text{Hall}}}{V_{\text{Hall}(l/w > 1)}} = \int_{-w/2}^{+w/2} \{G(y, 0, w/2, w, l) - G(y, 0, -w/2, w, l)\} \, dy \] (11)

This formula neatly reproduces the result of Lippmann and Kuhrt, in the limit of small Hall angles.

For \( l/w \gg 1 \), that is, for relatively long specimens, series (6) degenerates into

\[ G(y, 0, y', w, l) = \begin{cases} -2\pi y/w - \pi & \text{for } -w/2 < y < y' \\ -2\pi y/w + \pi & \text{for } y' < y < w/2 \end{cases} \] (12)

for the interior and into

\[ G(y, 0, w/2, w, l) = -2\pi y/w - \pi \quad (l/w \gg 1) \] (13)
\[ G(y, 0, -w/2, w, l) = -2\pi y/w + \pi \quad (l/w \gg 1) \]

for the endpoints \( y' = \pm w/2 \). Contrary to the case \( l/w \ll 1 \), the expressions for the endpoints are included in the expressions for the interior and eqn (12) is not restricted to values of \( y' \) not close to the edges.

The \( G \)-functions for intermediate values of \( l/w \) cannot be expressed in single terms. However, for the cases \( y' = \pm w/2 \) we have found (by trial and error) the following single-term interpolation formula

\[ G(y, 0, -w/2, w, l) = 2\pi \frac{\operatorname{artg}\{\sinh \pi(w - 2y)/2l\}}{\cosh \pi w/l} \operatorname{artg}(\tgh \pi w/l) \] (14)

but this result is not exact. (The expression for \( y' = w/2 \) can simply be derived from eqn (14) with \( G(y, 0, w/2, w, l) = -G(-y, 0, -w/2, w, l) \). The approximation can be made better even if eqn (14) is used for \( l/w \ll 3 \) and eqn (13) for \( l/w > 3 \). In that case the error does not exceed 1% of \( 2\pi \).

In Fig 3 we give the set of \( G \)-functions for \( y' = -w/2 \) with \( l/w \) as a parameter.
3 Transducers based on the normal Hall effect

It is known that as a consequence of the form effect, a normal Hall transducer shows magnetoresistance [3]. This magnetoresistance thus is an overall effect and not the consequence of an intrinsic property. This means that the potentials defined in Section 2 can readily be appointed as Hall potentials without ambiguity and as a second-order approximation eqn (5) can be used for any field inhomogeneous in the y-direction with the \( G \)-functions as computed in Section 2. For large fields an iterative procedure must be followed, while for fields inhomogeneous in the x-direction as well, a more complicated \( G \)-function has to be computed [1]. Implicit to the formulation of the boundary conditions is that the Hall potentials are zero at the current contacts so that the total voltage over the transducer does not change if a magnetic field is applied. The extrinsic magnetoresistance finds expression in the change of the current through the device.

In the case of an intrinsic magnetoresistivity Hlásnik and Kokavec’s procedure can also be applied if certain measures are taken to prevent non-zero boundary terms “at infinity” (see Appendix 1).

4. Transducers based on the planar Hall effect

In ferromagnetic thin films, like permalloy, a mechanism is effective leading to the so-called planar Hall effect. The origin of this effect is an anisotropy of the resistivity with respect to the direction of magnetization in a single domain. We will give a short derivation of the behaviour of such a device. Figure 4 shows the device consisting of a permalloy thin film with only one single domain and having an axis of magnetic anisotropy directed

![Diagram](image)

**Fig 3** \( G \)-functions at the median \( x' = 0 \) and \( y' = -w/2 \), showing the form-factor effect. The values of the parameter \( l/w \) are in ascending order 0.063, 0.125, 0.25, 0.5, 0.75, 1, 1.5, 2, > 4 starting with the most bent curve at bottom left for \( l/w = 0.063 \) and then going upwards and to the right until the straight line for \( l/w = > 4 \).

**Fig 4** In a ferromagnetic thin film, consisting of one single domain, the initial magnetization may be oriented along the \( x \)-axis and rotated over an angle \( \alpha \) under the influence of an in-plane field \( B \). The shaded contact areas are symbolically extended in order to prevent edge domains growing into the effective area ABCD.
along the strip axis. Under the influence of a magnetic field oriented perpendicular to the strip axis but in the plane of the strip, the magnetization vector will rotate away from its direction of equilibrium. Now let us express the fact that the resistivity is anisotropic with respect to the direction of magnetization by means of a resistivity tensor

\[
\begin{pmatrix}
E_1 \\
E_2
\end{pmatrix} =
\begin{pmatrix}
\rho_1 & 0 \\
0 & \rho_2
\end{pmatrix}
\begin{pmatrix}
\sigma_1 \\
\sigma_2
\end{pmatrix}
\]  

(15)

The subscripts 1 and 2 refer to orientations perpendicular to or parallel with the magnetization vector. Let the latter be rotated over an angle \( \alpha \) with respect to the strip axis. To compute the field components parallel and perpendicular to the strip axis, we have to transform eqn (15) by a rotation of the coordinate axes over an angle \( \alpha \). The result is

\[
\begin{pmatrix}
E_x \\
E_y
\end{pmatrix} =
\begin{pmatrix}
\rho_1 \sin^2 \alpha + \rho_2 \cos^2 \alpha & (\rho_2 - \rho_1) \sin \alpha \cos \alpha \\
(\rho_2 - \rho_1) \sin \alpha \cos \alpha & \rho_1 \cos^2 \alpha + \rho_2 \sin^2 \alpha
\end{pmatrix}
\begin{pmatrix}
\sigma_x \\
\sigma_y
\end{pmatrix}
\]  

(16)

If we again approximate the current density by the current density at \( B = 0 \), we have \( \sigma_y = 0, \sigma_x = \sigma_0 \) and consequently find

\[
E_y = \sigma_0 (\rho_2 - \rho_1) \sin \alpha \cos \alpha,  
\]

(17)

\[
E_x = \sigma_0 \rho_2 - \sigma_0 (\rho_2 - \rho_1) \sin^2 \alpha
\]

(18)

It is seen from eqn (17) that a phenomenon like the Hall effect is present. However, the generating field is in the plane of the film (that is why it is called the planar Hall effect) and notwithstanding the name, the phenomenon clearly has nothing to do with the effect originally discovered by Hall.

The normal Hall effect, which must also be present, is negligible at field values at which the planar effect is effective.

Equation (18) describes the magnetoresistivity which is exploited nowadays in a recording read head and as a detector of magnetic bubbles. The approximations (17) and (18) are very good because the Hall angle \( \Theta \) is always very small (\( \tan \Theta = E_y / E_x \approx 2/100 \)). In practice, \( \rho_2 - \rho_1 \) is only a few percent of \( \rho_2 \) or \( \rho_1 \).

If the right precautions are taken, eqn (2) can again serve as a starting point for the derivation of Hall potentials, while, restricting ourselves to magnetic fields inhomogeneous in the \( y \)-direction, we can have \( E_x = 0, E_y = \sigma_0 (\rho_2 - \rho_1) \sin \alpha \cos \alpha \) as a first-order approximation for \( E_B \) (see Appendix 1). So we now have

\[
\phi_H(\mathbf{r}') = \frac{(\rho_2 - \rho_1)V}{2\pi \omega t} \int_{-\infty}^{+\infty} \sin \alpha \cos \alpha G(y, x', y', l, w) dy,
\]

(19)

comparable to the case of the normal Hall effect. Since the Hall angles never exceed a few percent of a radian, this second-order approximation of \( \phi \) will
be sufficient for the entire range of \( \alpha \). We again consider the potential of eqn (19) as a Hall potential (expressed by the subscript \( H \)), because it is generated by the "planar" Hall term from eqn (17).

Since \( \sin \alpha = M_y/M_s \) and consequently \( \cos \alpha = \sqrt{1 - (M_y/M_s)^2} \), with \( M_y \) the component of the (saturation) magnetization \( M_s \) along the \( y \)-axis, we have analogous to eqn (5)

\[
\phi_H(r') = \frac{(\rho_s - \rho_\perp)I}{2\pi wt} \int_{-w/2}^{w/2} \frac{M_y}{M_s} \cdot \sqrt{1 - (M_y/M_s)^2} G(y, x', y', l, w) \, dy
\]

(20)

In this expression, the dependence on the applied field is implicitly contained in the dependence of \( M_y \) on \( B \), so we must read \( M_y(B) \), where \( B \) is a function of \( y \) in general.

The function \( M_y(B) \) cannot usually be given in closed form because the magnetic field, effective inside the film, does not depend on the applied field in a simple way. The reason for this is that demagnetizing fields add to the applied field and these demagnetizing fields in turn depend on the distribution of \( M_y \) in the \( y \)-direction. We do not go into the details of this problem (which has been treated in the literature [4, 5]), because that is outside the scope of this paper. The \( G \)-function in eqn (20) is the same function as operative for the normal Hall effect. Since \( G \) contains the effect of the form factor, we can conclude that the short-circuiting effect of the current leads is effective for the planar Hall effect in the same way as it is for the normal effect. This also suggests that an extrinsic magnetoresistive effect should exist additional to the intrinsic effect. Such an effect is present indeed, but negligible due to the limited values of the Hall angle. For a more detailed calculation we refer to the literature [6].

5. Some applications

Let us consider the potential distribution along the median of a normal Hall transducer placed in a homogeneous field. For relatively long specimens we have

\[
\phi_H(y') = \frac{R_HIB}{2\pi wt} \left\{ \int_{-w/2}^{y} (-2\pi y'/w - \pi) \, dy + \int_{y}^{w/2} (-2\pi y'/w + \pi) \, dy \right\}
\]

(21)

giving

\[
\phi_H(y') = \frac{R_HIB(w - 2y')}{2wt},
\]

(22)
a simple linear relationship as is intuitively expected.

For relatively wide specimens the result is not so simple, but a look at the graph for the \( G \)-function (Fig 2(a)) immediately shows that the potential
in the inner region of the transducer will be close to zero as a consequence of the antisymmetric structure of $G$. Only at the endpoints the potentials differ from zero over a range which is of the order of the specimen length. In Fig. 5 we have plotted these potential functions together with the potential functions for these extremal geometries for a transducer placed in an inhomogeneous field (we have taken a field that decays linearly from one side of the device to the other). It is seen how the potential distributions are deformed by this asymmetry.

The Hall voltage as it is measured traditionally is given by

$$V_{\text{EF}} = \frac{R_H I}{2\pi w t} \int_{-w/2}^{+w/2} B(y)K(y)dy$$  \hspace{1cm} (23)$$

with

$$K(y) = G(y, 0, -w/2, w, l) - G(y, 0, w/2, w, l) + G(-y, 0, -w/2, w, l)$$

$$+ G(-y, 0, w/2, w, l),$$  \hspace{1cm} (24)$$

where we have made use of the equality $G(y, 0, w/2, w, l) = -G(-y, 0, -w/2, w, l)$. The function $K(y)/2$ is the one originally computed by Hlásník and Kokavec [1]. In the case of the planar Hall effect, an analogous expression is found

$$V_{\text{EF}} = \frac{(\rho_\perp - \rho_\parallel)I}{2\pi w t} \int_{-w/2}^{+w/2} \frac{M_y}{M_s} \sqrt{1 - (M_y/M_s)^2} K(y)dy$$  \hspace{1cm} (25)$$

It can easily be derived that in the case of "long" specimens we have

$$K(y) = 2\pi,$$  \hspace{1cm} (26)$$

and consequently
\[ V_{EF} = \frac{R_H I}{wt} \int_{-w/2}^{+w/2} B(y) dy = \frac{R_H \bar{B}}{t} \]  

(27)

for the normal Hall effect \( \bar{B} \) is the mean induction. In the case of the planar effect we find

\[ V_{EF} = \frac{(\rho_y - \rho_z)}{w} \int_{-w/2}^{+w/2} \frac{M_y}{M_s} \sqrt{1 - (M_y/M_s)^2} dy \]  

(28)

Again there is the problem of relating \( M_y \) with the applied field \( B \). Let us confine ourselves to the case of small fields where we have \( M_y \ll M_s \). Then we have

\[ V_{AC} = \frac{(\rho_y - \rho_z)}{M_s} \bar{M}_y \]  

(29)

with \( \bar{M}_y \) the mean value of \( M_y \). Only in the case that \( H_k \gg tM_s/w \) (see ref. [4] or [5]) can we transform this expression into a relation containing the applied fields, since under this condition we have \( M_y/M_z = H/H_k \) and

\[ V_{AC} = \frac{(\rho_y - \rho_z)}{\mu_0 H_k} \frac{I}{t} \bar{B}, \quad H_k \gg \frac{tM_s}{w} \]  

(30)

\( H_k \) is the intrinsic magnetic anisotropy field.

Finally, we will treat an example using the procedure presented above at the hand of a Hall transducer with a layout as shown in Fig. 6. In this layout the voltage leads are at one side of the conductor, which may be of advantage for certain applications [7]. However, the response

\[ V_{AB} = \phi_H(B) - \phi_H(A) \quad (= \frac{2}{\mu_0} \phi_B(B) - \frac{2}{\mu_0} \phi_B(A) \quad \text{in the case of symmetry}) \]  

(31)

is expected to be relatively small if the main portion of the field is located at that part of the transducer which is opposite to the part where the voltage contacts are. One can wonder what, for a given transducer length, the effect of widening will be, given a constant current density (so that net current is proportional to \( w I = I_0 w \)) and a magnetic field distribution that is fixed in position with respect to the side of the transducer opposite to the voltage contacts.

Such a situation may occur when the transducer cannot be pushed any further into the region where the fields are large. The effect of increasing the width of the transducer with constant current density may be an increase of the response at A, because one expects the Hall voltage to cumulate on widening. On the other hand, as A moves away from the field region, the decreasing \( l/w \) ratio may cause a reduced "sight" from A leading to a decrease in the response.

In Fig. 7 the response to a homogeneous field is shown. It is seen that the response at A increases on widening, but this increase tends to saturate as
Fig 6 Hall transducer with the voltage leads on one side of the conductor [7]. This differential configuration is of advantage in cases where the design must leave the space underneath the transducer free of leads.

Fig 7 (Normal) Hall voltage at point A for three field distributions (1) homogeneous, (2) linearly decaying over a distance 2l, (3) linearly decaying over a distance l, as a function of width (w) for fixed length l. The position of the field distribution is fixed with respect to the left edge of the transducer opposite to A.

the transducer becomes small, so that the $G$-functions no longer range to the opposite side. Next it is seen how the response to an inhomogeneous field is in relatively small transducers, seen from point A, not only the opposite side of the transducer is going out of sight, but all of the field. This means that the response even goes down on widening, so that an optimum width exists under the conditions mentioned.

This example shows how the theory can be used to predict the behaviour of a certain design. Any response to any field distribution can be computed this way as has been done already for a number of situations. Hall potentials which result as a consequence of exponential fields have been computed for the normal [7] as well as for the planar [8] effect. Also the case of a shielded planar Hall device (a recording read head) has been treated [9]. Experimental confirmation of the theory has also been given in [7] and [8].

6. Conclusion

We have presented a computational procedure which can be used to derive Hall potentials at any point of rectangular devices. Formulae and graphs are given for potentials at the median of the device. Procedure and results are applicable for the case of fields inhomogeneous in the $y$-direction for the normal as well as the planar Hall effect, while the influence of the form factor $l/w$ is naturally included. Some simple situations are worked out. It is concluded that the data, graphs, and formulae are of interest to anyone who wants to have general insight into the effect of inhomogeneous fields on Hall plates.

Appendix 1

Following Stratton [10], for a two-dimensional problem the following equation is valid for the electrostatic potential $\phi$. 

\[
\phi = \int_S \ln r \nabla^2 \phi \, dS - \int_C (\ln r \nabla \phi - \phi \nabla \ln r) \cdot \vec{n} \, ds
\]  

(32)

with \(C\) the contour enclosing a finite surface \(S\) and \(\vec{n}\) a vector of unity length normal to the contour.

Using \(\vec{E} = -\nabla \phi\) and the vector equations

\[
\nabla \cdot \vec{p} = \vec{p} \nabla \cdot \nabla + \nabla \phi \cdot \vec{q}, \quad \int S \nabla \cdot \vec{p} \, dS = \int_C \vec{p} \cdot \vec{n} \, ds,
\]

we find

\[
\phi = \int_S \vec{E} \cdot \nabla \ln r \, dS + \int_C \phi \nabla \ln r \cdot \vec{n} \, ds
\]

(33)

where we recognize eqn (2) apart from the last term and the finiteness of \(S\).

Now consider two situations, one with the magnetic field on, giving rise to the electrostatic potential \(\phi\) and one with the magnetic field off giving \(\phi^*\) as the starting potential function. Then we have

\[
\phi - \phi^* = \int_S (\vec{E} - \vec{E}^*) \cdot \nabla \ln r \, dS + \int_C (\phi - \phi^*) \nabla \ln r \cdot \vec{n} \, ds
\]

(34)

If we now call \(\phi - \phi^* = \phi_B\) and \(\vec{E} - \vec{E}^* = \vec{E}_b\) we have

\[
\phi_B = \int_S \vec{E}_B \cdot \nabla \ln r \, dS + \int_C \phi_B \nabla \ln r \cdot \vec{n} \, ds
\]

(35)

So we find that eqn (2) is valid if

\[
\int_C \phi_B \nabla \ln r \cdot \vec{n} \, ds = 0
\]

(36)

in the limit that \(C\) is at infinity. It is at the heart of Hlásník and Kokavec's method that this is true for the pure Hall condition, where \(\phi_B\) is an oscillating function over the rectangles. In the case of an intrinsic magnetoresistivity, the method can be applied by changing the current density in such a way that eqn (36) is true again (at least at contours enclosing a whole number of rectangles and implying that \(\int E_{Bx} \, dx = 0\) between any pair of current leading edges). We can omit a number of complications, however, by restricting ourselves to the case of magnetic fields only inhomogeneous in the \(y\)-direction. In that case we can take \(E_{Bx}\) to be a constant in first approximation and because \(\int E_{Bx} \, dx = 0\) over any rectangle, we have \(E_{Bx} = 0\) (\(E_x = 0\) in Section 4).
Appendix 2

Starting from eqn (4) and assuming homogeneity in the \( x \)-direction the integration in the \( x \)-direction can be carried out

\[
\phi_H(r') = \frac{R_H I}{2\pi wt} \int_{-\infty}^{+\infty} B(y) \, dy \int_{-\infty}^{+\infty} \left\{ \pm \right\} \frac{y - y'}{(x - x')^2 + (y - y')^2} \, dx \tag{37}
\]

where \( \{\pm\} \) in the integrand symbolizes the fact that \( B \) is periodically positive and negative in the \( x \)-direction. The result is

\[
\phi_H(r') = \frac{R_H I}{2\pi wt} \int_{-\infty}^{+\infty} B(y)F(y, y', x', l) \, dy \tag{38}
\]

with

\[
F(y, y', x', l) = \sum_{n=-\infty}^{+\infty} (-1)^n \left\{ \arctan \frac{(2n + 1)}{2} \frac{l - x'}{y - y'} - \right. \\
\left. \arctan \frac{(2n - 1)}{2} \frac{l - x'}{y - y'} \right\} \tag{39}
\]

\((l \text{ is the length of the device})\)

Making use of the periodicity in the \( y \)-direction, the remaining integral in eqn (38) can be changed into one that is bounded between \(-w/2\) and \(+w/2\)

\[
\phi_H(r') = \frac{R_H I}{2\pi wt} \int_{-w/2}^{+w/2} B(y)G(y, x', y', w, l) \, dy \tag{40}
\]

with

\[
G(y, x', y', w, l) = \sum_{m=-\infty}^{+\infty} \left\{ F(y + 2mw) - F(-y + (2m + 1)w) \right\} \, dy, \tag{41}
\]

or

\[
G(y, x', y', w, l) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} (-1)^n \left\{ \arctan \frac{K_n}{L_m} - \arctan \frac{K^*_n}{L_m} - \\
- \arctan \frac{K_n}{L_m} + \arctan \frac{K^*_n}{L_m} \right\} \tag{42}
\]

with
We have not succeeded in summing the double series analytically, but have used a partial solution with the help of the following equations, given by Bromwich [11]

\[
\sum_{n=-\infty}^{+\infty} (-1)^n \left\{ \text{artg} \left( \frac{y}{n + x} \right) - \frac{y}{n} \right\} = -\text{artg} \left( \frac{\sinh \pi y}{\sin \pi x} \right) + \text{artg} \left( \frac{\sinh \pi y}{\sin \pi x} \right)
\]  

(43)

\[
\sum_{m=-\infty}^{+\infty} \left\{ \text{artg} \left( \frac{y}{n + x} \right) - \frac{y}{n} \right\} = -\text{artg} \left( \frac{\tgh \pi y}{\tgh \pi x} \right) + \text{artg} \left( \frac{\tgh \pi y}{\tgh \pi x} \right)
\]  

(44)

Both equalities can be used to reduce the expression for $G$ leading to expressions (6) and (7).

(Warning: Renumberation of the series, in order to simplify them for computational purposes, should keep the two artg-functions in the terms of the series of eqn (39) together, otherwise divergence of the series may be introduced.)

References

9 J H J Fluitman and J P J Groenland, Comparison of a shielded "one-sided" planar Hall transducer with an MR-head, *paper 174*, *INTERMAG Conference, Grenoble, 1981*
