On the problem of generating small convergent systems.

(English. English summary)


Let $\Sigma$ be an alphabet and $\Sigma^*$ the set of all words over $\Sigma$ including the empty word. If $>_l$ is a total order on $\Sigma$ and $>_\text{lex}$ the induced lexicographical ordering on $\Sigma^*$, then the “length-lexicographical ordering” $>_l$ on $\Sigma^*$ is defined by: $u >_l v \iff |u| > |v|$ or $(|u| = |v|$ and $u > v$).

Here $|x|$ is the length of the word $x$.

A “string-rewriting system” $R$ on $\Sigma$ is a subset of $\Sigma^* \times \Sigma^*$ and its size equals $\sum_{(x,y) \in R} (|x| + |y|)$. Let $\Rightarrow$ denote the rewrite relation of $R$ and $\Rightarrow^*$ its reflexive and transitive closure. $R$ is called “convergent” if (i) $R$ is Noetherian, i.e., there is no infinite sequence of rewrite steps, and (ii) $R$ is confluent, i.e., for all $u, v, w \in \Sigma^*$, $u \Rightarrow^* v$ and $u \Rightarrow^* w$ imply that $v$ and $w$ have a common descendant. A string $u$ is “irreducible” with respect to $R$ if $u \Rightarrow v$ holds for no string $v$. $R$ is “normalized” if for each rule $(x, y)$ in $R$, $y$ is irreducible with respect to $R$ and $x$ is irreducible with respect to $R - \{(x, y)\}$.

The authors establish the existence of a sequence $(R_{n,m})_{n, m \in \mathbb{N}}$ of normalized string-rewriting systems on a fixed alphabet $\Sigma$ such that for all $n, m \in \mathbb{N}$, (1) $R_{n,m}$ contains 44 rules, is of size $O(n + m)$ and is compatible with $>_l$, i.e., $(x, y) \in R$ implies $x >_l y$; (2) given $R_{n,m}$ and $>_l$ as input, the Knuth-Bendix completion procedure will generate more than $A(n, m)$ intermediate rules before a finite convergent system $S_{n,m}$ of size $O(n + m)$ is delivered. ($A$ denotes Ackermann’s function.)

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