Impedance Analysis of Subwoofer Systems*

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The electrical impedance of four low-frequency loudspeaker systems is analyzed. The expression for this impedance is obtained directly from the acoustical analogous circuit. Formulas are derived for calculating the small-signal parameters from the frequencies of impedance minima and maxima of two fourth-order and two sixth-order systems. Results of measurements are presented for two sixth-order systems.

0 INTRODUCTION

With the advent of digital sound processing and recording there has been a strong need for compact loudspeakers which are able to reproduce the lowest audible frequencies with sufficient power [1]. At low frequencies the excursion capability of the loudspeaker membrane often determines the maximum acoustic power of the system [2]–[4]. Often one or more Helmholtz resonators are used in order to reduce the excursion of the membrane over a certain frequency range.

Some of these loudspeaker systems exhibit a natural rolloff for higher frequencies [5], [6]. These loudspeakers sometimes are called bandpass loudspeakers. The natural rolloff of bandpass loudspeakers allows the use of crossovers with less steep filter slopes. Furthermore the pressure level of loudspeaker distortion will be reduced by the low-pass filtering of the enclosure.

The performance of most loudspeaker systems depends critically on the parameters of the loudspeaker and the enclosure [4]. The sensitivity for parameter variations generally increases with the order of the total system. For higher order systems, as discussed in the present paper, slight misalignments can have a large effect on the total system performance. Total system performance here is expressed in terms of maximum electrical and acoustical power, peak cone excursion, and flatness of the frequency response.

The impedance as a function of the frequency of the loudspeaker system is directly related to the parameters of the system. For conventional closed-box and vented-box systems, the expressions enabling the calculation of the important parameters from the impedance of the system are well known [3], [4]. The cone excursion and the sound pressure response of these systems can be predicted with reasonable accuracy if the parameters are known. As far as the author knows, for the sixth-order bandpass systems described in the present paper, no derivation relating the impedance to parameters has appeared in press.

Knowledge of the system parameters can be an aid in predicting the system performance. As the parameters of a loudspeaker vary considerably within a production process, methods must be found to adjust the box optimally to the different loudspeakers, in order to obtain acceptable performance. The system parameters can be valuable in the development phase too, because, for instance, the air mass loading on the loudspeaker membrane can vary for different box types and sizes. For bandpass systems with multiple vents contributing to the total output volume velocity, standard near-field sound pressure measurements [7] can be inconvenient. This is because, as for conventional vented-box systems, the pressure contributions from the individual vents must be scaled to the vent diameter and have to be summed in a phase-sensitive way.

Parameter estimation is difficult for enclosures which are heavily damped with lining. Often these configurations are not very attractive because of their low efficiency. This paper will be restricted to those loudspeaker systems for which the enclosure losses may be neglected in the first approximation. This means that the losses which are present in the system do not have a considerable influence on the performance and impedance of the system. Also the effects of mutual radiation [8], [9] are neglected in the analysis. Only if the spacing between the radiating elements is very small should mutual radiation effects be included in

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are represented by electrical symbols is shown in Fig. 1. The inductance of the voice coil has been neglected, which seems reasonable in the frequency range of interest. The acoustical impedance of the enclosure is represented by the element \( Z_a \). The complete circuit is based on the works of Beranek [2] and Small [3]. The electrical and mechanical elements can be transformed to the acoustical side of the circuit. The resulting circuit can then be simplified to the acoustical analogous circuit of Fig. 2. The relations between the components of Figs. 1 and 2 are given later. According to Small a normalized cone excursion \( X(s) \) can be defined as

\[
X(s) = \frac{1}{sC_{as}} \frac{U_d(s)}{P_e(s)}
\]

Using the relations

\[
T_s^2 = M_{ms}C_{ms} = M_{as}C_{as}
\]

\[
C_{as} = S_{\phi}^2 C_{ms}
\]

\[
T_s = C_{ms}B^2 l^2
\]

\[
Q_{es} = \frac{R_e}{T_s}
\]

\[
\frac{T_s}{Q_{ms}} = R_{ms}C_{ms}
\]

\[
\frac{1}{Q_{ls}} = \frac{1}{Q_{es}} + \frac{1}{Q_{ms}}
\]

we get

\[
Z_{ve}(s) = \frac{R_e}{s^2 T_s^2 + s(T_s/Q_{ls} + C_{as}Z_a) + 1}
\]

\[
\frac{T_s}{Q_{ls}} = R_a C_{as}
\]

Fig. 1. General analogous circuit for loudspeaker systems at low frequencies.

Fig. 2. Analogous circuit of Fig. 1 with all electrical and mechanical elements transformed to the acoustical side.

Fig. 3. Analogous circuit of Fig. 1 with all mechanical and acoustical elements transformed to the electrical side.
the analysis.

An outline of the paper is as follows. In Section 2 a general method is given for the determination of the electrical impedance function. It will be shown that this function can be formed directly from the characteristic polynomial. The electrical impedance function can also be formed directly from the function that relates cone excursion to input voltage, that is, the displacement function. In Section 3 an example of the method is presented. The method is applied to the conventional vented-box system, for which some impedance-parameter relations are derived. Afterward the single-vented bandpass system is discussed in Section 4. In Sections 5 and 6 an analysis is made of two sixth-order bandpass systems. Expressions are derived which enable the calculation of the most important small-signal parameters from the electrical impedance. Finally in Section 7 results of measurements on two sixth-order systems are presented.

1 GLOSSARY OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$B$</td>
<td>effective induction of loudspeaker magnet</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of sound</td>
</tr>
<tr>
<td>$C_{ab}$</td>
<td>acoustical compliance of enclosure volume, $= V_b/pc^2$</td>
</tr>
<tr>
<td>$C_{ab1}$</td>
<td>acoustical compliance of compartment 1 volume, $= V_{b1}/pc^2$</td>
</tr>
<tr>
<td>$C_{ab2}$</td>
<td>acoustical compliance of compartment 2 volume, $= V_{b2}/pc^2$</td>
</tr>
<tr>
<td>$C_{as}$</td>
<td>acoustical compliance of membrane suspension, $= V_{as}/pc^2$</td>
</tr>
<tr>
<td>$C_{at}$</td>
<td>combined acoustical compliance of membrane suspension and enclosure volume</td>
</tr>
<tr>
<td>$C_{ms}$</td>
<td>mechanical compliance of membrane suspension</td>
</tr>
<tr>
<td>$D(s)$</td>
<td>characteristic polynomial</td>
</tr>
<tr>
<td>$D'(s)$</td>
<td>as $D(s)$, but with $Q_{ts}$ replaced by $Q_{ms}$</td>
</tr>
<tr>
<td>$D_1$</td>
<td>diameter of compartment 1 port</td>
</tr>
<tr>
<td>$D_2$</td>
<td>diameter of compartment 2 port</td>
</tr>
<tr>
<td>$e_g$</td>
<td>open-circuit output voltage of source</td>
</tr>
<tr>
<td>$F$</td>
<td>force of voice coil on loudspeaker cone</td>
</tr>
<tr>
<td>$i$</td>
<td>effective current flowing through loudspeaker coil</td>
</tr>
<tr>
<td>$l$</td>
<td>effective length of voice coil in magnetic gap</td>
</tr>
<tr>
<td>$L_1$</td>
<td>length of compartment 1 port</td>
</tr>
<tr>
<td>$L_2$</td>
<td>length of compartment 2 port</td>
</tr>
<tr>
<td>$M_{ap}$</td>
<td>acoustical mass of port, including air load</td>
</tr>
<tr>
<td>$M_{ap1}$</td>
<td>acoustical mass of compartment 1 port, including air load</td>
</tr>
<tr>
<td>$M_{ap2}$</td>
<td>acoustical mass of compartment 2 port, including air load</td>
</tr>
<tr>
<td>$M_{as}$</td>
<td>acoustical mass of loudspeaker membrane, including air load</td>
</tr>
<tr>
<td>$M_{ms}$</td>
<td>mechanical mass of loudspeaker membrane, including air load</td>
</tr>
<tr>
<td>$p_g$</td>
<td>pressure source of acoustical analogous circuit</td>
</tr>
<tr>
<td>$Q_{es}$</td>
<td>electrical $Q$ of loudspeaker</td>
</tr>
<tr>
<td>$Q_{ms}$</td>
<td>mechanical $Q$ of loudspeaker</td>
</tr>
<tr>
<td>$Q_{ts}$</td>
<td>total $Q$ of loudspeaker, including all loudspeaker resistances</td>
</tr>
<tr>
<td>$R_{at}$</td>
<td>total resistance component of loudspeaker in acoustical analogous circuit</td>
</tr>
<tr>
<td>$R_e$</td>
<td>electrical dc resistance of voice coil</td>
</tr>
<tr>
<td>$R_{ms}$</td>
<td>mechanical resistance of membrane suspension</td>
</tr>
<tr>
<td>$s$</td>
<td>complex frequency variable, $= j\omega + \sigma$</td>
</tr>
<tr>
<td>$S_d$</td>
<td>membrane surface area, $m^2$</td>
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<tr>
<td>$u$</td>
<td>induced voltage by velocity of voice coil</td>
</tr>
<tr>
<td>$U_d$</td>
<td>volume velocity of loudspeaker membrane, $m^3/s$</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity of voice coil, $m/s$</td>
</tr>
<tr>
<td>$X(s)$</td>
<td>displacement function, relating cone excursion to input voltage</td>
</tr>
<tr>
<td>$X'(s)$</td>
<td>as $X(s)$, but with $Q_{ts}$ replaced by $Q_{ms}$</td>
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<tr>
<td>$Z_a$</td>
<td>acoustic impedance of loudspeaker enclosure seen by loudspeaker membrane</td>
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<tr>
<td>$Z_{vc}$</td>
<td>electrical impedance at loudspeaker terminals of total loudspeaker system</td>
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<tr>
<td>$\alpha$</td>
<td>ratio of loudspeaker compliance to enclosure compliance</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>ratio of loudspeaker compliance to compartment 1 compliance</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>ratio of loudspeaker compliance to compartment 2 compliance</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>ratio of combined loudspeaker compartment 1 compliance to compartment 2 compliance</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of air</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>angular frequency of first minimum in electrical impedance, $= 2\pi f_1 = 1/T_1$</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>angular frequency of second minimum in electrical impedance, $= 2\pi f_2 = 1/T_2$</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>angular frequency of vented-box enclosure resonance, $= 2\pi f_0 = 1/T_b$</td>
</tr>
<tr>
<td>$\omega_{b1}$</td>
<td>angular frequency of compartment 1 resonance, $= 2\pi f_{b1} = 1/T_{b1}$</td>
</tr>
<tr>
<td>$\omega_{b2}$</td>
<td>angular frequency of compartment 2 resonance, $= 2\pi f_{b2} = 1/T_{b2}$</td>
</tr>
<tr>
<td>$\omega_{c1}$</td>
<td>angular frequency of loudspeaker resonance when loaded by volume of compartment 1, $= 2\pi f_{c1} = 1/T_{c1}$</td>
</tr>
<tr>
<td>$\omega_L$</td>
<td>angular frequency of first maximum in electrical impedance, $= 2\pi f_L$</td>
</tr>
<tr>
<td>$\omega_M$</td>
<td>angular frequency of second maximum in electrical impedance of double-vented bandpass systems, $= 2\pi f_M$</td>
</tr>
<tr>
<td>$\omega_H$</td>
<td>angular frequency of second maximum for single-vented systems or third maximum for double-vented systems in electrical impedance, $= 2\pi f_H$</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>angular frequency of loudspeaker resonance, $= 2\pi f_s = 1/T_s$</td>
</tr>
</tbody>
</table>

2 GENERAL RELATIONSHIP BETWEEN ACOUSTICAL ANALOGOUS CIRCUIT AND ELECTRICAL IMPEDANCE

A general analogous circuit in which the mechanical and acoustical elements as well as the electrical elements...
then the normalized cone excursion $X(s)$ can be calculated with Eq. (1).

$$X(s) = \frac{1}{s^2 T_s^2 + s(T_s/Q_{ts}) + C_m Z_a + 1} \quad (10)$$

$$X(s) = \frac{s^2 T_b^2 + 1}{s^4 T_s^2 T_b^2 + s^3 T_s T_b^2 + s^2 [T_s^2 + T_b^2 (1 + \alpha)] + s T_s + 1} \quad (13)$$

Comparing Eqs. (8) and (10) leads to the conclusion that the input impedance $Z_{vc}(s)$ of a loudspeaker system can be found very easily from the expression for $X(s)$ by

$$Z_{vc}(s) = R_e \frac{D(s)}{D'(s)} \quad (11)$$

where $D(s)$ is the denominator of $X(s)$. $D'(s)$ can be found from $D(s)$ by replacing every appearance of $Q_{ts}$ by $Q_{ms}$. This formula is convenient because no time-consuming transformations to the electrical domain have to be applied once the characteristic polynomial $D(s)$ is known. Notice that in general $Z_a$ is an expression with both denominator and numerator expressed in $s$. Using Eq. (7), Eq. (11) can also be written as

$$Z_{vc}(\omega) = R_e \left\{ 1 + \frac{1}{Q_{es}} \left[ \omega T_s T_b^2 \left( -j \omega T_s + j \omega T_b \right) - \omega^2 [T_s^2 + T_b^2 (1 + \alpha)] + j \omega \frac{T_s}{Q_{ms}} + 1 \right] \right\} \quad (16)$$

For reasons of convenience Eq. (12) will be preferred to Eq. (11) in determining the voice-coil impedance $Z_{vc}(s)$. Applying Eq. (12), changing every $Q_{ts}$ appearing in Eq. (13) to $Q_{ms}$ according to Section 2, and making the substitution $s = j \omega$ results in an expression for the impedance as a function of the angular frequency $\omega$.

To illustrate the results of the preceding section, an analysis is made of the impedance of the conventional vented-box system [4] of Fig. 4. The acoustic impedance seen by the loudspeaker membrane is represented by the circuit of Fig. 5. Standard circuit analysis of Fig. 2 then leads to an expression for the volume velocity $U_d$ of the membrane. Use of Eq. (1) results in the following expression for the cone excursion:

$$T_{b}^2 = M_p C_{ab} \quad (14)$$

and

$$\alpha = \frac{C_{ms}}{C_{ab}} \quad (15)$$

For reasons of convenience Eq. (12) will be preferred to Eq. (11) in determining the voice-coil impedance $Z_{vc}(s)$. Applying Eq. (12), changing every $Q_{ts}$ appearing in Eq. (13) to $Q_{ms}$ according to Section 2, and making the substitution $s = j \omega$ results in an expression for the impedance as a function of the angular frequency $\omega$.

3 IMPEDANCE-PARAMETER RELATIONS OF A CONVENTIONAL VENTED-BOX SYSTEM

In order to illustrate the results of the preceding section, an analysis is made of the impedance of the con-

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Minima occur when the numerator of the second part becomes zero. The two solutions $\omega_{1,2}$ are

$$\omega_{1,2} = \frac{1}{T_b} = \omega_b .$$  \hspace{1cm} (17)

The maxima are determined as follows. For minimum-phase systems it is known that at frequencies of magnitude extrema the phase is zero. As the imaginary parts of numerator and denominator are identical except for a factor $Q_m$, an impedance maximum occurs if the real part of the denominator is zero. This observation leads directly to the value of the maxima,

$$Z_{nc,\text{max}} = R_e \left(1 + \frac{Q_m}{Q_e}\right).$$  \hspace{1cm} (18)

From the requirement for the real part of the denominator a polynomial of the second degree in $\omega^2$ is obtained. The angular frequencies of the maxima of Eq. (16) will be called $\omega_1$, $\omega_2$. The following two relations can then be found,

$$\omega_1^2 \omega_2^2 = \frac{1}{T_s^2 T_b^2}$$  \hspace{1cm} (19)

$$\omega_1^2 + \omega_2^2 = \frac{T_s^2 + T_b^2 (1 + \alpha)}{T_s^2 T_b^2} .$$  \hspace{1cm} (20)

From these equations $T_s$ and $\alpha$ can be solved. Subsequently the equations are expressed in frequencies $f$ instead of angular frequencies $\omega$ and time constants $T$. The indices remain unchanged. This leads to the following well-known [4] solutions for the parameters:

$$f_b = f_1$$  \hspace{1cm} (21)

$$f_s = \frac{f_1 f_H}{f_b}$$  \hspace{1cm} (22)

$$\alpha = \frac{(f_H - f_b^2)(f_s^2 - f_1^2)}{f_1^2 f_H^2} .$$  \hspace{1cm} (23)

An example of a typical impedance curve can be found in Fig. 6. Note that for this impedance curve port losses, enclosure absorption, and leakage losses were included. This means Eq. (18) cannot be used for the curve of Fig. 6. The other equations give fairly good approximations provided that the losses are not too large [4]. It is important to note that $f_s$ is the resonance frequency of the loudspeaker in the particular enclosure, that is, the resonance frequency with the air load of the pertinent enclosure included. This resonance frequency $f_s$ usually is shifted somewhat relative to the free-air resonance frequency. Due to this resonance shift the $Q$ factors of the loudspeaker change also. Note further that in Eqs. (21)–(23) only the frequencies of the minima and maxima appear and not the amplitudes. The analysis of lower order systems, such as closed-box systems, can be performed in a similar way, but will not be discussed here.

4 IMPEDANCE-PARAMETER RELATIONS OF A SINGLE-VENTED BANDPASS SYSTEM

The analysis of the impedance of the single-vented bandpass system [5] of Fig. 7 resembles the analysis of the vented-box system of the previous section. Only the results of the calculations will be given. The acoustic impedance seen by the loudspeaker membrane is represented by the circuit of Fig. 8. Compliance $C_{ab1}$ is combined with loudspeaker compliance $C_{as}$ into the single compliance $C_{at}$,

$$C_{at} = \frac{C_{as} C_{ab1}}{C_{as} + C_{ab1}} .$$  \hspace{1cm} (24)

![Fig. 6](image)

Typical electrical impedance curve of conventional vented-box system of Fig. 4 and single-vented bandpass system of Fig. 7 showing one minimum and two maxima.

![Fig. 7](image)

Single-vented bandpass system.

![Fig. 8](image)

Acoustical analogous circuit of single-vented bandpass enclosure of Fig. 7.
The resonance frequency of the loudspeaker loaded by the volume of compartment 1 is defined by

$$\frac{1}{(2\pi f_{e1})^2} = T_{e1}^2 = M_{as}C_{ut}.$$  \hspace{1cm} (25)

The compliance ratios are defined as

$$\alpha_1 = \frac{C_{as}}{C_{ab1}},$$  \hspace{1cm} (26)
$$\alpha_2 = \frac{C_{as}}{C_{ab2}}.$$  \hspace{1cm} (27)

A convenient parameter is the total compliance ratio

$$\alpha_t = \frac{C_{ut}}{C_{ab2}}.$$  \hspace{1cm} (28)

Then the following solutions for parameters $f_{b2}$, $f_{c1}$, and $\alpha_t$ can be found:

$$f_{b2} = f_1$$ \hspace{1cm} (29)
$$f_{c1} = f_1 \sqrt{1 + \alpha_1} = \frac{f_1 f_H}{f_{b2}}$$ \hspace{1cm} (30)
$$\alpha_t = \frac{(f_H^2 - f_{b2}^2)}{(f_1 f_H)}$$ \hspace{1cm} (31)

An example of a typical impedance curve is given in Fig. 6.

### 5 IMPEDEANCE-PARAMETER RELATIONS OF DOUBLE-VENTED SYSTEM TYPE 1

The loudspeaker system of Fig. 9 will be called double-vented system type 1. The acoustic impedance $Z_a$ seen by the loudspeaker membrane can be represented by the equivalent circuit of Fig. 10 [6], [8]. Analyzing the circuit of Fig. 2 and using Eqs. (1) and (12) results in an expression for the electrical impedance $Z_{vc}(\omega)$ of the total system,

$$Z_{vc}(\omega) = R_e \left\{ \begin{array}{l}
1 + \frac{1}{Q_e} \\
\quad + j \omega^5 T_{e1}^2 T_{e2} T_s - j \omega^3 (T_{e1}^2 + T_{e2}^2) T_s + j \omega T_s \\
\quad - \omega T_s T_{e1}^2 T_{e2}^2 + j \omega^5 \frac{T_s}{Q_{ms}} T_{e1}^2 T_{e2}^2 \\
\quad + \omega^4 [T_{e1}^2 T_{e2}^2 + T_s T_{e2}^2 + T_{e1}^2 T_{e2}^2 (1 + \alpha_1 + \alpha_2)] \\
\quad - \omega^3 \left( \frac{T_s}{Q_{ms}} T_{e2}^2 + \frac{T_s}{Q_{ms}} T_{e1}^2 \right) \\
\quad + \omega^2 [T_{e1}^2 + T_{e2}^2 (1 + \alpha_1) + T_{e2}^2 (1 + \alpha_2)] + \omega \frac{T_s}{Q_{ms}} + 1
\end{array} \right\}$$  \hspace{1cm} (32)
\[ \omega_2 = \frac{1}{T_{b2}} = \omega_{b2}. \] (36)

From the requirement for zero real part of the denominator for the impedance maxima, a polynomial of the third degree in \( \omega^2 \) is obtained. A simple way to calculate the roots of this polynomial is to look at the relations between combinations of the roots and the coefficients of the polynomial. The frequencies of the maxima of Eq. (32) will be called \( \omega_L, \omega_M, \omega_H \). The following three relations can then be found:

\[ \omega_L^2 + \omega_M^2 + \omega_H^2 = \frac{T_s^2T_{b1}^2 + T_s^2T_{b2}^2 + T_{b1}^2T_{b2}^2 (1 + \alpha_1 + \alpha_2)}{T_s^2T_{b1}^2T_{b2}^2}. \] (37)

\[ \omega_L^2\omega_M^2 + \omega_L^2\omega_H^2 + \omega_M^2\omega_H^2 = \frac{T_s^2 + T_{b1}^2 (1 + \alpha_1) + T_{b2}^2 (1 + \alpha_2)}{T_s^2T_{b1}^2T_{b2}^2}. \] (38)

\[ \omega_L^2\omega_M^2\omega_H^2 = \frac{1}{T_s^2T_{b1}^2T_{b2}^2}. \] (39)

From these equations \( T_s, \alpha_1, \) and \( \alpha_2 \) can be solved. Using the same conventions for notation as in the preceding section leads to the following solutions for the parameters:

\[ f_{b1} = f_1 \] (40)

The formula for \( \alpha_2 \) is the same as for \( \alpha_1 \) except that \( f_{b1} \) and \( f_{b2} \) must be interchanged. A typical impedance curve is shown in Fig. 11.

Enclosure losses were not considered in the analysis of this section. It is fairly easy to calculate the value of, for instance, leakage resistance [4] from the impedance curve. Additional research is required regarding the topic of enclosure losses in bandpass systems. It is not quite clear whether the vented-box model of Small [4] is directly applicable to bandpass systems.

### 6 Impedance-Parameter Relations of Double-Vented System Type 2

The loudspeaker system of Fig. 12 will be called double-vented system type 2. The equivalent enclosure impedance circuit \( Z_e \) is shown in Fig. 13. Analyzing the circuit of Fig. 2 and using Eqs. (1) and (12) results in an expression for the electrical impedance \( Z_{ve}(\omega) \) of the total system.

\[ Z_{ve}(\omega) = R_e \left\{ \begin{array}{c}
1 + \frac{1}{Q_{es}} \\
- \omega_0 T_s^2 T_{b1}^2 T_{b2}^2 + \frac{\omega_0^2 T_s^2 T_{b1}^2 T_{b2}^2}{T_{b1}^2 T_{b2}^2} \\
+ \omega_0^2 \left[ T_s^2 T_{b1}^2 + T_s^2 T_{b2}^2 \left( 1 + \frac{\alpha_2}{\alpha_1} \right) + T_{b1}^2 T_{b2}^2 \left( 1 + \alpha_1 + \alpha_2 \right) \right]
- \omega_0^2 \left[ T_s^2 T_{b2}^2 \left( 1 + \frac{\alpha_2}{\alpha_1} \right) + \frac{T_s^2}{Q_{ms}} T_{b1}^2 \right]
- \omega_0^2 \left[ T_s^2 + T_{b1}^2 \left( 1 + \alpha_1 \right) + T_{b2}^2 \left( 1 + \frac{\alpha_2}{\alpha_1} \right) \right] + \frac{T_s^2}{Q_{ms}} + 1
\end{array} \right\} \] (44)

with the same definitions of \( T_{b1}, T_{b2}, \alpha_1, \) and \( \alpha_2 \) as in Eqs. (33), (34) and (26), (27), respectively.

Minima in this expression occur for the case that the numerator of the final term becomes zero. This leads

\[ f_{b2} = f_2 \] (41)

\[ f_s = \frac{f_{b1} f_{b2}}{f_{b1} f_{b2}} \] (42)

\[ \frac{\alpha_1}{\alpha_2} = \frac{f_s^2 f_{b2}^2 + f_{b1}^2 (f_{b1} + f_{b2}) - f_{b1} f_{b2} - f_{b1} f_{b2} + f_{b1} f_{b2}}{f_s^2 (f_{b1}^2 - f_{b2}^2)}. \] (43)
to the following relations between the roots, that is, the minima $\omega_1$ and $\omega_2$ and the polynomial coefficients,

$$\omega_1^2 + \omega_2^2 = \frac{T_{b1}^2 + T_{b2}^2 \left( 1 + \frac{\alpha_2}{\alpha_1} \right)}{T_{b1}^2 T_{b2}^2} \triangleq (2\pi)^2 d_1$$  \hspace{1cm} (45)

$$\omega_1^2 \omega_2^2 = \frac{1}{T_{b1}^2 T_{b2}^2} \triangleq (2\pi)^4 d_2.$$  \hspace{1cm} (46)

For the formulas resulting from the maxima the same procedure as before is used. The following three relations can then be found:

$$\omega_1^2 + \omega_3^2 + \omega_4^2 = \frac{T_{s}^2 T_{b1}^2 + T_{s}^2 T_{b2} (1 + \frac{\alpha_5}{\alpha_1}) + T_{b1}^2 T_{b2}^2 (1 + \alpha_1 + \alpha_2)}{T_{s}^2 T_{b1}^2 T_{b2}^2} \triangleq (2\pi)^2 d_3$$ \hspace{1cm} (47)

$$\omega_1^2 \omega_3^2 + \omega_1^2 \omega_4^2 + \omega_3^2 \omega_4^2 = \frac{T_{s}^2 + T_{b1}^2 (1 + \alpha_1) + T_{b2}^2 \left( 1 + \frac{\alpha_2}{\alpha_1} \right)}{T_{s}^2 T_{b1}^2 T_{b2}^2} \triangleq (2\pi)^4 d_4$$  \hspace{1cm} (48)

$$\omega_1^2 \omega_3^2 \omega_4^2 = \frac{1}{T_{s}^2 T_{b1}^2 T_{b2}^2} \triangleq (2\pi)^6 d_5.$$  \hspace{1cm} (49)

$$f_{b1} = \frac{\sqrt{d_2}}{d_{b1}}$$ \hspace{1cm} (50)

$$f_{b2} = \frac{\sqrt{d_2}}{d_{b1}}$$ \hspace{1cm} (51)

$$\alpha_1 = \frac{d_3 d_4 - d_2^2}{d_3 d_5} f_{b1}$$ \hspace{1cm} (52)

$$\alpha_2 = \frac{d_3 d_4 - d_2 d_5}{d_5} - 1 - \alpha_1.$$ \hspace{1cm} (53)

A typical impedance curve is shown in Fig. 11.

7 Measurements

7.1 Subwoofer Systems

Two subwoofer systems were built, a double-vented system type 1 and a double-vented system type 2. In both systems a Philips AD70654W4 loudspeaker was
used. The relevant parameters of this loudspeaker are \( f_1 = 47.2 \text{ Hz} \), \( V_{ss} = 25.4 \text{ L} \), \( R_e = 3.37 \text{ Ω} \), \( Q_{ms} = 3.6 \), and \( Q_{ts} = 0.27 \). The cabinets were constructed from 22-mm medium density fiberboard.

### 7.1.1 Double-Vented System Type 1

The volumes of the compartments of this system are \( V_{b1} = 22.5 \text{ L} \) and \( V_{b2} = 9.0 \text{ L} \). The port lengths are \( L_1 = 200 \text{ mm} \) and \( L_2 = 200 \text{ mm} \). The port diameters are \( D_1 = 68 \text{ mm} \) and \( D_2 = 68 \text{ mm} \).

### 7.1.2 Double-Vented System Type 2

The volumes of the compartments of this system are \( V_{b1} = 11.0 \text{ L} \) and \( V_{b2} = 20.5 \text{ L} \). The port lengths are \( L_1 = 260 \text{ mm} \) and \( L_2 = 70 \text{ mm} \). The port diameters are \( D_1 = 68 \text{ mm} \) and \( D_2 = 68 \text{ mm} \).

### 7.2 Methods

The impedance curves were obtained with a 16-bit AD–DA board installed in a 80486-based PC. The output of the measurement system was connected to a power amplifier. The output of the power amplifier was loaded by a series connection of a 47-Ω resistor and the loudspeaker system. The input signal was a sinusoidal signal with frequency swept logarithmically from 5 to 300 Hz. The sampling frequency was 8 kHz and the total number of samples was 100 000, resulting in a total signal length of 12.5 s. Two sweeps were used. During the first sweep the voltage across the loudspeaker terminals was measured. Subsequently the loudspeaker system was replaced by a resistor of 4.7 Ω, and the voltage across the 4.7-Ω resistor was measured during the second sweep.

Of both measured signals a discrete Fourier transform was calculated. The impedance \( Z_v(\omega) \) of the loudspeaker system was obtained by straightforward circuit analysis of the two measurement conditions. The impedance \( Z_v(\omega) \) then can be expressed in the two Fourier transformed signals and the two resistor values (here 47 and 4.7 Ω). The output voltage of the amplifier was assumed to be independent of the load impedance.

### 7.3 Results

The design parameters \( f_1, f_{b1}, f_{b2}, \alpha_1, \) and \( \alpha_2 \), which are directly related to the parameters of Section 7.1, were compared to the parameters obtained from the measured impedance curves.

#### 7.3.1 Design Parameters

The compliance ratios \( \alpha_1 \) and \( \alpha_2 \) are calculated with \( \alpha_1 = V_{ss}/V_{b1} \) and \( \alpha_2 = V_{ss}/V_{b2} \). The port masses \( M_{sp1} \) and \( M_{sp2} \) are calculated by considering that one end of the port terminates in a flange [10], with the exception of port 1 of the double-vented system type 2, where no end terminates in a flange. Then Eqs. (33) and (34) are used for calculating the resonance frequencies \( f_{b1} \) and \( f_{b2} \). The design parameters are collected in Table 1 for double-vented system type 1 and in Table 2 for double-vented system type 2.

#### 7.3.2 Parameters Obtained from Impedance Measurements

The impedance curve for double-vented system type 1 is shown in Fig. 14. From this curve the following extrema were found:

- Minima: \( f_1 = 44.3 \text{ Hz} \), \( f_2 = 68.3 \text{ Hz} \)
- Maxima: \( f_L = 23.3 \text{ Hz} \), \( f_M = 52.6 \text{ Hz} \), \( f_H = 117.0 \text{ Hz} \)

Substitution of these frequencies in Eqs. (40)–(43) gives values for the parameters \( f_1, f_{b1}, f_{b2}, \alpha_1, \) and \( \alpha_2 \). These measured parameters are collected in Table 1.

The impedance curve for double-vented system type 2 is shown in Fig. 15. From this curve the following extrema were found:

- Minima: \( f_1 = 42.7 \text{ Hz} \), \( f_2 = 84.2 \text{ Hz} \)
- Maxima: \( f_L = 26.6 \text{ Hz} \), \( f_M = 52.2 \text{ Hz} \), \( f_H = 128.9 \text{ Hz} \)

Substitution of \( f_1, f_2, f_L, f_M, \) and \( f_H \) in Eqs. (45)–(49) yields

\[
\begin{align*}
d_1 &= 8.9129 \times 10^3, \\
d_2 &= 1.2926 \times 10^3, \\
d_3 &= 2.0048 \times 10^4, \\
d_4 &= 5.8958 \times 10^6,
\end{align*}
\]
Substitution of \( d_1 \) to \( d_5 \) in Eqs. (50)–(54) then gives values for the parameters \( f_s, f_{b1}, f_{b2}, \alpha_1, \) and \( \alpha_2 \). These measured parameters are collected in Table 2.

### 7.4 Discussion

The design parameters and the measured parameters for both the double-vented system type 1 (Table 1) and the double-vented system type 2 (Table 2) show good agreement. In order to get an estimate of the desired accuracy of the frequency measurements the sensitivity of Eqs. (40)–(43) and (50)–(54) with respect to the measured frequencies was calculated. The sensitivity of the compliance ratios for errors in the frequency measurements is quite high. The relative error in \( \alpha_1 \) and \( \alpha_2 \) typically is three to ten times higher than the relative error in the measured frequencies \( f_s, f_{b1}, f_{b2}, f_1, \) and \( f_2 \). The relative error in \( f_s, f_{b1}, \) and \( f_{b2} \) is about the same as the relative error in the measured frequencies.

Simulations showed that the parameters had no abnormally high sensitivity with respect to leakage losses to the outside, leakage losses between compartments, port losses, and box absorption losses.

### 8 CONCLUSION

Expressions were derived for calculating the small-signal parameters of four low-frequency loudspeaker systems from the frequencies of minima and maxima in the electrical impedance. The expression for the electrical impedance was obtained directly from the normalized displacement function without transformation of analogous circuit elements from the acoustical to the electrical domain. Measured parameters of two sixth-order systems showed good agreement with design parameters. For future work, an analysis of enclosure losses can be made.

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### 10 REFERENCES


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