Application of acoustically tuned resonators for the improvement of sound insulation in aircraft

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Abstract One of the aims of the EU project FACE (Friendly Aircraft Cabin Environment) is to reduce aircraft interior noise. For modern aircraft flying at cruise conditions, the turbulent boundary layer is the main source for cabin noise. Normally, the turbulent boundary layer causes the trim panels to vibrate, and hence to radiate sound into the aircraft cabin. The purpose of the present work is to reduce this kind of noise by means of sound insulating trim panels with tuned acoustic resonators. The length and the radius of these resonators are tuned in such a way that the volume velocities at the vibrating panel surface and at the entrance of the resonators are equal in magnitude but opposite in phase. In this way, maximum reduction of the radiated sound can be achieved for a specified frequency range. Because of the repetitive pattern of the resonators in the panel, the influence of the resonators on the sound radiated in normal direction by the panel is studied with a one-dimensional model. The so-called low reduced frequency model is extended to describe the viscothermal wave propagation in the vibrating resonators. An advantage of the viscothermal effects is that, in the low frequency range, more sound reduction is obtained than if these effects are not present or very small. Calculations show that a large reduction of the radiated sound can be achieved. The model is also validated by experiments in an impedance tube. Good agreement is found between theory and measurements.

1. INTRODUCTION

When a trim panel in an aircraft is excited, either structurally or acoustically, sound is radiated from the panel. In this paper, it is shown that cylindrical resonators can be used to reduce the radiated sound. The principle is based on minimising locally the volume velocity of small partitions of the panel; a method that is also used in active acoustic control [2]. Ross [3] applied a similar principle for passive noise control by means of so-called weak radiating cells.

1On the method and construction a patent is pending.
Because of the repetitive pattern of the resonators in the panel, the panel can be divided into a number of so-called characteristic areas, each area containing one resonator; see Figure 1. Since all characteristic areas are identical and assumed to be small compared to the total area of the panel, the boundaries of these characteristic areas can be regarded as symmetry planes. Consequently, the velocity $v$ normal to the symmetry planes is assumed to be zero. This means that the boundaries of the characteristic areas can be modelled as semi-infinite prismatic tubes with rigid walls. The effect of the resonators on the sound that is radiated in normal direction from the panel can therefore be studied with a one-dimensional model of one characteristic area. The effects of the boundaries of the panel are omitted in this paper.

In the next sections, the acoustic model of the characteristic area is described in two steps. Firstly, the model is presented that is used to describe the propagation of sound waves in the vibrating resonators. Subsequently, the expressions derived for the pressure and velocity perturbations are used to calculate the sound that is radiated from the panel. Finally, the model is validated by means of experiments in an impedance tube.

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![Figure 1: Part of a panel with resonators divided into characteristic areas.](image)

## 2. VISCOThermal WAVE PROPAGATION

The propagation of sound waves in the resonators is described by the so-called low reduced frequency model of Zwikker and Kosten [6]. An extensive overview of different analytical solutions for viscothermal wave propagation, given by Tijdeman [4], shows that the low reduced frequency model is a very accurate and efficient model. When the panel is excited perpendicular to the surface, the resonators will vibrate in axial direction. In this paper, the low reduced frequency model is therefore extended to include the effects of the axially vibrating walls on the viscothermal wave propagation. The low reduced frequency solution is written in terms of the same dimensionless parameters as those used by Tijdeman [4].

**Low reduced frequency solution** Figure 2 shows the coordinate system used for the cylindrical tube. The vibration of the resonator is taken into account in the model by prescribing a harmonic axial velocity perturbation $u_s e^{i\omega t}$ at the wall of the tube, with $u_s$ the amplitude of the velocity perturbation, $\omega$ the angular frequency and $t$ the time. The pressure is assumed to be constant over the cross-section and the solution for the amplitude $p$ of the pressure perturbation remains the same as for a stationary prismatic
tube:

\[ p(x) = Ae^{\Gamma kx} + Be^{-\Gamma kx} \]  

where \( k = \omega / c_0 \) is the wave number, with \( c_0 \) the speed of sound. \( A \) and \( B \) are the complex amplitudes of the backward and forward travelling waves, respectively, determined by the boundary conditions. For a cylindrical tube, the propagation constant \( \Gamma \) is given by:

\[ \Gamma = \sqrt{\frac{J_0(i^{3/2}s)}{J_2(i^{3/2}s)}} n, \quad \text{with} \]

\[ n = \left[ 1 + \frac{\gamma - 1}{\gamma} \frac{J_2(i^{3/2}s\sigma)}{J_0(i^{3/2}s\sigma)} \right]^{-1}, \quad s = R\sqrt{\frac{\rho_0\omega}{\mu}} \quad \text{and} \quad \sigma = \sqrt{\frac{\mu C_p}{\lambda}} \]

where \( J_0 \) and \( J_2 \) are the Bessel functions of the first kind of order 0 and 2, \( i \) is the imaginary unit, \( s \) is the shear wave number, \( n \) is the polytropic constant, \( \gamma \) is the ratio of specific heats, \( \sigma \) is the square root of the Prandtl number, \( R \) is the radius of the tube, \( \rho_0 \) is the mean density, \( \mu \) is the dynamic viscosity, \( C_p \) is the specific heat at constant pressure and \( \lambda \) is the thermal conductivity. With the following (no-slip) boundary condition:

\[ u = u_s \quad \text{at} \quad r/R = 1 \]

the basic equations of the low reduced frequency model can be solved in a similar way as described by Tijdeman [4] and the solution for the amplitude \( u \) of the velocity perturbation in axial direction can be written as:

\[ u(x, r) = \frac{i\Gamma}{\rho_0 c_0} \left[ 1 - \frac{J_0(i^{3/2}s r/R)}{J_0(i^{3/2}s)} \right] \left[ Ae^{\Gamma kx} - Be^{-\Gamma kx} \right] + u_s \frac{J_0(i^{3/2}s r/R)}{J_0(i^{3/2}s)} \]

The first term of this equation is identical to the solution for the amplitude of the velocity perturbation in axial direction as given by Tijdeman [4]. The second term in equation (5) accounts for the effect of the vibrating wall. This term only depends on the viscosity, which means that in the present model no additional effects of heat condition are introduced by the vibrating wall.

\[ \text{Figure 2: Coordinate system for a cylindrical tube.} \]

**Velocity profile** Figure 3 shows the influence of the shear wave number on the two parts of equation (5) that determine the shape of the velocity profile. The magnitude of the expressions is plotted as function of the dimensionless radius \( r/R \). It is noted that the equation for the amplitude of the velocity perturbation is complex, which means that not all points pass the equilibrium position at the same time. The shear wave number is a measure for the ratio of the inertial effects and the viscous effects. For small values of \( s \), the viscous effects dominate and the velocity profile is parabolic. In
this case, the magnitude of the expression due to the vibration of the tube approaches one and the prescribed velocity at the walls influences the velocity profile over the entire cross-section; see Figure 3 (right). For large values of $s$, the inertial effects dominate and a nearly flat velocity profile is obtained. The magnitude of expression due to the vibration of the tube approaches zero now, which means that the prescribed velocity at the walls does not influence the velocity profile anymore. Figure 3 (left) shows that the magnitude of the first expression approaches one. Hence, for large values of the shear wave number, equation (5) converges to the solution for standard acoustic wave propagation.

For the model described in the next section, the amplitude of the velocity perturbation in axial direction as defined by equation (5) is averaged over the cross-section. This leads to the following expression:

$$\bar{u}(x) = \frac{\gamma}{\Gamma n} \frac{1}{\rho_0 c_0} \left[ Ae^{\Gamma k x} - Be^{-\Gamma k x} \right] + u_s \left[ \frac{\gamma}{\Gamma^2 n} + 1 \right]$$

(6)

3. RADIATION OF SOUND BY A RIGID CHARACTERISTIC AREA

In this section, the one-dimensional wave propagation theory as presented above is used to model the sound radiated by a characteristic area, vibrating with a harmonic velocity $u_\omega e^{i\omega t}$. Both the part of the plate and the resonator are assumed to be rigid and no acousto-elastic interaction is taken into account.

Model for radiation of sound  Figure 4 shows the model of a characteristic area. The model consists of two parts: the resonator and a semi-infinite cylindrical tube, which represents the acoustic far-field to which the sound is radiated. $A_1$ and $B_1$ are the pressure amplitudes of the backward and forward travelling waves in the resonator, respectively, and $B_2$ is the pressure amplitude of the radiated sound wave. Since in the semi-infinite tube no reflection takes place, the amplitude $A_2$ of the backward travelling wave equals zero. Given that the structure vibrates harmonically with a normal velocity amplitude $u_\omega$, three boundary conditions can be formulated. The first boundary condition states that the acoustic velocity at the back of the resonator is equal to the
velocity of the structure (no-slip condition). At the entrance of the resonator the pressure perturbation is assumed to be continuous and conservation of mass is applied for the control volume indicated by the dashed area in Figure 4. The boundary conditions can be written as:

\[ \bar{u}_1(x_1 = 0) = u_s \]  
\[ p_1(x_1 = L_{\text{res}}) = p_2(x_2 = 0) \]  
\[ \bar{u}_1(x_1 = L_{\text{res}})S_{\text{res}} + u_s[S - S_{\text{res}}] = \bar{u}_2(x_2 = 0)S \]

where \( L_{\text{res}} \) is the effective length of the resonator, \( S_{\text{res}} \) is the cross-sectional area of the resonator, \( S \) is the characteristic area, subscript 1 refers to the resonator, and subscript 2 refers to the semi-infinite tube. The pressure perturbations and the axial velocity perturbations in the two parts are described by equations (1) and (6). By substituting these equations into the relations for the boundary conditions, the unknown pressure amplitudes \( A_1, B_1 \) and \( B_2 \) can be solved. For the semi-infinite tube, the velocity \( u_s \) in equation (6) equals zero, because the walls of the tube (i.e. the symmetry planes) do not move.

Results – influence of the porosity  With the model described above, the working principle of the vibrating resonators can be studied in more detail. Figure 5 shows the influence of the porosity \( \Omega = S_{\text{res}}/S \) on the sound radiated by the rigid characteristic area\(^2\). The results are plotted for a resonator of length \( L_{\text{res}} = 0.14 \text{ m} \) and a characteristic area of \( S = 2.25 \cdot 10^{-4} \text{ m}^2 \). Since the amplitude \( B_2 \) of the sound pressure that is radiated from the characteristic area is linearly dependent on the excitation velocity \( u_s \), the magnitude of the transfer function \( H_{B_2u_s} = B_2/u_s \) is used as a measure for the sound radiated from the panel. The sound that is radiated from a rigid characteristic area without a resonator (\( \Omega = 0 \)) is used as reference here. In this example, the resonator radius is so large that viscothermal effects hardly play a role.

The locations of the minima of the transfer functions for the different porosities can be explained with Figure 6, where the magnitude of the acoustic velocity perturbation in axial direction is plotted over the length of the resonator. The sound radiated from the characteristic area is minimal if the volume velocities at the vibrating panel surface \( u_s[S - S_{\text{res}}] \) and at the entrance of the resonators \( \bar{u}_1(x_1 = L_{\text{res}})S_{\text{res}} \) are equal in magnitude.

\(^2\)For all calculations the following air conditions were used: \( c_0 = 343 \text{ m/s} \), \( \rho_0 = 1.2 \text{ kg/m}^3 \), \( \mu = 18.2 \cdot 10^{-6} \text{ Ns/m}^2 \), \( \gamma = 1.4 \) and \( \sigma = 0.845 \).
but opposite in phase. If the porosity is very small, this is achieved when the axial acoustic velocity at the entrance of the resonator is very large in comparison with the velocity of the structure. Figure 6(a) shows that this happens at the frequencies for which a quarter or three quarters of the acoustic wavelength $\lambda = 2\pi c_0/\omega$ corresponds with the length of the resonator, i.e. $L_{\text{res}} = \lambda/4$ or $L_{\text{res}} = 3\lambda/4$. In case the porosity equals 0.5, the radiated sound is minimal when the axial acoustic velocity at the entrance of the resonator is equal to the velocity of the structure. Figure 6(b) shows that this happens at the frequency for which half of the acoustic wavelength corresponds with the length of the resonator, i.e. $L_{\text{res}} = \lambda/2$. When the porosity increases from 0.5 upwards, the radiated sound approaches the sound radiated by a rigid characteristic area without a resonator again. When the curves for the different porosities in Figure 5 are compared with the sound radiated by a rigid characteristic area without a resonator, it can be seen that the radiated sound can be reduced considerably in a specified frequency range by tuning the dimensions of the resonators.

\begin{align*}
\text{(a)} & \quad \Omega \ll 1 \text{ at } f = c_0/4L_{\text{res}} \text{ and } f = 3c_0/4L_{\text{res}}. \\
\text{(b)} & \quad \Omega = 0.5 \text{ at } f = c_0/2L_{\text{res}}.
\end{align*}

Figure 6: Distribution of the magnitude of the axial acoustic velocity over the length of the resonator for different porosities at the frequencies for which maximum sound reduction is achieved.

**Results – viscothermal effects** Figure 7 shows the influence of the viscothermal effects on the sound radiated by the rigid characteristic area. The resonator length is again $L_{\text{res}} = 0.14$ m and the porosity is held constant at $\Omega = 0.45$, while the resonator radius is varied. If the resonator radius decreases, the viscothermal effects increase. This is shown by a decrease of both the amplifications and the reductions of the radiated sound. The advantage is that the frequency range over which the noise is reduced broadens and that it is possible to obtain reductions at lower frequencies without lengthening the
resonators. Since the shear wave number decreases with frequency, it is noticed that at lower frequencies more viscothermal effects are present. Furthermore, it is seen that the locations of the minima shift slightly to lower frequencies. This is caused by the fact that due to viscous effects, the effective speed of sound $c_{\text{eff}} = c_0 / \text{Im}(\Gamma)$ is lower than the undisturbed speed of sound $c_0$.

![Figure 7: Sound radiated by a characteristic area for different resonator radii.](image)

### 4. EXPERIMENTAL VALIDATION

The model described in the previous sections is validated by means of experiments in an impedance tube. In this section, the experiments and their results are discussed.

**Experimental setup and procedure** Figure 8 shows the experimental setup that is used to validate the model. A sample with resonator is harmonically excited in an impedance tube by a shaker. The shaker is driven by a random signal, so that a broadband sound field is generated. The sound pressures $p_1$ and $p_{\text{II}}$ are measured at two positions in the tube and the excitation velocity $u_s$ is measured with an accelerometer which is attached to the sample. To make a comparison with the model, the transfer $H_{p1u_s} = p_1/u_s$ of the pressure $p_1$ in the impedance tube and the excitation velocity $u_s$ is determined. The end of the tube is provided with a baffle to have a determined end condition. The radius of the impedance tube is $R = 0.025$ m, and the distances of the pressure transducers to the end of the impedance tube are $L_{\text{I}} = 0.4945$ m and $L_{\text{II}} = 0.4495$ m, respectively. All measurements are performed in the frequency range of 1000-2000 Hz.

**Model of the experimental setup** The difference between the experimental setup and the model described in the previous section is that the impedance tube is not infinitely long. Therefore, a part of the forward travelling sound waves are reflected at the end of the impedance tube. The pressure amplitude $A_2$ of the reflected sound waves (see Figure 4) is calculated by imposing an impedance condition at the end of the impedance tube. This boundary condition can be written as:

$$\frac{\text{Im}(\Gamma) \rho c_0}{\gamma n_2} \frac{1}{p_2(x_2 = L)} = \zeta$$

(10)
with $\zeta$ the prescribed dimensionless impedance and $L$ the distance from the front of the sample to the end of the impedance tube. The dimensionless impedance $\zeta$ is determined from the transfer $H_{21} = \frac{p_{II}}{p_I}$ of the pressures measured with the two pressure transducers, and can be calculated as follows:

$$\zeta = -\frac{H_{21} \sinh(\Gamma_2 k L_I) - \sinh(\Gamma_2 k L_{II})}{H_{21} \cosh(\Gamma_2 k L_I) - \cosh(\Gamma_2 k L_{II})}$$ (11)

By combining equation (10) with the other boundary conditions (7), (8) and (9), the unknown pressure amplitudes $A_1$, $B_1$, $A_2$ and $B_2$ can be solved.

Due to inlet effects, the effective length of the resonator $L_{res} = L_{phy} + \delta$ is larger than the physical length $L_{phy}$ of the resonator. For a tube centrally located in another tube, the end correction $\delta$ is given by [1]:

$$\delta = \frac{8R_{res}}{3\pi} \left[ 1 - 1.25 \frac{R_{res}}{R} \right], \quad \text{with} \quad \frac{R_{res}}{R} < 0.6$$ (12)

**Results** Figure 9 shows the two samples with resonators that are tested. The dimensions of the resonators, as well as the distance from the front of the sample to the end of the impedance tube, are listed in Table 1. The radius of sample 1 is chosen small enough to perceive the viscothermal effects and large enough to observe the difference between the results of the sample with resonator and the reference. The radius of sample 2 is tuned to achieve a large reduction of the sound pressure over a broad frequency range. For radiation to the far-field, a reduction in sound pressure level of at least 15 dB is predicted in the frequency range of 1000-2000 Hz.

<table>
<thead>
<tr>
<th>Sample 1</th>
<th>Sample 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{res}$</td>
<td>0.0060</td>
</tr>
<tr>
<td>$L_{phy}$</td>
<td>0.0615</td>
</tr>
<tr>
<td>$L$</td>
<td>0.6630</td>
</tr>
</tbody>
</table>

Table 1: Dimensions [m].

Figure 9: Photos of the samples for experimental validation.
Figures 10 and 11 show the results of both measurements compared with the numerical results. The sound radiated from a rigid characteristic area without resonator is used as reference. If the magnitude of the transfer function $H_{p_{1,us}}$ is lower than the transfer function of the characteristic area without resonator, reduction of the radiated sound is obtained.

The porosity of sample 1 is $\Omega = 0.06$. In the previous section it was shown that if $\Omega \ll 1$, maximum reduction of the sound pressure is achieved at the frequency for which a quarter of the acoustic wavelength corresponds with the effective length of the resonator. For the resonator in sample 1, this frequency is 1374 Hz. In Figure 10 it can be seen that at that frequency indeed most reduction is achieved. The porosity of sample 2 is $\Omega = 0.41$. In this case, most reduction of the radiated sound is achieved in a broad range around the frequency for which half of the acoustic wavelength corresponds with the effective length of the resonator. For the resonator in sample 2, this frequency is 1524 Hz. The peaks in the transfer functions of the two samples are caused by resonances in the impedance tube. It should be noted that Bolt [1] proved that the end correction defined by equation (12) is valid for $R_{\text{res}}/R < 0.6$. However, for sample 2 the ratio of the resonator radius and the radius of the impedance tube equals 0.64. Nevertheless, good agreement is obtained between model and measurements for both samples.

**5. CONCLUSIONS**

In this paper, the application of resonators for the improvement of sound insulation in aircraft was examined. To describe the viscothermal wave propagation in the resonators, the low reduced frequency model was extended by including the effects of the vibrating walls. By dividing the panel into a number of identical characteristic areas, the sound radiated by such area could be analysed with a one-dimensional model.
It was seen that by tuning the dimensions of the resonators, very large reductions of the radiated sound can be obtained. The resonator length and the porosity determine the frequencies at which maximum reduction of the radiated sound is achieved. The amount of viscothermal effects is determined by the radius of the resonator. Viscothermal effects cause a decrease of both the amplifications and the reductions of the radiated sound, and broaden the frequency range in which reduction is obtained.

Validation of the model by means of experiments with two different samples in an impedance tube showed there is a good agreement between model and measurements. Therefore, it can be concluded that the model presented here is appropriate to describe the radiation of sound by a rigid characteristic area. Also, the extended low reduced frequency model is proven to be accurate for the applications presented in this paper.

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