Prediction of sheet necking with shell finite element models

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ABSTRACT: In sheet forming simulations, the prediction of localised necking is an important goal. A pragmatic approach is to compare calculated principal strains with a forming limit curve (FLC). However, the FLC’s are known to depend on the strain path and most experimental FLC’s are determined for straight deformation paths. Localisation can also be determined numerically with a Marciniak–Kuczynski analysis (M–K). It is recognised that a FEM analysis with shell elements resembles the M–K analysis very much. For uniform deformations a band with slightly reduced thickness is necessary to trigger localisation. In practical forming conditions, however, the non-uniformity of the process automatically triggers localisation and an arbitrary initial imperfection is not needed. FEM models have the additional benefit that boundary conditions, non-proportional deformation and e.g. friction with the tools are completely included.

Key words: yield locus, localisation, necking, finite elements

1 INTRODUCTION

An important type of failure in sheet forming processes is necking. Necking is a deviation from locally uniform deformation due to a tensile instability. Two important material parameters that influence the forming limits through necking are the work hardening and the shape of the yield locus. The deformation of a number of points in a sheet product can graphically be presented in a forming limit diagram. All states in which the deformation becomes unstable can be connected and then form a forming limit curve (FLC). The FLC gives an impression of the formability of a sheet material. Calculated strains, e.g. from FEM analysis, can be compared with an FLC to determine the feasibility of the analysed sheet forming process.

In sheet forming processes in-plane deformations are often more or less kinematically determined by the tool displacements and diffuse necks cannot develop. Local necking does not directly influence the in-plane deformation. These necks typically have a neck size of the order of the sheet thickness and rapidly lead to fracture. If FLC’s are determined theoretically, local necking is usually assumed to be the determining factor.

2 LOCALISED NECKING

Marciniak and Kuczynski [1] introduced an analysis to determine localised necking in biaxial extension. In this analysis a sheet is considered with an initial groove with reduced thickness (Figure 1(a)). Quantities referring to the groove are given an index B and quantities referring to the rest of the sheet are given an index A. The initial thickness of the groove is \( t_{B,0} \) and outside the groove \( t_{A,0} \). In practice, for \( \varepsilon_2 \geq 0 \) a neck is observed perpendicular to the direction of the major strain. Therefore, the initial groove is aligned with the minor strain in the Marciniak–Kuczynski (M–K) analysis. The M–K analysis is reviewed here briefly. A more comprehensive description is given in [2].

2.1 Marciniak–Kuczynski analysis

Outside the groove a proportional deformation path is assumed. For isotropically hardening models then also the stress path will be proportional. This is described by

\[
\sigma_{2A} = \alpha_0 \sigma_{1A} \quad \sigma_{3A} = 0
\]  

(1)

\[
\varepsilon_{2A} = \beta_0 \varepsilon_{1A} \quad \varepsilon_{3A} = -(1 + \beta_0) \varepsilon_{1A}
\]  

(2)
Compatibility between the uniform part $A$ and the groove $B$ requires that
\[ d_2 = d_1 \] (3)
The force per unit sheet length in direction 1 must be transmitted through the groove, hence
\[ T_1 = \sigma_1 A t_A = \sigma_1 B t_B \Rightarrow \sigma_1 B = \sigma_1 A / f \] (4)
Where $f = t_B / t_A$ is the current thickness ratio. As long as $f \approx 1$, the stress ratio $\sigma_0$ approximately holds for both regions $A$ and $B$. Since the stress $\sigma_1 B$ in the groove is larger than $\sigma_1 A$ in the uniform part, the material in the groove reaches the yield surface first. In Figure 1(b) this is approximately at position $P$. Because of the constraint equation (3) no yielding takes place, since the uniform region is still fully elastic. The stress state in region $B$ will move along the yield locus to position $Q$, until also region $A$ reaches the yield locus at position $P$. This situation is depicted in Figure 1(b). In that period $\sigma_1 B$ has increased and $\sigma_2 B$ has decreased, hence the stress ratio $\alpha$ has decreased and proportional deformation is not possible in the groove.

With the stress state in $A$ and $B$ at two different positions on the yield locus, the normal to the yield surface is different and because of the constraint (3), the strain perpendicular to the neck must be larger in the neck than in the uniform part. As a consequence, the thickness decreases more in region $B$ ($f < f_0$). The analysis of the deformation can be further developed numerically. The drawing region can be included in the analysis by assuming an inclined groove, as predicted by Hill [3].

In Figure 2 the strain paths in the plate (region $A$) and in the groove (region $B$) are presented in a forming limit diagram for $f_0 = 0.99$. It can be seen that in the stretching region, the strain ratio $\varepsilon_{1 B} / \varepsilon_{1 A}$ keeps increasing until the rate of deformation in the groove approaches a plane strain situation.

### 3.1 Grooved plate

A biaxially loaded grooved plate is simulated with a finite element model as presented in Figure 3(a). On the left and lower edges symmetry conditions are prescribed and on the right and upper edge, displacements perpendicular to the edges are prescribed. The ratio of displacements in the $x$- and $y$-directions is constant within every simulation and is comparable with the strain-ratio $\beta$ in the M–K analysis. At the nodes on the lower boundary the plate is 1% thinner than all other nodes. The interpolation of nodal thickness to the integration points of the elements results in a thickness of the lower row of elements of about 0.995 times the nominal thickness.

The loading of the model differs somewhat from the M–K analysis. In the FEM model the strain incre-
ments perpendicular to the groove in the uniform region decrease upon localisation. In the M–K analysis, the strain increments remain constant. This means that in the FEM analysis the strain ratio $\beta$ changes when the strain localises in the groove: the strain perpendicular to the groove decreases to zero and in the direction of the groove it remains constant. The analysis is stopped if the strain increment in an element at the lower edge is larger than 5 times the strain increment in an element in the centre of the plate. Upon failure, the total strain in the element in the upper right corner is used to determine a point on the FLC.

For comparison, an analysis is first performed with a Von Mises yield criterion and Nadai hardening, as in the previous section. The results are presented in Figure 3(b). The FLC from the FEM analysis falls nicely between the curves for $f_0 = 0.99$ and $f_0 = 0.999$ in the M–K analysis, in spite of the different boundary conditions.

In Figure 3(c) the predicted FLC’s for the Von Mises and Vegter yield functions are compared. The parameters for the Vegter model are optimised for an investigated AA 5754-O sheet [4]. The curvature of the yield locus between equi-biaxial stress and plane strain states is now much larger than with the Von Mises model and it is seen immediately that the limit strains in the stretching region decrease. By using a Voce hardening law instead of Nadai hardening, the limit strains decrease further. From uniaxial and biaxial tests, the Vegter yield function with Voce type hardening was selected as an appropriate continuum model for the investigated alloy. The FLC that is predicted in this way resembles the experimentally obtained FLC quite well. In Figure 3(d) the experimental FLC and a corrected experimental FLC are depicted. The correction, performed by Vegter [5], accounts for the difference between the strain on the mid-surface and on the outer surface and for an equi-biaxial pre-strain in the Nakazima tests. The most notable difference between the corrected proportional deformation and the Nakazima strain path is that the lowest point of the FLC shifts in the direction of the plane strain point. The similarity between the Vegter/Voce model and the raw and corrected experimental data is satisfactory.

3.2 Plane strain tensile test

In plane strain tension experiments the strain distribution is not completely uniform. Near the edges of the specimen a uniaxial stress-state prevails. In the experiments it was observed that necking starts in the centre of the specimen [4]. This behaviour can be explained with the Hill local necking criterion, which predicts a higher uniform strain in uniaxial deformation than in
a plane strain deformation.
In the simulation of a completely uniform plate, an initial imperfection is necessary to trigger necking instability. Since the plane strain test is not completely uniform, we investigated numerically how the strain distribution localises, with no further imperfection. The finite element mesh is presented in Figure 4. A quarter of the deformation zone is modelled. Symmetry conditions are applied at the bottom and the right-hand boundary of the mesh. The left-hand boundary is free and a vertical displacement is prescribed at the top boundary where horizontal displacements are suppressed. The strain in the vertical direction is followed for the numbered positions. In Figure 5, the true strain $\varepsilon_y$ is presented as a function of the vertical displacement of the upper edge. It can be seen that at positions 3–5 strain localisation starts at a top displacement of 0.43 mm and at position 6 a little later. This shows that necking starts at the centre of the specimen and grows towards the outer edges, corresponding to the experiments. The magnitude of the strain at positions 3–6, after localisation, is mesh dependent and cannot be used quantitatively. The ‘uniform strain’ at positions 1 and 2 is 0.156 and 0.161, respectively. This fits the experimentally determined force maximum in the plane strain test, that appeared at a strain of 0.16–0.18. It is noted that the strain at the free edge (position 7) is lower than in the rest of the specimen, and it increases uniformly for the presented displacement range. The edge part of the specimen can still bear increasing loads while the centre part is already necking. 

The results for the onset of localisation in a dominating plane strain situation are realistic, even without an initial groove in the model. It suggests that, apart from ‘ideal’ uniform situations, local necking will be triggered by FEM models with membrane and shell elements, without special adaptations to the model.

4 CONCLUSION

In this paper the relation between the material model and forming limits was demonstrated. In the stretching region the work hardening and the shape of the yield function both influence the FLC significantly. For this region, the relevant part of the yield locus is the relatively small part between plane strain and equi-biaxial stress states.

Finite element models with membrane and shell elements can spot the onset of local necking. For models with uniform strain distributions an initial imperfection is necessary to trigger strain localisation. This is comparable to the Marciniak–Kuczynski analysis. In such cases the determined forming limit strains are similar. In situations where the strain is not uniformly distributed, an imperfection is not necessary. This even holds for the weak non-uniformity of the plane strain test. In many industrial forming simulations the strain or thickness distribution can therefore be used directly, to view potential necking zones. In large uniform areas, there is a risk that the simulation will follow an unstable deformation path. In that case, a comparison of strains with an FLC is useful.

FEM models have the additional benefit that boundary conditions, non-proportional deformation and e.g. friction with the tools are completely included. For a correct localisation prediction, however, the accuracy of the material model is critical. After localisation, the numerical results become mesh dependent and the results should be interpreted with great care.

REFERENCES