WRINKLING PREDICTION WITH ADAPTIVE MESH REFINEMENT

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Abstract: An adaptive mesh refinement procedure for wrinkling prediction analyses is presented. First the critical values are determined using Hutchinson’s bifurcation functional. A wrinkling risk factor is then defined and used to determine areas of potential wrinkling risk. Finally, a mesh refinement is operated.

Keywords: Wrinkling Prediction, Wrinkling Indicator (Risk Factor) and Adaptivity.

1. Introduction

Wrinkling is becoming one of the most troublesome modes of failure in sheet forming mainly because of the trend towards thinner, high-strength sheet metals. Therefore, the critical conditions which promote the initiation of wrinkling are of fundamental significance to the design of deep drawn components and are incorporated in a predictive model for numerical simulations.

The methods used in the past to predict wrinkling failures in sheet metals have been mostly empirical. These methods have, unfortunately, proved to be inadequate for predicting observed trends.

A more recent approach for the analysis of local wrinkling has been presented by Hutchinson and Neale (1985). It consists of formulating the problem within the context of plastic bifurcation theory for thin shell elements.

In this work the analysis of Hutchinson and Neale (1985) and its extension by Neale (1989) to account for more general constitutive models is used.

2. Wrinkling Analysis

We consider a contact free sheet element which, in the current stage of forming, has attained a doubly curved state with principal radii of curvature and thickness assumed to be constant over the region of the sheet being examined for susceptibility to local wrinkling. Moreover, the stress state prior to wrinkling is assumed to be a uniform membrane state. Simplifications arise from the fact that the anticipated short-wavelength modes are shallow and can be analysed using Donnell-Mushtari-Vlasov (DMV) shallow shell theory.

The basic theory of plastic buckling and relevant relations for the DMV shallow shell theory have been developed by Hutchinson (1974). The application of this theory to sheet wrinkling was first carried out by Hutchinson and Neale (1985).

To determine the critical values for buckling we use Hutchinson’s (1974) bifurcation functional. For the pre-wrinkling geometry and stress state considered here, wrinkling will in most cases be aligned with one of the principal curvatures (stress) directions. In such a case the analysis simplifies considerably and a simple wrinkling criterion can be obtained.

3. Wrinkling indicator

Using the wrinkling critical stress or strain values, we define a wrinkling risk factor (perpendicular to the first principal direction) as

\[ f_\sigma = \frac{\sigma_{ij}}{\sigma_{ij}} \quad \text{or} \quad f_\varepsilon = \frac{\varepsilon_{ij}}{\varepsilon_{ij}} \]  

(1)

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Therefore a wrinkling risk exists whenever \( f_\sigma \) or \( f_\varepsilon \) is larger than 1. The risk is more important with larger values of the risk factors.

4. Adaptive strategy

In the adaptive procedure we need to detect the zones (elements) to be refined and determine a new size for those zones (elements). The zones detection is obtained by the use of the wrinkling risk factors (1). The new mesh size is obtained by the use of the wavelength of the wrinkles

\[
L_i = \frac{l}{\lambda_i} = \frac{\sqrt{R_i \cdot l}}{\lambda_i}
\]

(2)

to get a new weighted wrinkling mesh size as

\[
L^w_i = \frac{L_i}{m} f^{-1}_\sigma \quad \text{or} \quad L^w_i = \frac{L_i}{m} f^{-1}_\varepsilon
\]

(3)

with \( m \) being the number of elements we wish to discretise a single wave with.

For wrinkling perpendicular to the second principal direction, a careful indices interchange is to be operated.

5. General Algorithm for wrinkling prediction

Input the FE model (co-ordinates and connectivity) and principal stresses (strains)

For each element evaluate if necessary the principal curvatures

Depending on the signs of the principal stresses (strains), go to one of the four situations depicted in Figure 1. If Case I is encountered, go to next element. Else

Evaluate the loading (straining) proportionality factor

Evaluate the critical stress (strain) and wavelength number for wrinkling perpendicular to the 1-direction (Case II)

Evaluate the critical stress (strain) and wavelength number for wrinkling perpendicular to the 2-direction (Case III)

Evaluate the critical stresses (strains) in both directions for Case IV and determine the predominant direction for wrinkling. Once the wrinkling direction is found evaluate the corresponding wavelength number.

Evaluate the risk factor

Evaluate the wrinkling mesh size and go to next element.

![Figure 1. Possible wrinkling directions.](image)
6. Numerical Example

The performance of the wrinkling prediction procedure with adaptive mesh refinement is here demonstrated. The deep drawing of a hemispherical product is considered. The punch has a radius of 146.5mm and the die shoulder a radius of 30mm. The initial sheet thickness is 1mm and the product depth 100mm. Hollomon’s hardening law is used with $K = 542$, $n = 0.228$ and $r = 2.2$.

Three numerical simulations are performed with a coarse mesh of approx. 2000 Elts, a fine mesh of approx. 10000 Elts and an adapted mesh starting with a coarse mesh of approx. 2000 Elts and converging toward an adapted mesh of approx. 10000 Elts. Figure 2 shows different meshes used.

To visualise the appearance of wrinkles in all three simulations, the curvature in the first principal direction is presented in Figure 3. The wrinkling risk factor is shown in Figure 4.

Clearly the coarse mesh is unable to properly predict the wrinkling occurrence. When the fine grid is used, the lower part and the flat part under the blank holder of the product are obviously over meshed. However, when the adapted mesh is used, a proper mesh distribution is observed.

![Figure 2. Deformed meshes (A: Coarse mesh - B: Fine mesh - C: Refined mesh)](image1)

![Figure 3. Curvature in the first principal direction](image2)
7. Conclusion

It has been demonstrated that the use adaptive mesh refinement in wrinkling prediction analysis is a necessary approach for reducing the computational cost and better describing the wrinkling phenomena. However, it is also found that this has to be linked with an error estimation routine to properly approximate the curvatures as these play a major role in the wrinkling process.

8. References

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