MODELING AND CHARACTERIZATION OF THE SENSITIVITY OF A HOT-WIRE PARTICLE VELOCITY SENSOR

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Abstract — The sensitivity of an innovative micromachined acoustic sensor consisting of four hot-wires is analyzed theoretically and experimentally. An analytical model is presented that describes both the air flow and the temperature profile in and around the probe. The presence of the chip surface in the vicinity of the wires influences the acoustic flow, while also affecting the temperature distribution in the probe. Both effects result into a specific angular dependence of the sensor sensitivity. Acoustic flow measurements are compared with the theory and with numerical simulations on the device, showing good qualitative and quantitative correspondence.

Key Words: particle velocity sensor, acoustics, flow sensors, MEMS-modelling

I INTRODUCTION

For a complete characterization of a three-dimensional sound field, both the acoustic particle velocity and the pressure have to be determined. By means of a particle velocity sensor complementary to the conventional pressure microphone, a complete determination of the three-dimensional acoustic field, including intensity streamlines and different source contributions becomes possible. To this purpose a micromachined particle velocity sensor, composed of two parallel heated wires, of about 1 mm length and spaced about 300 µm apart, has been developed [1,2,3,4]. When subject to an acoustic flow, forces convection leads to a small temperature difference between the wires, which is a measure of the particle velocity. To exploit fully its abilities to measure the three-dimensional particle velocity vector, we designed a new device in which four hot-wires in perpendicular directions are integrated in one probe [5] (Fig.1).

For a very accurate determination of the direction of the involved particle velocities, a good understanding of the directional sensitivity of the probe is needed. However, the current designs show a deviation in their directional sensitivity pattern from the ideal ‘figure-of eight’ response [2,6] that would be the case for free standing wires. Up to now, this deviation has not been fully understood. The purpose of this paper is to analyze this effect and model the sensitivity for the four wire probe, with the sensor wires placed within a square orifice in the chip surface.

In the analysis we take into account all physical aspects that play a role: both the acoustic flow profile around the wires, that is disturbed due to the presence of the printed circuit board acting as a disturbance for the flow, and the local temperature distribution close to the heaters. The chip surface acts as a heat sink that affects the heat transport due to convection and consequently influences the sensor response to particle velocities.

We present an analytical model for the directional sensitivity of this novel device, and compare this with experiments and numerical calculations on the flow and temperature distribution in the probe.

Figure 1. (upper:) The four wire particle velocity probe, containing four 1 mm long and 400 µm wide sensor wires suspending the rectangular orifice in the chip surface. (lower:) schematic picture of the probe.

II THEORY

Ideally, the two parallel heated wires of a particle velocity sensor exhibit a cos(θ)-dependent response to a fluid flow imposed under an angle θ with the plane of the wires, since the sensor wires detect the velocity component directed in the plane of the wires only. This cos(θ) angular dependence on the incoming flow is
sometimes referred to as a ‘figure-of-eight’ response [2,6]. However, in practice free standing wires without any connections to the probe do not exist, and as a consequence of the surrounding material the angular dependence of the sensitivity is significantly influenced. For the probe as shown in figure 1, where the thin wires (1 mm long silicon nitride beams of 400 µm thickness with 100 nm thick platinum layers on top) are placed within a square orifice in a 380 µm thick silicon support, measurements show a significant constant deviation from the figure-of-eight pattern. Is is observed that the measured angle deviates from the angle \( \theta \) of the incoming flow; but also that this angular shift of approximately 18° appears to be \( \theta \)-independent as well, which results into a constant rotation of the entire ‘figure-of-eight’ curve of about 18° in the polar plane.

Two possible causes for the observed effect can be propounded. First, the presence of the chip surface in the close proximity of the wires disturbs the air flow, thereby altering the direction of the local particle velocity. Second, the silicon chip acts as a ‘heat sink’ resulting in a preferred direction of the heat flow generated by the sensor wires, to the silicon. Consequently, the temperature distribution around the wires is not completely symmetric anymore as it would be for wires in free space.

II.1 THE TEMPERATURE PROFILE

To analyze the response of the sensor due to an imposed acoustic flow, the temperature profile around the heated wires has to be calculated. At first, the static temperature distribution in the device is calculated, when no gas flow is present. It has been proven [7,8] that an acoustic wave gives only a small perturbation to this temperature profile, and also that the output signal of the sensor is linearly proportional to the local temperature gradient at the place of the sensor wires. Therefore, the static temperature distribution only provides sufficient information to deduce the sensitivity properties we are interested in in this paper, in particular the directionality (the polar dependence) of the sensitivity of the device.

In the absence of a moving gas, the temperature distribution around the wires follows from the stationary heat equation

\[
-\nabla (k \nabla T) = Q
\]

where \( k = k(T) \) is the thermal conductivity of air, that is in principle a function of temperature, and \( Q \) the heat produced by the heater per unit volume per unit time. We now solve the static temperature \( T(x,z) \) for the geometry as shown in Fig. 2. Taking into account the appropriate boundary conditions for \( T \) and the heat flux at \( x = \pm l_x \), we can obtain an expression for the temperature in the device as an expansion in harmonics in normalized variables \( \xi = x/l_x \), \( \zeta = z/l_x \). The resulting expansion converges quickly and can be easily calculated numerically. For a specific set of parameters the temperature distribution is shown in figure 3, also compared with numerical simulations described in section III.

![Figure 2. Cross-sectional view of the model geometry for the calculation of the temperature distribution. The thickness of the substrate (the chip) is \( d \); its full width is \( 2D \), the horizontal distance between the wires is also \( d \).](image)

![Figure 3. Temperature distribution in the channel along the normalised x-axes, according to the calculated expansion for \( T \) with \( \xi=0.2 \), \( l_x = 1 \) mm and \( l_y=1.5 \) mm. For comparison, results from numerical simulations (section III) are also shown. The temperatures are normalised by \( T_v \), the temperature at the place of the heaters.](image)
angle of the incident flow. It is seen that the representation of $S(\theta)$ in the polar plane, the polar figure, is shifted with respect to the 'ideal' figure-of-eight of the sensitivity of free standing wires, by an angle $\Delta$, determined by the ratio of the $x$- and $z$- derivatives of $T$ at the wires (Fig. 4). The calculated shift $\Delta$ due to this temperature effect, as a function of the gap distance is shown in Fig. 7.

II.2 THE AIR FLOW AROUND THE PROBE

Due to the presence of the chip surface, the fluid flow itself is influenced as well, and the local angle $q$ will in general not be equal to the angle of incidence of the acoustic wave far away from the probe. To analyse therefore the air flow around the probe, one should solve the full Navier-Stokes equations for the three-dimensional probe geometry to obtain the complete particle velocity field around the probe. For the relatively complicated geometry of the real probe as depicted in figure 1, an exact analytic solution cannot be found. Therefore, we model the sensor chip by a pair of long and thin parallel plates, of which the cross section is shown by figure 2. The problem becomes then two-dimensional. It has been shown [8, 10] that for propagating acoustic waves, the fluid can be considered as incompressible, so that the Navier-Stokes equations yield

$$\frac{\partial \bar{v}}{\partial t} + (\bar{v} \cdot \nabla) \bar{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \bar{v}$$  \hspace{1cm} (9)$$

with $p$ the pressure and $\nu$ the kinematic viscosity. It was shown before [8,10], that for the current values of $\nu$, the nonlinear term in the equation can be neglected. Using elliptic coordinates [11], the equation can be solved analytically for thin, infinitely long, plates of width $D-l_z$ ($d \ll (D-l_z)$ ) For a specific value of $\theta$, the streamline pattern is seen in Fig. 5. From the obtained streamline pattern, we can determine the local velocity of the air at the place of the wires, and determine the angular shift $\Delta$. The values of $\Delta$ due to this flow profile around the probe, are also depicted in Fig. 7.

III NUMERICAL CALCULATIONS

We used a finite-elements numerical solver, CFD RC®, to analyze the temperature distribution and the air flow in and around the three-dimensional probe geometry. Both the Navier-Stokes equations and the heat diffusion equations were solved numerically by constructing a fine mesh of discrete small control volumes in the solution space, defining the appropriate boundary and initial conditions, and solving the equations for each cell. A plane propagating wave of incident angle $\theta$ was defined as entering the outer boundaries, and $\theta$ was varied in the simulations. The dissipated power per wire was set to be $20\, \text{mW}$. Devices with varying gap distances, ranging from $0.2\, \text{mm} < l_x < 1.8\, \text{mm}$, were investigated. Additionally, the heat

![Figure 6](image.png)
conductivity of the chip material was varied, ranging from the heat conductivity of air, $k_{\text{air}} = 0.0263$ W/m·K, to that of silicon, $k_{\text{Si}} = 130$ W/m·K, in five linear steps. As described in the next section, several probes made of different heat conducting materials were also actually designed (in particular, of wood and of silicon) and to comply with these experiments, those geometries were also analysed in the simulations. The results of the calculations on two suchlike probes are illustrated by Fig. 6; Fig. 7 gives a comparative summary of numerical, theoretical and experimental results.

IV EXPERIMENTS

The particle velocity probe was placed in a 25 cm long standing wave tube [2,6] with a pressure microphone connected in the tube end, and at the other end a loudspeaker generating a broad band (0-10 kHz) signal. The probe measured the generated standing wave pattern and was rotated in steps of 3.75°. The measured frequency response was in correspondence with previous measurements on the sensitivity of the particle velocity sensor [1,9,4], showing a low-pass frequency response with a roll-off above 2 kHz. The measurements showed further that the polar pattern of the directional sensitivity was nearly frequency independent in the range from 50-1000 Hz for all investigated probes. To distinguish between the temperature induced and flow induced effects, two different series of probes were designed, one set with silicon as the surrounding material, the other with balsa wood. This material was chosen because of its extremely low heat conductivity compared to that of silicon. Each set of probes comprised of devices with varying gap distances, with the full gap distance $2l_x$ varying between 0.4 and 2.0 mm. Comprehensive comparison of these measurement results with both the numerical simulations and the theoretical calculations is made in Fig. 7. We see that the balsa based probes show significantly less temperature-related effects, resulting in a smaller angular shift.

V CONCLUSIONS

The sensitivity of a novel micromachined acoustic sensor consisting of four heated wires was analyzed theoretically and experimentally. The presented model is seen to be appropriate to describe the angular dependence and the magnitude of the sensor response and in satisfying correspondence with simulations and experiments. Besides, the observed angular dependence of the sensitivity is frequency independent and can therefore easily be corrected for. It is also concluded that the perturbed air flow due to the chip surface is the dominant reason for the observed angular sensitivity.

REFERENCES