DETERMINISTIC AND ROBUST OPTIMISATION STRATEGIES FOR METAL FORMING PROCESSES

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ABSTRACT

Product improvement and cost reduction have always been important goals in the metal forming industry. The rise of Finite Element simulations for metal forming processes has contributed to these goals in a major way. More recently, coupling FEM simulations to mathematical optimisation techniques has shown the potential to make a further contribution to product improvement and cost reduction.

Mathematical optimisation consists of the modelling and solving of optimisation problems. Although both the modelling and the solving are essential for successfully optimising metal forming problems, much of the research published until now has focussed on the solving part, i.e. the development of a specific optimisation algorithm and its application to a specific optimisation problem for a specific metal forming process.

In this paper, we propose a generally applicable optimisation strategy which makes use of FEM simulations of metal forming processes. It consists of a structured methodology for modelling optimisation problems related to metal forming. Subsequently, screening is applied to reduce the size of the optimisation problem by selecting only the most important design variables. Screening is also utilised to select the best level of discrete variables, which are in such a way removed from the optimisation problem. Finally, the reduced optimisation problem is solved by an efficient optimisation algorithm. The strategy is generally applicable in a sense that it is not constrained to a certain type of metal forming problems, products or processes. Also any FEM code may be included in the strategy.

However, the above strategy is deterministic, which implies that the robustness of the optimum solution is not taken into account. Robustness is a major item in the metal forming industry, hence we extended the deterministic optimisation strategy in order to be able to include noise variables (e.g. material variation) during optimisation. This yielded a robust optimisation strategy that enables to optimise to a robust solution of the problem, which contributes significantly to the industrial demand to design robust metal forming processes. Just as the deterministic optimisation strategy, it consists of a modelling, screening and solving stage.

The deterministic and robust optimisation strategies are compared to each other by application to an analytical test function. This application emphasises the need to take robustness into account during optimisation, especially in case of constrained optimisation. Finally, both the deterministic and the robust optimisation strategies are demonstrated by application to an industrial hydroforming example.

Keywords: Metal forming, FEM, optimisation, robustness, reliability

1. INTRODUCTION

During the last decades, Finite Element (FEM) simulations of metal forming processes have become important tools for designing feasible production processes. In more recent years, coupling FEM simulations to mathematical optimisation techniques evolved to address two industrial needs: (i) Designing optimal metal forming processes instead of only feasible ones (better products, lower costs); and (ii) Solving problems in manufacturing.

Mathematical optimisation consists of two major phases: the modelling and the solving of the optimisation problem. The modelling phase consists of:

1. Selecting a number of design variables the user is allowed to adapt;
2. Choosing an objective function, i.e. the optimisation aim;
3. Taking into account possible constraints.

The modelled optimisation problem can subsequently be solved using an appropriate optimisation algorithm. Modelling and solving are both crucial parts when applying optimisation techniques: if the problem is not modelled well, optimisation will not yield an improvement with respect to the industrial metal forming problem; if the algorithm does
not suit the optimisation model, the problem will not be
solved efficiently or not solved at all [1].

In this paper, we propose an optimisation strategy for
metal forming processes that addresses both the modelling
and the solving part. This “deterministic” optimisation
strategy is introduced in Section 2. A major item in
industrial metal forming is robustness. For instance, material
variation is causing many costly problems in the metal
forming industry. Therefore, it is important to take into
account the influence of noise variables [2]. The
“deterministic” optimisation strategy is extended to a
“robust” optimisation strategy that also takes into account
these noise variables. The “robust” strategy is introduced in
Section 3. In Section 4, we compare deterministic and robust
optimisation by their application to an analytical test
function. To demonstrate the applicability of both strategies
to industrial metal forming processes, they are applied to a
hydroforming process in Section 5.

2. DETERMINISTIC OPTIMISATION STRATEGY

The proposed optimisation strategy is published in detail
in [3]. This section contains a summary.

The strategy consists of three stages:
1. Modelling the optimisation problem;
2. Screening to reduce the optimisation problem’s
   size;
3. Solving the optimisation problem using an
   optimisation algorithm.

2.1. Modelling
The first stage is to model the optimisation problem. It
is quite a challenge to design a structured methodology that
is on the one hand applicable to any kind of metal forming
problem, product and process, but on the other hand yields a
specific mathematical formulation of the optimisation
problem.

We attempted to overcome this paradox by consulting
specialists at several large metal forming companies. This
resulted in a large number of industrially relevant objectives,
constraints and design variables. Subsequently, these
quantities have been structured using the Product
Development Cycle [4], which has been applied to metal
products and their manufacturing processes [3]. The final
result of this research is the following 7 step methodology:

1. Determine the appropriate optimisation stage;
2. Select only the necessary responses;
3. Select one response as objective function, the
   others as implicit constraints;
4. Quantify the objective function and implicit
   constraints;
5. Select possible design variables;
6. Define the ranges on the design variables;
7. Identify explicit constraints.

Without going into detail, we conclude the modelling
stage by emphasising that following this 7 step methodology
is applicable to any metal forming problem and yields a
specific mathematical optimisation model, which can
subsequently be solved using a suitable optimisation
algorithm. The 7 step methodology is further demonstrated
in Section 5 when it is applied to an industrial hydroforming
process.

2.2. Screening
The modelling stage yields a specific optimisation
model. However, many design variables may be present,
which makes the problem time consuming to solve.
Additionally, discrete design variables may be present,
which cannot be solved by the selected optimisation
algorithm. It is worthwhile to invest some time in reducing
the number of design variables and removing discrete design
variables before applying the optimisation algorithm. This is
done in the screening stage.

For reducing the number of variables, we propose to
screen the importance of the design variables by applying a
Design Of Experiments (DOE) plan. Applying DOE, one
cleverly selects a couple of combinations of the design
variables at which one would like to evaluate the responses
(objective function and implicit constraint values in case of
optimisation). These response measurements can
subsequently be used to estimate the effect of the design
variables on the responses.

In case of screening, we propose to use a Resolution III
fractional factorial DOE strategy [5]. Resolution III designs
allow for independently estimating the linear effects of the
design variables on the responses. After having run the
corresponding FEM simulations, the linear effects can be
estimated by applying statistical techniques such as
ANalysis Of Variance (ANOVA) [5]. The amount and
direction of the effect of each variable on each response can
be nicely displayed in Pareto and Effect plots. An example
of a Pareto plot is presented in Figure 1. Using these
techniques, the variables with the largest effects may be kept
in the optimisation model whereas the variables having less
effect may be omitted. In such a way, the amount of design
variables may be significantly reduced while maintaining
control over objective function and implicit constraints
during optimisation.

Discrete design variables are removed by applying
Mixed Arrays [6] that provide a DOE for combined
continuous and discrete variables. After having run the

Figure 1: Pareto plot
corresponding FEM simulations, one can determine the average response for each level of the discrete variable. The level providing the lowest average objective function value (mean effect) provides the best setting for the discrete design variable. That is, if the objective function is minimised. In such a way, a discrete variable may be replaced by its estimated best level, which removes the discrete variables from the optimisation model.

After screening, the model contains only a few continuous design variables, which can subsequently be solved efficiently using an appropriate optimisation algorithm.

2.3. Solving

Details on the specific algorithm we developed have been presented in several publications, see e.g. [7-9]. An overview of the algorithm is presented in Figure 2(a). It comprises a spacefilling Latin Hypercubes Design Of Experiments (DOE) strategy, Response Surface Methodology (RSM) and Kriging metamodelling and validation techniques, and a multistart SQP algorithm for optimising the metamodels. The algorithm allows for sequential improvement of the accuracy and can thus be denoted as a Sequential Approximate Optimisation (SAO) algorithm.

The efficiency of the algorithm has been assessed by comparing it to other optimisation algorithms and applying all algorithms to two forging processes, see [10,11].

3. ROBUST OPTIMISATION STRATEGY

The robust optimisation strategy differs from the deterministic strategy in the modelling, optimisation and evaluation parts.

Concerning the modelling, noise variables are included in addition to deterministic control variables. For the noise variables, a normal distribution is assumed. For each response (objective function or implicit constraint), one now obtains a response distribution \((\mu_y, \sigma_y)\) instead of a deterministic response value \(y\). As objective function \(f\) one can optimise \(\mu_y\), \(\sigma_y\), or a weighted sum \(\mu_y \pm w\sigma_y\). If \(\mu_y\) or \(\sigma_y\) are optimised, it is advised to include the weighted sum as a constraint: this takes into account process reliability in the optimisation problem. Also other constraints \(g\) are taken into account as a weighted sum \(\mu_g \pm w\sigma_g\).

Figures 2(b) and (c) compare the differences in the optimisation algorithms and optimum evaluation for the deterministic and robust optimisation strategies. The difference in optimisation is the determination of the separate metamodels for \(\mu_y\) and \(\sigma_y\). For this, we employ a Single Response Surface technique, which fits one metamodel in both the control and noise design variable space, e.g. the following RSM metamodel which is quadratic in the design variable space and linear + interaction in the noise variable space [5]:

\[
y(x, z) = \beta_0 + x^T \beta + x^T B x + z^T \gamma + x^T \Delta z + \epsilon \quad (1)
\]
where $y$ is a single metamodel of a response dependent on the control variables $x$ and noise variables $z$. $\beta_0$, $\beta$, $B$, $\gamma$ and $\Delta$ denote the fitted regression coefficients and $\varepsilon$ is the random error term. From Equation 1, one can analytically determine two RSM metamodels for mean and variance [5]:

$$
\mu_y = E[y(x,z)] = \beta_0 + x^T \beta + x^T B x \\
\sigma_y^2 = \text{var}[y(x,z)] = \sigma^2 \left( \gamma^T + x^T \Delta \right) \left( \gamma + \Delta^T x \right) + \sigma^2
$$

with $\mu_y$ and $\sigma_y^2$ the metamodels for mean and variance of the response.

When Kriging is employed instead of RSM, an analytical derivation of $\mu_y$ and $\sigma_y^2$ is not possible. In this case a Monte Carlo Analysis (MCA) is run on the fitted metamodel as shown in Figure 2(c). Single Response Surface techniques are a relatively efficient way of robust optimisation [5].

The difference between robust and deterministic optimisation (see Figure 2) in the evaluation of the optimum $X^*$ is that, in the deterministic case, this can be done by running one final FEM calculation. In case the robustness and reliability need to be assessed after optimisation, it is necessary to run an MCA using FEM calculations, which is quite time consuming.

4. DETERMINISTIC VS. ROBUST OPTIMISATION

The robust optimisation problem is modelled as follows:

$$
\min_{\mu_f} \\
\text{s.t. } \mu_f + 3\sigma_f \leq 50 \\
\mu_g + 3\sigma_g \leq 0 \\
1 \leq x_1, x_2 \sim \mathcal{N}(\mu, 0.4) \leq 10
$$

Again the unconstrained ($g$ omitted) and the constrained problem have been optimised, this time using the robust optimisation strategy. 100 function evaluations have been run for each optimisation. Both corresponding optima are again displayed in Figure 3(b). After optimisation, the reliability of all optima has been evaluated using an MCA of 20000 function evaluations.

Figure 4 compares the results of deterministic and robust unconstrained optimisation. The scrap rate has been reduced from 0.92% for the deterministic optimum to <<0.005% for the robust optimum.

The improvement of the robust optimisation strategy with respect to the deterministic one is even much more dramatic in constrained cases as depicted in Figure 5. For the deterministic optimum, the scrap rate due to violation of the constraint $g$ is 50.3% (Figure 5(b)). For the robust optimum, Figure 5(d) shows that the scrap rate has been reduced to 0.1%, which nicely corresponds to the $3\sigma$ reliability level modelled in Equation 4.

5. APPLICATION TO HYDROFORMING

Both the deterministic and robust optimisation strategies will now be applied to optimise the hydroforming process of an automotive part designed by Corus, which is depicted in Figure 6. Figure 6 shows the FEM model of half the part. AutoForm has been used as FEM code.

5.1. Deterministic optimisation

For deterministic optimisation, we follow the three stage optimisation strategy: modelling, screening and solving.
Figure 4: Response distributions: (a) Deterministic unconstrained optimum; (b) Robust unconstrained optimum

Figure 5: (a) Deterministic optimum $f$; (b) Deterministic optimum $g$; (c) Robust optimum $f$; (d) Robust optimum $g$;
5.1.1 Modelling

We follow the 7 step methodology for modelling the optimisation problem.

**Step 1:** Determine the appropriate optimisation stage: Aim of optimisation is to design the manufacturing process in order to produce the part presented in Figure 6;

**Step 2:** Select only the necessary responses: An essential product property is the outer shape accuracy of the part (filling) and necking/crack defects should not occur;

**Step 3:** Select one response as objective function, the others as implicit constraints: The objective is to prevent necking, the constraint is the outer shape which has been formulated as the distance between the final product and the die;

**Step 4:** Quantify the objective function and implicit constraints: Final quantification of the responses is based on several properties of the responses and is performed by a table proposed in [3];

**Step 5:** Select possible design variables: The Process Design stage identified in Step 1 implies that the so-called Process Variables (PVs) are possible design variables. PVs can be categorised further, see [3]:

- Part, Workpiece and Tool Geometries: The Part Geometry is fixed by the designer, as is the Tool Geometry since the manufacturing process contains only one forming stage. Remaining is the Workpiece Geometry, in this case the initial tube radius and thickness;
- Workpiece and Tool Material: For the Workpiece Material, one can choose between 7 steel types, which makes this a discrete variable. The Tool Material is assumed rigid in the FEM simulation, thus no design variables are taken into account from this category;
- Load paths and other process parameters: The typical internal pressure and axial feeding load paths for hydroforming are presented in Figure 7 and can be described by nine design variables. Another process parameter taken into account is the friction between product and die;

**Step 6:** Define the ranges on the design variables: Upper and lower bounds have been defined on all design variables;

**Step 7:** Identify explicit constraints: Explicit constraints are defined by impossible combinations of the design variables. Explicit constraints for the hydroforming application are related to the axial feeding and internal pressure load paths shown in Figure 7.

The 7 step methodology yielded the following mathematically formulated optimisation model:

\[
\begin{align*}
\min \max_N & \left( \frac{e_1(e_j)}{e_1^M(e_j)} - 1 \right) \\
\text{s.t.} & \max\left( \text{dist}_{\text{tool-product}} \right) - 3 \leq 0 \\
& T_1 - T_2 \leq 0 \\
& T_2 - T_3 \leq 0 \\
& P_1 - P_3 \leq 0 \\
& P_2 - P_3 \leq 0 \\
& \nu_1 - \nu_2 \leq 0 \\
& \nu_3 - \nu_4 \leq 0 \\
56 \text{ mm} & \leq R \leq 60 \text{ mm} \\
1.5 \text{ mm} & \leq t_1 \leq 3.5 \text{ mm} \\
M & = 0,1,\ldots,6 \\
0.1 & \leq T_j \leq 0.1 \text{ s} \\
0.1 & \leq T_j \leq 0.2 \text{ s} \\
0.2 & \leq T_j \leq 0.3 \text{ s} \\
0 \text{ MPa} & \leq p_i \leq 25 \text{ MPa} \\
25 \text{ MPa} & \leq p_i \leq 50 \text{ MPa} \\
50 \text{ MPa} & \leq p_i \leq 300 \text{ MPa} \\
0 \text{ mm/} \text{s} & \leq \nu_i \leq 150 \text{ mm/} \text{s} \\
0 & \leq \nu_i \leq 150 \text{ mm/} \text{s} \\
0.05 & \leq \mu \leq 0.12
\end{align*}
\]

Where R and t_1 are the initial tube radius and thickness and M is the discrete variable denoting the 7 materials: Material
The load paths are described by the pressure \( p \), axial feeding velocity \( v \) and time \( T \) parameters. \( \mu \) denotes the coefficient of friction.

Hence, the total optimisation model consists of 2 responses (1 objective function, 1 implicit constraint), 6 explicit constraints, 12 continuous design variables and 1 discrete variable.

### 5.1.2 Screening
Before the Sequential Approximate Optimisation algorithm can be applied, the discrete variable \( M \) needs to be removed and the number of continuous design variables need to be reduced. Screening techniques as introduced in Section 2.2 are used for this.

Applying an MA.28.12.7.1 Mixed Array DOE (see [6]), running the corresponding 28 FEM simulations and calculating the mean effects yielded the discrete variable selection, in this case material selection.

Subsequently, a 16 run fractional factorial DOE has been executed to screen the importance of the 12 continuous design variables.

The resulting Pareto plot for the objective function is presented in Figure 1. A similar Pareto plot can be generated for the implicit constraint. In such a way, screening techniques assisted in reducing the 12 variables to only 2: the initial tube radius \( R \) and the filling pressure \( p_3 \). The reduced optimisation model is now:

\[
\min f = \max N \frac{E_i (\epsilon_2)}{E_i^2 (\epsilon_2)} - 1 \\
\text{s.t.} \quad g = d_{\text{filling}} - 3 \leq 0 \\
56 \leq x_1 = R \leq 60 \text{ mm} \\
50 \leq x_2 = p_3 \leq 300 \text{ MPa}
\]

### 5.1.3 Solving
Applying the deterministic SAO algorithm from Figure 2, the optimum has been found after 53 FEM simulations. The convergence behaviour of SAO is shown in Figure 8. The optimisation results in Table 1 show that a 31% margin below the FLC has been reached, while a negative value for the constraint \( g \) denotes the filling of the product satisfies the demands.

However, it is well-known that material parameters such as R-values display variation. This input variation is transferred to the objective function and constraint. One can check the robustness and reliability of the obtained deterministic optimum by running a Monte Carlo Analysis (MCA). Figures 9(a) and (b) present the response histograms of a 200 FEM run MCA for both \( f \) and \( g \), respectively. As shown in Table 1, the scrap rate is 41.2%, which is totally due to violation of the filling constraint \( g \).

### 5.2. Robust optimisation
Let us now see whether the robust optimisation strategy is able to reduce this scrap rate. Following the robust optimisation strategy, the following robust optimisation problem has been modelled and subsequently solved.

#### 5.2.1 Modelling
As a basis for the robust optimisation model, we take the reduced deterministic optimisation model. The material variation (R-values) is added to the model as noise variables, and – as introduced in Section 3.1 – the responses are taken into account as stochastic variables instead of deterministic values. The robust optimisation model is now:

\[
\min \mu f \\
\text{s.t.} \quad \mu f + 3\sigma f \leq 0 \\
\mu g + 3\sigma g \leq 0 \\
56 \leq x_1 = R \leq 60 \text{ mm} \\
50 \leq x_2 = p_3 \leq 300 \text{ MPa} \\
\mu_z - 4\sigma_z \leq R_0, R_{45}, R_{90} \sim N(\mu_z, \sigma_z) \leq \mu_z + 4\sigma_z
\]

where \( R_0, R_{45} \) and \( R_{90} \) denote the material's R-values in the rolling, 45 degrees and transverse directions. Hence, it is tried to minimise the mean value of the objective function while putting a 3 sigma reliability demand on both the objective function and constraint.

#### 5.2.2 Solving
The robust optimisation algorithm has been applied to solve the robust optimisation problem. 200 FEM simulations have been run in the 5D combined control-noise variable space and a Kriging metamodel has been fitted and optimised. A 200 FEM analysis MCA was used to validate the robust optimum: its results are presented in Table 1, the response histograms are included in the Figures 9(c) and (d).

Although the scrap rate has been reduced to 27.9%, the table and the figures show that the constraint \( \mu_z + 3\sigma_z < 0 \) is not satisfied. This may be due to inaccuracy of the metamodel. Just as was the case for the analytical test...
function in the previous section, an accurate metamodel would have further reduced the scrap rate, if possible to the required 3 sigma level, i.e. a scrap rate of maximum 0.3%.

6. CONCLUSIONS AND FUTURE WORK

Robustness, reliability, optimisation and Finite Element simulations are of major importance to improve product quality and reduce costs in the metal forming industry. In this paper, both a deterministic and a robust optimisation strategy for metal forming processes have been proposed. They have been compared to each other by applying them to an analytical test function: for constrained cases, deterministic optimisation will yield a scrap rate of about 50% whereas the robust optimisation strategy reduced this scrap rate to the demanded 3 sigma reliability level. Both strategies have also been applied to an industrial hydroforming process. Applying the robust strategy above the deterministic one also reduced the scrap rate in this case.

Table 1: Optimisation results hydroforming

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>p1</th>
<th>f</th>
<th>g</th>
<th>Scrap rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic optimum</td>
<td>58.1</td>
<td>239</td>
<td>-2.313</td>
<td>-0.16</td>
<td>41.2%</td>
</tr>
<tr>
<td>Robust optimum</td>
<td>59.3</td>
<td>258</td>
<td>-2.34</td>
<td>-0.036</td>
<td>27.9%</td>
</tr>
</tbody>
</table>

The application to hydroforming underlines the potential of both strategies to optimise metal forming processes. The robust strategy explicitly takes into account noise variables such as material variation and optimises probability distributions of objective function and constraints in order to achieve a robust and reliable metal forming process. Although the scrap rate has not been reduced as much as required, the importance of including robustness during industrial metal forming processes using time consuming FEM simulations has been clearly demonstrated.

The scrap rate can be further reduced when the accuracy of the metamodel in increased. Future work comprises improving the metamodel – at least in the vicinity of the optimum – by developing sequential improvement strategies for the robust algorithm (see Figure 2).

ACKNOWLEDGMENTS

This research has been carried out in the framework of the project “Optimisation of Forming Processes “MC1.03162”, which is part of the research programme of the Netherlands Institute for Metals Research (NIMR). The industrial partners co-operating in this project are gratefully acknowledged.

![Figure 9](image-url)
REFERENCES


