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CONSISTENT PLANE STRESS–3D CONVERSION OF HARDENING MODELS AND YIELD CRITERIA.

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Summary. Material models in FE-simulations are used both in 3D and plane stress situations. In this paper it is shown that for models that include kinematic hardening, the conversion from a 3D to a plane stress algorithm needs more adaptations than only eliminating the thickness components. An example and a consistent conversion are presented. Secondly it is discussed how to convert a 2D plane stress yield criterion into a full 3D yield criterion. This is essential for using solid-shell elements with 2D yield criteria, typically used in sheet metal forming.

1 INTRODUCTION

Elasto-plastic plane stress material models in FE-software for simulations of sheet forming processes have proven to be very time efficient compared to 3D algorithms and are therefore widely applied. Eliminating the thickness-direction from the calculations is usually not a problem because the stress in this direction approximates zero. However, in some cases the stresses in thickness direction have to be taken into account. This paper discusses two issues where the loss of the thickness direction in the material model is not desirable. Firstly, the conversion from a 3D material model to a plane stress algorithm is discussed. Classical material models are based on (combinations of) isotropic and kinematic hardening laws. A 3D material model with only isotropic hardening is consistently converted to a plane stress (PS) algorithm, by eliminating the z-direction and adapting the elasticity-matrix, as is illustrated in many textbooks [2, 3, 5]. For kinematic hardening models however, it is shown that the thickness direction cannot be simply eliminated. The back stress in this direction needs to be taken into account to give consistent results. Even if the yield criterion is only defined in plane stress, a compensation is required. It is noted that the yield functions do not need to be adapted because the compensation acts on the return mapping algorithm itself. Secondly, a trend to use solid-shell elements is developing, but more advanced anisotropic material models used in sheet-forming are usually based on plane stress situations [1, 4], which cannot be applied in 3D elements. Therefore, yield criteria defined as plane stress algorithms need to be extended to 3D criteria. Under some assumptions an extension with Von Mises like shear contributions yields a full 3D yield function based on a plane stress criterion.

2 CONSISTENT DIMENSION CONVERSION IN KINEMATIC HARDENING MODELS.

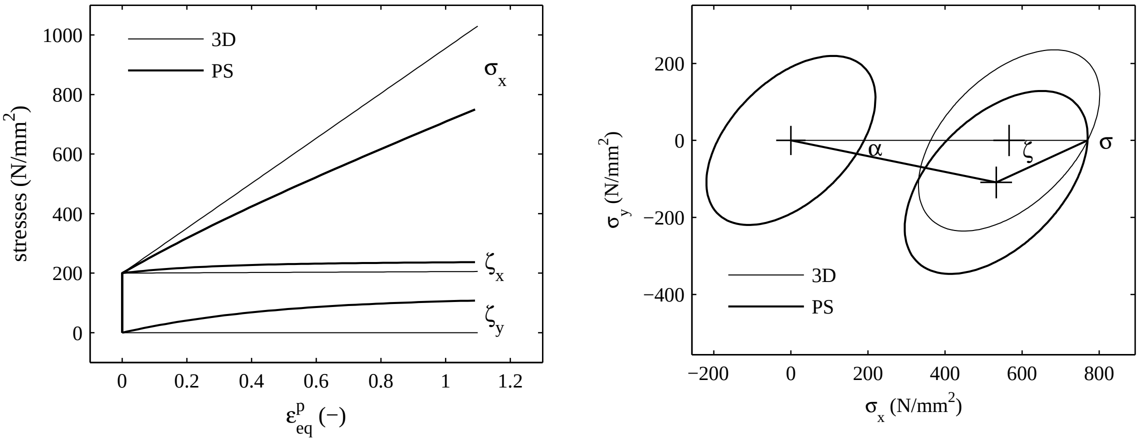
The calculations that are performed in this section take place at integration point level in a regular FE-simulation. At this level the stress increment is calculated from a predicted strain increment and the stiffness-matrix at that point needs to be determined. Calculations in this article are in vector-matrix format with a full 3D stress vector represented by $\boldsymbol{\sigma}^{\mathrm{3D}} = [\sigma_x,\ \sigma_y,\ \sigma_z,\ \tau_{xy},\ \tau_{xz},\ \tau_{yz}]^{\mathrm{T}}$ and the PS stress vector by $\boldsymbol{\sigma}^{\mathrm{ps}} = [\sigma_x,\ \sigma_y,\ \tau_{xy}]^{\mathrm{T}}$. Linear hardening is adopted for the kinematic hardening model and isotropic hardening is not included. Kinematic hardening is implemented via the concept of Ziegler, by adopting a back stress $\boldsymbol{\alpha}$ and an effective stress $\boldsymbol{\zeta}$ with $\boldsymbol{\sigma} = \boldsymbol{\zeta} + \boldsymbol{\alpha}$. These are all combined in the well known Hill'48 yield criterion for demonstration

$$\varphi = \sigma_{\mathrm{eq}}\left(\boldsymbol{\zeta}\right) - \sigma_{\mathrm{f}}\left(\varepsilon^{\mathrm{p}}_{\mathrm{eq}}\right) \tag{1}$$

The flow stress (σ_{f}) equals the uniaxial stress in a tensile test under plastic deformation. Essential for this paper is that the evolution of the back stress is determined via the Prager assumption; the direction of $\dot{\boldsymbol{\alpha}}$ and the plastic deformation are parallel:

$$\dot{\boldsymbol{\alpha}} = f\left(\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{eq}}\right)\frac{\partial\varphi}{\partial\boldsymbol{\sigma}} \tag{2}$$

To illustrate the differences in results for the 3D algorithm and the converted PS algorithm which is derived from the 3D model with the thickness components simply eliminated, a calculation is performed that describes a uniaxial test. Figure 1(a) shows that the uniaxial test for the PS algorithm has a lower hardening rate. Furthermore, the effective stress state in the 3D algorithm remains at the uniaxial point on the yield locus, whereas in the PS algorithm, the effective stress state moves to the plane strain point, which is illustrated in Figure 1(b). Clearly, the two algorithms show a different evolution of the back stress. In the plane stress algorithm the direction of the back stress is taken parallel to the two strain components, at the uniaxial point, hereby neglecting the third component. This causes a deviating shift of the yield locus, which on its turn causes that the stress state of the effective stress finally arrives at the



(a) Stresses as a function of the equivalent plastic strain. (b) The shift of the yield surfaces for the two algorithms.

Figure 1: The resulting stresses in a uniaxial test and the positions in stress space.

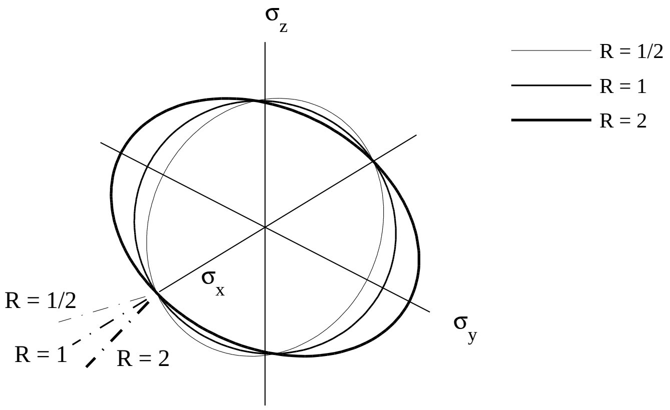


Figure 2: Different R-values for the Hill'48 yield criterion in the deviatoric plane. The dash-dotted lines represent the derivatives of the Hill'48 yield criteria for the different R-values.

plane strain point of the locus. In the 3D case, the shift of the yield locus is apparently not perpendicular to the yield surface at the uniaxial point, but this is because α_z compensates for this. Furthermore, $\boldsymbol{\alpha}$ and $\boldsymbol{\zeta}$ seem to be parallel, but they are not because $\alpha_z = -\zeta_z$, which is not visible in this cross section of the stress space. For other yield surfaces, where the derivative of the yield surface at the uniaxial point is different from the Von Mises derivative, similar effects can be observed, although still working with a 3D algorithm. This is illustrated in Figure 2, where 3 Hill'48- yield loci are depicted with different R-values. The derivatives at the uniaxial point show different orientations.

To arrive at a situation where the PS and 3D algorithm are consistent, α_z and ζ_z have to be taken into account in the PS algorithm. For a 3D algorithm to come to a plane stress situation, a hydrostatic pressure is added to $\boldsymbol{\zeta}$, such that $\sigma_z = \zeta_z + \alpha_z = 0$. In the PS algorithm this is not done, and thus, a consistent conversion from 3D to PS is made when the equivalent stress and the derivative of the yield function are determined with the additional component ζ_z:

$$\sigma_{\text{eq}} = \sigma_{\text{eq}}\left([\zeta_x,\ \zeta_y,\ \zeta_z,\ \zeta_{xy}]^{\text{T}}\right) \qquad \text{and} \qquad \frac{\partial\varphi}{\partial\boldsymbol{\sigma}} = \frac{\partial\varphi}{\partial\boldsymbol{\sigma}}\left([\zeta_x,\ \zeta_y,\ \zeta_z,\ \zeta_{xy}]^{\text{T}}\right) \tag{3}$$

The value of ζ_z is easily determined because $\boldsymbol{\alpha}$ is deviatoric and thus:

$$\alpha_z = -\left(\alpha_x + \alpha_y\right) \qquad \Longrightarrow \qquad \zeta_z = -\alpha_z = \left(\alpha_x + \alpha_y\right) \tag{4}$$

The rest of the material model remains the same, only Equations (3) and (4) need to be implemented.

It is noticed that the yield criteria defined for sheet metal forming are often plane stress models that do not include the required z-component. These models show the same phenomenon described in the example and therefor require the adaptation. If the yield criterion is independent of hydrostatic pressure, then so are the definitions of the equivalent stress and the derivatives, and it is easy to shift the stress state to the plane stress level by simply adding a hydrostatic pressure to the normal components of $\boldsymbol{\zeta}$ with the magnitude of $-\boldsymbol{\zeta}_z$:

$$\boldsymbol{\zeta}_{\text{c}} = \left\{ \begin{array}{c} \zeta_x - \zeta_z \\ \zeta_y - \zeta_z \\ \zeta_{xy} \end{array} \right\} \qquad \Longrightarrow \sigma_{\text{eq}} = \sigma_{\text{eq}}\left(\boldsymbol{\zeta}_c\right), \quad \frac{\partial\varphi}{\partial\boldsymbol{\sigma}}\left(\boldsymbol{\zeta}_c\right) \tag{5}$$

3 INTEGRATING A PLANE STRESS YIELD CRITERION IN A 3D MATERIAL MODEL.

The used yield criteria in FE-simulations of sheet metal forming processes are often specifically developed for plane stress situations, which implies that the stress in thickness direction do not influence the yield function. A full 3D yield criterion is developed that calculates the equivalent stress based on an arbitrairly plane stress hydrostatic pressure independent yield criterion, with supplements from the Von Mises yield criterion. The definitions of equivalent stress according to Von Mises in 3D and PS are:

$$\sigma_{\text{eq}}^{\text{3D}}\left(\boldsymbol{\sigma}^{\text{3D}}\right) = \sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2-\sigma_x\sigma_y-\sigma_y\sigma_z-\sigma_z\sigma_z+3\tau_{xy}^2+3\tau_{yz}^2+3\tau_{zx}^2} \tag{6}$$

$$\sigma_{\text{eq}}^{\text{ps}}\left(\boldsymbol{\sigma}^{\text{ps}}\right) = \sqrt{\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_{xy}^2} \tag{7}$$

Equation (7) is easily implemented in Equation (6) by interchanging the corresponding x and y-stress components. The stress in z-direction is not directly captured in the definition of $\sigma_{\text{eq}}^{\text{ps}}$, but can be implemented by adding a hydrostatic pressure to the stress vector with the size of σ_z, analogue to Equation (5):

$$\sigma_{\text{eq}}^{\text{3D}}\left(\boldsymbol{\sigma}^{\text{3D}}\right) = \sqrt{\sigma_{\text{eq}}^{\text{ps}}\left(\sigma_x-\sigma_z,\sigma_y-\sigma_z,\tau_{xy}\right)^2+3\tau_{yz}^2+3\tau_{zx}^2} \tag{8}$$

The derivatives of the yield function with respect to the stresses are easily calculated from this equation. However, the derivative with respect to σ_z needs to be determined otherwise, because this component is not explicitly included in the definition of $\sigma_{\text{eq}}^{\text{3D}}$. This is done via the assumption that the implemented PS yield criterion is independent of hydrostatic pressure and thus the derivatives of the yield function are in the deviatoric plane:

$$\frac{\partial\varphi}{\partial\sigma_z}^{\text{3D}} = -\left(\frac{\partial\varphi}{\partial\sigma_x}^{\text{ps}}+\frac{\partial\varphi}{\partial\sigma_y}^{\text{ps}}\right) \tag{9}$$

The new yield criterion is generic with respect to the used plane stress yield criterion and gives the same convergence in the return mapping algorithm as the used plane stress yield criterion.

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