Towards Efficient Modelling Of Macro And Micro Tool Deformations In Sheet Metal Forming

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Abstract. During forming, the deep drawing press and tools undergo large loads, and even though they are extremely sturdy structures, deformations occur. This causes changes in the geometry of the tool surface and the gap width between the tools. The deep drawing process can be very sensitive to these deformations. Tool and press deformations can be split into two categories. The deflection of the press bed-plate or slide and global deformation in the deep drawing tools are referred to as macro press deformation. Micro-deformation occurs directly at the surfaces of the forming tools and is one or two orders lower in magnitude.

The goal is to include tool deformation in a FE forming simulation. This is not principally problematic, however, the FE meshes become very large, causing an extremely large increase in numerical effort. In this paper, various methods are discussed to include tool elasticity phenomena with acceptable cost. For macro deformation, modal methods or ‘deformable rigid bodies’ provide interesting possibilities. Static condensation is also a well known method to reduce the number of DOFs, however the increasing bandwidth of the stiffness matrix limits this method severely, and decreased calculation times are not expected. At the moment, modeling Micro-deformation remains unfeasible. Theoretically, it can be taken into account, but the results may not be reliable due to the limited size of the tool meshes and due to approximations in the contact algorithms.

Keywords: Sheet Metal Forming, Die Design, Tool Elasticity, Press deformation

TOOL AND PRESS DEFORMATION

Deformation of the press and the tools occurs at various locations in the press. Four different categories are identified (see figure 1):

1. press-frame deformation
2. press deformation: deflection of the bed-plate and slide
3. tool deformation
4. tool-surface deformation

Press-frame deformation (1) is not considered to be significant for the deep drawing process. However, the deflection of the bed-plate and slide (2) can be quite substantial (in the order of magnitude of several millimeters), especially in large presses with a large tonnage that are used for parts made of high strength steels. The slide not only deflects, but it can also tilt slightly. Via the blank, the force of the slide and punch and the blankholder load are transferred to the die. This results in deformation of these tools (3), even though they are very heavy structures: The geometry of the die, blankholder and punch is in most cases optimized for casting only, structural calculations and optimizations are not carried out. The effects of tool deformations are not often documented in literature. The article [1] nicely shows how important this problem can be. Here different results were found for the cross-die deep drawing benchmark, when different arrangements of the tool supporting pins were used. The slightly different deformation of the tools during forming resulted in different benchmark results.
of the material that was tested. However, in some cases the blankholder is also designed to be flexible [2] on purpose, to be able to apply locally varying blankholder loads.

The previous deformations are regarded as 'macro-deformations' which means that they are global deformations with a large wavelength. Tool-surface deformation is a local deformation. It is mainly caused by thickening of the blank, for example at the die shoulder. These deformations are one or two orders of magnitude lower (max. 0.1 mm). However, friction forces are very large, so small changes in geometry may cause loss of contact and hence friction force so will have a large influence on the simulation results, blank draw-in and springback.

This phenomenon is the focus of almost all present publications on elastic deformations in tools [3, 4]. However, in most publications the tools were meshed so roughly that the resolution of the FE-models is much too low to capture these effects reliably. Also, the numerical side-effects of, for example, a penalty contact algorithm may be larger than the effect of tool deformation. There are strict hard-contact algorithms, but they can cause convergence problems in the highly nonlinear deep drawing simulation, or unrealistic friction peaks. The thesis [5] shows that vibration also occurs in the contact calculation. This has numerical reasons and damping is required, which may lead to artifacts as well. Concluding, extreme care has to be taken when such small deformations are assessed using FE analyses, and the results may not be reliable. This issue is reflected in the fact that most authors conclude that there is an influence but do not present quantitative results for realistic industrial problems.

The focus of this paper is to find a fast and effective way to incorporate press and tool deformation in the deep drawing simulation. Static condensation and modal methods are discussed as techniques for reducing the numerical cost of modeling tool and press macro deformation. Currently, reliable implementation of local deformation is considered not feasible on industrial scale.

### MACRO TOOL AND PRESS DEFORMATION

The research on macro tool and press deformation is relatively limited. Many publications have been focused on spring-damper models [6] which are particularly useful for dynamic (vibration) analyses in high-speed forming processes. Dynamic effects are also present in the relatively slow deep drawing process due to the strain-rate sensitivity of certain materials but they are not taken into consideration here. Additionally, the measurement and derivation of the model’s spring and damper constants has turned out to be complicated.

In other publications, the tools were modeled elastically within the forming simulation, or calculated externally during the simulation [4]. The tool meshes have to be extremely fine in order to preserve the contact surface smoothness, so the size of the problem is increased by an order of magnitude. The numerical cost becomes so high that such a calculation can only be carried out for simple academic geometries, and inclusion of the press structure is not feasible at the moment.

Tool and press deformation can be considered as a linear elastic problem. There are two techniques for reducing the size of a linear elastic FE calculation. Firstly, static condensation is a well-known method. The FE model can be reduced to generate only the output for a specific set of (retained) DOFs, the other DOFs are pre-solved on forehand. There is no loss of accuracy. This was demonstrated for tool elasticity modeling in [7]. The other method depends on modal reduction, and the amount of DOFs can be reduced much further, but as this is an approximate method, the accuracy is not retained. Therefore this method is useful for macro-deformations only. An academic example of modal reduction for tool elasticity was shown in [8]

### Static Condensation

The main idea of static condensation is to speed up the calculation by pre solving a part of the equation. The elastic FE problem is the following equation:

\[
[K]u = F
\]

Here \([K]\) is the \(n\) by \(n\) stiffness matrix, \(u\) is the displacement vector and \(F\) is the load vector. In the next equation, the ‘master’ displacements with the subscript \(a\) are to be retained, the displacements with subscript \(c\) are to be condensed out:

\[
\begin{bmatrix}
K_{aa} & K_{ac} \\
K_{ca} & K_{cc}
\end{bmatrix}
\begin{bmatrix}
u_a \\
u_c
\end{bmatrix} =
\begin{bmatrix}
F_a \\
F_c
\end{bmatrix}
\]

In case of the elastic tools, only the displacements of the nodes that are in contact with the blank are required, and the loads on the other nodes, which are condensed out, are zero, \(F_c=0\). Therefore equation 2 becomes:

\[
K' u_a = F_a
\]
with $K' = K_{aa} - K_{ac}K_{cc}^{-1}K_{ca}$. $K'$ has a smaller dimension, but the bandwidth has become much larger than the bandwidth of $K$. As the bandwidth is a major factor in the cost of the solution of the problem, static condensation is useful only when the amount of retained DOFs is much lower than the initial amount of DOFs. In the case of elastic tool modeling, still 20 to 40% of the DOFs are retained because the entire contact surface has to remain available, and therefore static condensation makes the calculation actually slower instead of faster.

For a blankholder of a roof panel deep drawing process by Daimler Chrysler [2], shown in figure 2, static condensation reduced the amount of DOFs by 62%. A set of loadcases was carried out with and without static condensation in ABAQUS/standard, and the CPU time for solving the system increased by more than a factor of 10 for the statically reduced calculation.

Figure 2. THE BLANKHOLDER MESH (TOP) AND THE RETAINED REGIONS (BOTTOM)

The reason why so many nodes have to remain is the meshing of solid bodies. In regular deep drawing simulations, the rigid tool surfaces are meshed with a large amount of elements that vary heavily in size and shape. This is done to keep the mesh size minimal while still capturing the fine geometrical details of the tool surfaces and to retain the surface smoothness. Because the elements are not deforming, the requirements to their shape are not very stringent. When the entire tool is meshed as a deformable solid, the element shape has to meet more geometrical conditions. To obtain the same smoothness on the contact surface, the mesh has to be dense at that location, and all these DOFs have are retained. This is demonstrated in figure 3.

Figure 3. A TYPICAL RIGID SURFACE MESH (TOP) AND A DEFORMABLE SOLID MESH (BOTTOM)

Modal Methods

Modal methods have the potential to reduce the amount of degrees of freedom (DOFs) much further. When the deformation is global, acceptable results can be obtained with as little as 10 DOFs, however the accuracy is slightly compromised. The modal reduction technique is far from new, but it is not available in (forming) simulation codes.

Instead of solving equation 1 directly, $[K]$ is decomposed into two matrices $[P]$ and $[D]$:

$$[K] = [P][D][P]^{-1}$$  \hspace{1cm} (4)

$[D]$ is a diagonal matrix, containing the eigenvalues $\lambda_i$ and $[P]$ is an (orthogonal) matrix, containing the eigenvectors $v_i$ or modes. In order to solve equation 1 the stiffness matrix needs to be inverted. When this so-called
eigen-decomposition has been carried out this can be calculated very quickly:

\[ [K]^{-1} = ([P][D][P]^T)^{-1} = [P][D]^{-1}[P]^T \]  

(5)

And \([D]^{-1}\) is directly calculated as follows:

\[ [D]^{-1} = \begin{bmatrix}
\lambda_1^{-1} & 0 & \ldots & 0 \\
0 & \lambda_2^{-1} & \ldots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \ldots & 0 & \lambda_m^{-1}
\end{bmatrix} \]  

(6)

The solution \(u\) can be approximated by taking into account only the lowest \(m\) eigenvalues and their corresponding eigenvectors. \([D]\) is now reduced to a \(m\) by \(m\) matrix \([\tilde{D}]\) and \([P]\) becomes a \(n\) by \(m\) matrix \([\tilde{P}]\): \[ u \approx [\tilde{P}]^T[\tilde{D}]^{-1}[\tilde{P}]F \]  

(7)

In figure 4 two modes are shown for a slide connection plate. With increasing eigenvalue, the mode becomes more complex, so it becomes possible to model more detailed deformations. Note that the modes do not contain boundary conditions, these are added later in the calculation, which will be discussed in the next section. Unlike the dynamic modes, these can be obtained from any standard FE code, the static modes do not have a physical meaning.

Figure 4. THE FOURTH (TOP) AND SIXTEENTH STATIC MODE (BOTTOM) OF A SLIDE CONNECTION PLATE

As an example, the die from [2] is loaded with the contact forces at the end of the forming stage, using ABAQUUS. The die is a solid mesh with 180,000 DOFs. Then, the same calculation was carried out using modal methods. Here, only 10 modes were used. The error in the nodal displacement is shown as a percentage of the maximum displacement (0.4mm) in the contour plot of figure 5.

Figure 5. Normalized error (%) in displacement

Modeling Interaction

To be able to model the press and tools as one system, the interaction between several (modally reduced) bodies needs to be modeled. Some bodies are connected to others, for example, pillars of the press frame carry the bed plate. This can be solved with regular boundary conditions. To model the interaction between the press frame and tools, for example the bed-plate and punch, also a contact algorithm is required. Because modally reduced bodies have such a small amount of DOFs, the deformation of the entire press and tool setup can be included with a negligible added cost. As an additional convenience, the modally reduced bodies can be archived for later use and recombed to check how a certain tool-set works on different presses.

The penalty method is used to enforce the boundary conditions. This is essential, as the two modally reduced bodies will never fulfill the boundary conditions exactly. The boundary conditions have to be transformed so, that they can be used in the modal approach too. The basic problem is to find a solution for the linear elastic problem, with a distance \(d\) between the DOFs \(\alpha\) and \(\beta\):

\[
\begin{cases}
[K]u = F \\
u_\alpha - u_\beta = d
\end{cases}
\]  

(8)

The following analysis is a variation on the calculation in [9], pages 194-197. The boundary function is rearranged into the following form:

\[ l^T p = d \]  

(9)
with \( \mathbf{I} = [0, \ldots, 0, l_a = -1, 0, \ldots, 0, l_b = 1, 0, \ldots, 0]^T \).

Equation 8 can be approximated in the following form:

\[
(\mathbf{K} + \mathbf{M}) \mathbf{u} = \bar{\mathbf{F}}
\]

with \( \mathbf{M} = k \mathbf{I}^T \) and \( \bar{\mathbf{F}} = \mathbf{F} - kd \). The variable \( k \) is the penalty constant. This new equation can be solved in a modal way, by decomposing the matrix \((\mathbf{K} + \mathbf{M})\). The boundary conditions are then 'embedded' in the modes, which is problematic for a contact algorithm, where boundary conditions have to be released or added during consequent iterations.

The decomposition of the stiffness matrix in equation 4 can be regarded as a change of basis for the equations. The solution is to transform the boundary condition matrix to this new basis. The combined problem looks like this:

\[
(\mathbf{P}^T \mathbf{D} \mathbf{P}^T + \mathbf{M}) \hat{\mathbf{u}} = \hat{\mathbf{F}}
\]

Now, the parameters \( \mathbf{u} \) and \( \bar{\mathbf{F}} \) are transformed using the \( \mathbf{P} \) matrix:

\[
\mathbf{u} = \mathbf{P} \hat{\mathbf{u}} \quad (12)
\]

\[
\bar{\mathbf{F}} = \mathbf{P} \hat{\mathbf{F}} \quad (13)
\]

With this transformation, equation 11 can be rewritten as

\[
\mathbf{P}^T (\mathbf{P}^T \mathbf{D} \mathbf{P}^T + \mathbf{M}) \mathbf{P} \hat{\mathbf{u}} = \hat{\mathbf{F}}
\]

Because \( \mathbf{P} \) is orthogonal, \( \mathbf{P}^T \mathbf{P} = \mathbf{I} \) so this can be rewritten as

\[
(\mathbf{D} + \mathbf{P}^T \mathbf{M} \mathbf{P}) \hat{\mathbf{u}} = \hat{\mathbf{F}}
\]

When only the first \( m \) modes and eigenvalues are calculated \( \mathbf{D} \) is now reduced to a \( m \) by \( m \) matrix \( \mathbf{D} \). \( \mathbf{P} \) is a \( n \) by \( m \) matrix \( \mathbf{P} \), so \( \mathbf{P}^T \mathbf{M} \mathbf{P} \) also becomes \( m \) by \( m \). Generally the number of calculated modes \( m \) is significantly lower than the amount of DOFs \( n \), typically in the order of magnitude of 10-100, so this inversion is very inexpensive.

**Example**

As an example, the die that was previously shown is now placed on a slide connection plate and again loaded with the process forces. The plate and die were modeled with 20 modes each. The deformation is shown in figure 6.

**CONCLUSION**

Two technologies were demonstrated to include the effects of elastic tool deformation in a regular FE forming code with a small amount of added DOFs. Theoretically, static reduction represents a significant reduction in problem size without affecting the accuracy, however the amount of retained nodes is still large and the resulting matrix problem becomes more expensive to solve.

Using modal methods, it becomes possible to include the global deformations of the entire press and tool set in the simulation at negligible cost. Each component can be pre-calculated and stored in a library, so different press/tool configurations can be tested in the process design phase already. This helps to identify geometric problems of the formed product, and it also has the potential to increase the accuracy of the press load prediction.

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**REFERENCES**


