

Note

Simple Perfect Squared Square of Lowest Order

A. J. W. DUIJVESTIJN

Technological University Twente, Enschede, The Netherlands

Communicated by N. G. de Bruijn

Received April 25, 1978

This note is to report the existence of a simple perfect dissection of a square into 21 unequal squares.

The dissection was found in the night of March 22, 1978 with the aid of the DEC-10 computer of the Technological University Twente, The Netherlands. Since no simple perfect squared squares were found of orders less

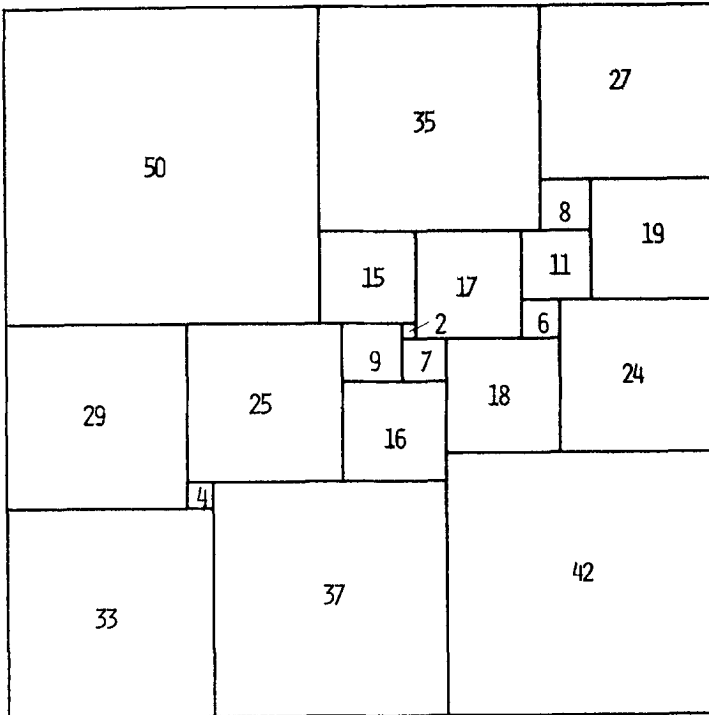


FIG. 1. Simple perfect squared square of order 21.

than 21, it is a simple perfect squared square of lowest order. Also, it is the only simple perfect squared square of order 21.

So far, the lowest order simple perfect squares known, are of order 25, the first one of which, due to Wilson, was published in [1].

In total 5 simple perfect squarings of order 25 were published in Wilson's thesis [2]. Later another 3 simple perfect squarings of order 25 were obtained by Federico [3].

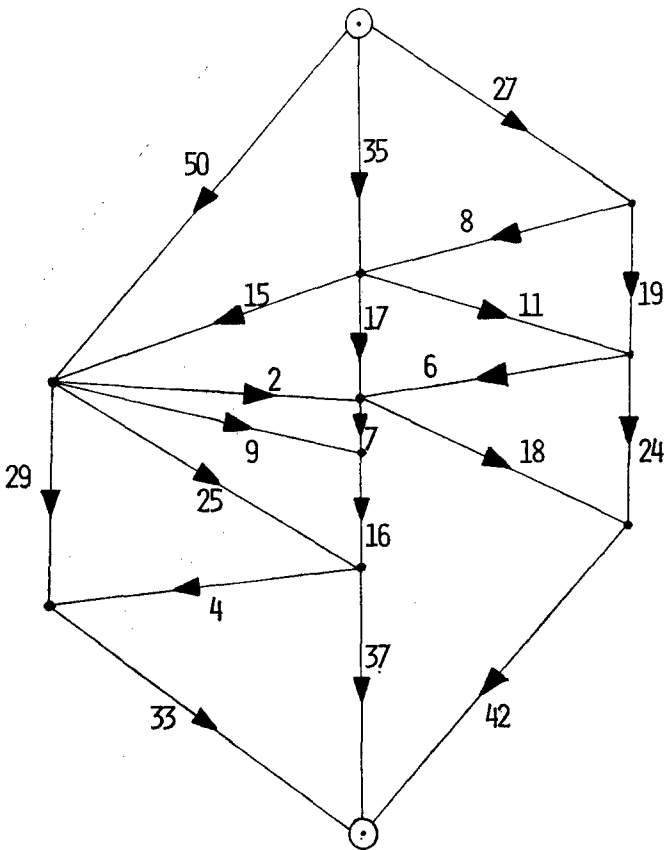


FIG. 2. Polar net from which the dissection can be obtained by calculating the current flow; \odot = pole of the net.

The lowest order compound perfect square is still the single 24 order perfect square found by Willcocks [4] in 1948.

In my thesis [5] the investigation of all 3-connected graphs of orders up to and including 20 was reported. No perfect squarings were found at that time. The Bouwkampcode [6] of the present squaring reads as follows

(50, 35, 27) (8, 19) (15, 17, 11) (6, 24) (29, 25, 9, 2) (7, 18) (16) (42) (4, 37) (33)
 The dissection was obtained from a 3-connected planar graph of order 22 with complexity 75264. The reduction factor is 336. The reduced side of the square is 112.

Figure 1 shows the simple perfect dissection of order 21. The figures 2 and 3

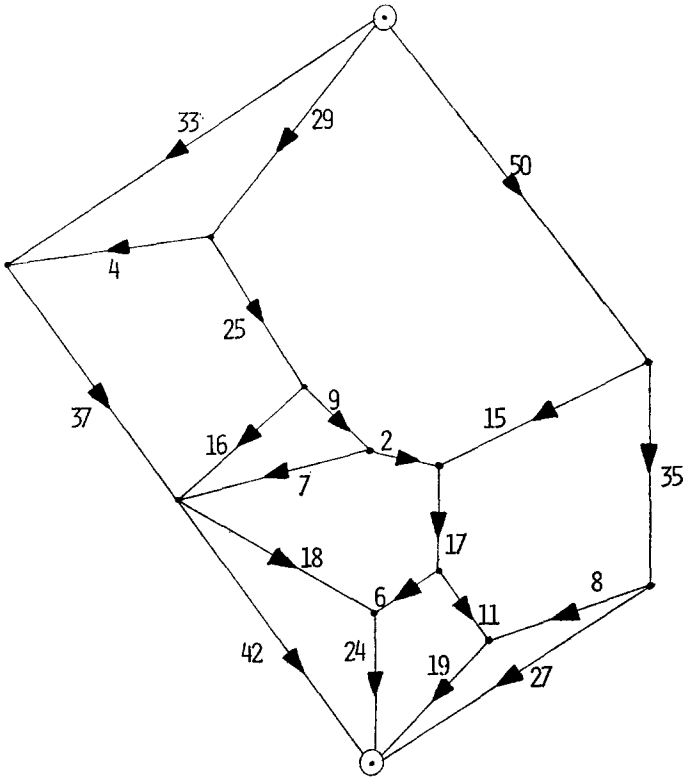


FIG. 3. Polar net from which the dissection can be obtained by calculating the current flow; \odot = pole of the net.

show the polar nets (originating from an appropriate 3-connected planar graph or its dual) from which the dissection can be obtained by calculating the current flow in the polar net using Kirchoff's laws.

REFERENCES

1. W. T. TUTTE, Squared rectangles, pp. 3-9 in "Proceedings of the IBM Scientific Symposium on Combinatorial Problems, held on March 16-18, 1964, at the Thomas J. Watson Research Center, Yorktown Heights, N.Y."

2. J. C. WILSON, "A Method for Finding Simple Perfect Squared Squarings," Thesis, University of Waterloo, 1967.
3. P. J. FEDERICO, private communication; Squaring rectangles and squares. A historical review with annotated bibliography, Submitted to the congress on graph theory held in July 1978 on the occasion of the 60th anniversary of Professor Tutte, to be published.
4. T. H. WILCOCKS, Problem 7795 and solution, *Fairy Chess Rev.* 7 (1948), 106.
5. A. J. W. DUIJVESTIJN, "Electronic Computation of Squared Rectangles," Thesis, Technological University, Eindhoven, The Netherlands, 1962; Philips Research Reports 17 (1962), 523-612.
6. C. J. BOUWKAMP, On the dissection of rectangles into squares, *Proc. Kon. Ned. Akad. Wetensch. Amsterdam* 49 (1946), 1176-1188; 50 (1947), 58-71, 72-78.