Aluminium sheet forming simulations: influence of the yield surface

A.H. van den Boogaard¹, P.J. Bolt², R.J. Werkhoven², M.H.F.M. van Hout²

Abstract: The accuracy of simulations of the plastic deformation of sheet metal depend to a large extend on the description of the yield surface, the hardening and the friction. In this paper simulations of deep drawing of an AlMg alloy with a shell model are presented. The yield surface is described by a Von Mises, a Hill ´48 and a Vegeter yield function. The parameters for the model are based on biaxial experiments. It is concluded that the shape of the yield locus has a minor influence on the prediction of the punch force–displacement diagram and a large influence on the prediction of the thickness strains. The Vegeter model performs much better than the Hill ´48 model, based on the same R-values.

Keywords: aluminium, yield locus, finite elements, thermo-mechanical, Vegeter model

1 Introduction

Deep drawing of a cylindrical cup can be simulated by axi-symmetric finite element models. On previous occasions, the measured and simulated punch force–displacement curves and the thickness distribution of cylindrical cups were presented for drawing at room temperature and at elevated temperatures [1, 2]. The material model described the rate-dependency as function of the temperature quite well. Also, the force–displacement curve of the punch during deep drawing was relatively well predicted. The thickness prediction however, was poor. In the simulations an isotropic Von Mises yield function was used and it was stated that results could possibly be improved by taking anisotropy into account.

To analyse the effect of the yield surface the cylindrical cup deep drawing is now simulated with a 3D shell element model, one isotropic and two different anisotropic yield surfaces. In addition to uniaxial experiments, biaxial experiments were performed to measure the yield stress in plane strain tension and simple shear. In the finite element model, the Vegeter yield criterion was used to represent the experimental results.

2 Experiments

An AA 5754-O alloy was used in experiments on cylindrical cup deep drawing, where the flange area was heated and the punch was cooled. In order to determine the biaxial response of the sheet material, uniaxial, plane strain and simple shear tests were performed. The uniaxial tests were performed in an ordinary tensile testing machine. The plane strain and simple shear tests were performed in a biaxial loading frame, described by Pijlman [3]. In this loading frame a sheet area of $45 \times 3$ mm can be deformed in plane strain tension, simple shear or any combination simultaneously. The strains for the plane strain and simple shear experiments are determined by recording the displacements of 4 dots on the specimen and subsequent image processing.

The basic stress–strain curves are presented in Figure 1. The true tensile or shear stress is plotted against the true tensile or shear strain. The spikes on the uniaxial stress–strain curve are accurately measured serrations, due to dynamic strain ageing. The wiggles on the shear curve and especially the plane strain curve are due to the limited accuracy of the optical strain measurement.

Figure 2 shows the equivalent stresses and strains, which are calculated from the measured stress and strain quantities according to Von Mises. It can be seen that the curves do not completely overlap.

The concept of equivalent stress–strain relations for arbitrary proportional deformation paths is valid, at least the first part of the equivalent stress–strain curves should overlap. An improved equivalent stress and strain measure was determined by fitting the initial 5% strain of the stress–strain curves to each other. With the constraint that the equivalent stress and...
strain are equal to the true stress and strain in the uniaxial case and that the equivalent stress and strain measures are energetically conjugate, we can define plane strain and shear factors $f_{ps}$ and $f_{sh}$ such that:

$$\sigma_{eq} = \sigma_{uni} \quad \varepsilon_{eq} = \varepsilon_{uni} \quad (1)$$

$$\sigma_{eq} = f_{ps} \sigma_{ps} \quad \varepsilon_{eq} = \frac{1}{f_{ps}} \varepsilon_{ps} \quad (2)$$

$$\sigma_{eq} = f_{sh} \tau_{sh} \quad \varepsilon_{eq} = \frac{1}{f_{sh}} \gamma_{sh} \quad (3)$$

The plane strain and shear factors, derived from the fitting of the initial part of the curve are given in Table 1. The values that follow theoretically from the Von Mises yield function and quadratic Hill yield function are also given. For the Hill function, an average $R$-value of 0.77 is used, based on the $R$-values as given by the supplier: $R_0 = 0.85$, $R_{45} = 0.67$ and $R_{90} = 0.70$. The resulting stress–strain curves are presented in Figure 3. It can be seen that the plane strain and shear curves are almost equal for the complete strain interval. The uniaxial curve deviates from the other two after approximately 6% strain. Apparently, the concept of equivalent stress and strain values is not valid for large strains. This may be due to texture or other microstructural developments during the deformation. The measured yield stress was 114 MPa for a uniaxial stress state. With the fitted plane strain and shear factors, the yield stress in plane strain is determined to be 130 MPa and in simple shear it is 69 MPa.

### Table 1: Plane strain and shear factors

<table>
<thead>
<tr>
<th></th>
<th>experimental</th>
<th>Von Mises</th>
<th>Hill ’48</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ps}$</td>
<td>1.14</td>
<td>1.15</td>
<td>1.11</td>
</tr>
<tr>
<td>$f_{sh}$</td>
<td>0.61</td>
<td>0.58</td>
<td>0.59</td>
</tr>
</tbody>
</table>

### 3 MATERIAL MODEL

Initially, an axi-symmetric model was used with a Von Mises yield function. The thickness prediction was poor and it was assumed that this was caused by the anisotropy in the material. As a first ‘improvement’ the available $R$-values were used to determine the parameters for an anisotropic quadratic Hill model. The results deteriorated, however, because the $R$-values less than one, that are found for aluminium, yield a much too ‘rounded’ yield surface.

Secondly the Vegter yield criterion was used [4]. As indicated in Figure 4 this criterion is based on the pure
shear, uniaxial, plane strain and equi-biaxial stress points and the gradients in these points. A third order Bezier polynomial is used to interpolate between two neighbouring points. Because the gradients are part of the interpolation, continuity of stresses and the gradients is guaranteed. Assuming the same flow stress for compression and tension, the opposite part of the yield locus is determined. By symmetry considerations or measurements under different specimen angles the total yield locus is found.

The uniaxial yield point is determined by the uniaxial experiment. The initial value of the pure shear point is determined by a simple shear experiment and subsequent rotation of the stresses over 45°. The stress in the tensile direction for the plane strain flow stress is determined by the plane strain experiment, however, the stress in the plane strain direction can not be determined in this way. Also, currently, no equi-biaxial experiments have been performed on this material. The missing data can be estimated, based on polycrystal calculations (see e.g. [5]) or by comparison with data for other f.c.c. metals.

![Figure 5: Yield loci based on different assumptions.](image)

In Figure 5 a predicted yield locus for an isotropic f.c.c. metal with 2000 grains is shown together with a Von Mises yield locus and a planar isotropic Hill ’48 locus with $R$-value of 0.77. Clearly, the polycrystal based yield locus has a shape that can not be described by any quadratic yield function. Especially in the equi-biaxial state, the polycrystal model predicts a much higher curvature and a higher flow stress value than the Von Mises or Hill (with $R < 1$) models. Note that the experimental plane strain and shear factor as given in Table 1 indicate a shear flow stress that is even higher than the Hill model and a plane strain flow stress that is nearer to the Von Mises locus than the Hill locus. In the simulations in the next section, the experimental factors were used to determine the Vegeter yield function. The equi-biaxial factor was set to 1.026, as determined in [3] and in reasonable accordance with the polycrystal yield locus. The gradients for the uniaxial point that are input parameters for the Vegeter model are derived from the measured $R$-values. The flow stress of the investigated aluminium-magnesium alloy is almost rate-independent at room temperature. Above 120 °C, the rate dependence increases. For warm forming of sheet material, the range of interest is between 150 and 250 °C. Above this range, the hardening becomes too low and tensile instabilities develop easily. At room temperature a Nadai hardening law can be used and an extension is possible for higher temperatures:

$$\sigma = C(T)(\varepsilon + \varepsilon_0)^n(T) \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^m(T)$$

(4)

It was demonstrated in [1] that exponentially decreasing $C$- and $n$-values and an exponentially increasing $m$-value yield good agreement with experiments.

4 SIMULATIONS

The cylindrical warm deep drawing process is schematically presented in Figure 6. By heating the flange area, the material draws into the wall area more easily and because the wall is cooled, the material is stronger where it is needed most.

![Figure 6: Cylindrical cup, warm deep drawing.](image)

A quarter of the cup was modelled with 3716 shell elements. Subsequently the analysis was run with a Von Mises, Hill ’48 and Vegeter material model. The same isotropic hardening model was used in all cases and the $R$-values were taken equal for the Hill ’48 and the Vegeter model. In Figure 7 it can be seen that the punch force–displacement curve for the Von Mises material is lower than for both other materials. This can be attributed to the lower shear factor as presented in Section 2. The material deformation takes place mainly in the flange area, and in this area shear deformation dominates. Hence a lower shear factor will result in a lower punch force.
In Figure 8, the prediction of the wall thickness is presented. On the horizontal axis, the arc-length is given, starting at the outer radius in transverse direction, going to the center, going to the outer radius in rolling direction and returning to the transverse direction along the outer radius.

All simulations predict too much thinning in the bottom of the cup. The Hill ‘48 model performs notably poor. This can be attributed to the bad prediction of the equi-biaxial stress by the Hill ‘48 model. Although a corner-like equi-biaxial point follows from the polycrystal analysis and other experiments for aluminium, the Hill ‘48 model predicts an almost circular locus between the two plane strain points. This is a result of the low R-values for aluminium and the basic shape of the Hill ‘48 yield locus.

In the bottom of the cup an equi-biaxial stress state dominates, hence a too low equi-biaxial flow stress will result in too much plastic strain in the bottom and hence, a too thin product. The Von Mises model and the Vegter model (with the same R-values as the Hill ‘48 model) perform better. The remaining difference is still quite large and should be investigated further.

For the warm deep drawing of the cup, the used finite element program was extended with thermal membrane elements. Calculations that were performed with a flange temperature of 175 °C showed a slightly higher punch force than was predicted with an axisymmetric Von Mises model. However, the ‘double bending’ effect just outside the punch radius cannot be modelled well with membrane elements. Therefore some essential details for the thickness prediction are missing and the thermal effects must be implemented in shell elements for better predictions.

5 CONCLUSIONS

It can be concluded from the presented analysis that the shape of the yield locus has an important effect on the calculated punch force–displacement curve and most notably on the thickness prediction. The anisotropy in the material is only of secondary importance as shown by the large difference between the results for the Hill ‘48 and the Vegter model, that both used the same R-values.

The calculated thickness strains are still not satisfactory. The results could be improved by adapting the friction coefficients. The used coefficients were obtained, however, by friction tests. It is recommended to investigate the friction more in detail, before adapting the measured data in order to get a better fit.

For the warm deep drawing, it was demonstrated that a membrane element can not tackle some of the important features of the process. Hence, for further analysis, a thermal shell element must be used.

ACKNOWLEDGEMENTS

The work, presented in this paper, was performed as part of project ME 97033 of the Netherlands Institute for Metals Research (www.nimr.nl).

REFERENCES