OPTIMISED SOUND ABSORBING TRIM PANELS FOR THE REDUCTION OF AIRCRAFT CABIN NOISE

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Abstract

The EU project FACE (Friendly Aircraft Cabin Environment) aims to improve the environmental comfort in aircraft cabins. As part of this project, this paper focuses on the reduction of noise in aircraft cabins. For modern aircraft flying at cruise conditions, this cabin noise is known to be dominated by turbulent boundary layer noise. The purpose of this work is to reduce the resulting sound pressure levels in the cabin by means of optimised sound absorbing trim panels with quarter-wave resonators.

Sound absorption with quarter-wave resonators is mainly realised by dissipation of sound energy as a result of viscous and thermal losses. The viscothermal wave propagation of the air inside the resonators is efficiently and accurately described by the so-called low reduced frequency model. By optimisation of the dimensions of the resonators, desired sound absorption characteristics can be obtained for different specified frequency ranges. This means that the panels can be tailored to different positions in the aircraft cabin with different prevailing sound pressure levels. Results of optimisations for various frequency ranges show that a very good agreement is obtained between the desired and the calculated absorption curves. With the same optimisation procedure, panels have also been tuned for the dominant frequency range of a sound spectrum measured in a modern aircraft. Experimental validation of the numerically predicted optimal configurations, by means of impedance tube measurements, shows that a fairly good agreement is obtained between the numerical and experimental results.

INTRODUCTION

In modern aircraft, boundary layer noise is known to dominate cabin noise at cruise conditions. In this paper a model and an optimisation procedure are presented that can
be used to design a sound absorbing trim panel with quarter-wave resonators, which optimally reduces this noise in the dominant frequency range. A schematic representation of the concept is shown in figure 1.

The absorption characteristics of the different resonator configurations are calculated by using the low reduced frequency model as presented by Zwikker and Kosten [6]. An extensive overview of literature on viscothermal wave propagation, presented by Tijdeman [4], has proven that this is both a very efficient and accurate way to describe the propagation of the air inside the resonators. Wijnant [5] already showed some preliminary results that can be obtained by optimising the dimensions of quarter-wave resonators for some simple cases. In this paper, the optimisation procedure is further explained and an analytically predicted optimal configuration is experimentally validated by small-scale experiments.

THEORY

Model

For a sound absorbing panel, the sound absorption coefficient $\alpha$, representing the fraction of the incident sound energy that is dissipated, is given by [2]

$$\alpha = 1 - |\mathcal{R}|^2, \quad \text{with} \quad \mathcal{R} = \frac{Z_w - \rho_0 c_0}{Z_w + \rho_0 c_0}.$$  \hfill (1)

$\mathcal{R}$ is the reflection coefficient, $\rho_0$ is the mean density of the air, and $c_0$ is the speed of sound in air (a list of symbols is given in appendix A). The relation for the impedance $Z_w$ of a panel with area $A_w$ containing $N$ resonators is derived from the conservation of mass for a control volume near the surface and is given by [2]

$$Z_w = \frac{1}{\sum_{j=1}^{N} \Omega_j Z_j}, \quad \text{with} \quad Z_j = \rho_0 c_0 \frac{\Gamma_j n_j}{1 + \gamma} \tanh^{-1} (\Gamma_j k L_j) \quad (2)$$

and $\Omega_j = A_j / A_w$ defined as the porosity of the resonator with a cross-sectional area $A_j$. $Z_j$ is the impedance at the entrance of a tube of length $L_j$ with a prescribed pressure.
perturbation at the entrance and the other side closed, where \( i = \sqrt{-1} \) is the imaginary unit, \( \gamma \) is the ratio of specific heats, and \( k \) is the wave number defined by \( \omega/c_0 \), with \( \omega \) the angular frequency. For a prismatic, cylindrical tube \( j \) the propagation constant \( \Gamma_j \) is given by \[ 4 \]

\[
\Gamma_j = \frac{\sqrt{J_0(i\sqrt{s_j} \gamma) \frac{\gamma}{J_2(i\sqrt{s_j} \gamma)}}}{n_j}, \quad \text{with} \quad (3)
\]

\[
n_j = \left(1 + \frac{\gamma - 1}{\gamma} \frac{J_2(i\sqrt{s_j} \gamma)}{J_0(i\sqrt{s_j} \gamma)}\right)^{-1}, \quad s_j = R_j \sqrt{\frac{\rho \omega}{\mu}} \quad \text{and} \quad \sigma = \sqrt{\frac{\mu C_p}{\lambda}}. \quad (4)
\]

\( J_0 \) and \( J_2 \) are the Bessel functions of the first kind of order 0 and 2, \( n_j \) is the polytropic constant, \( s_j \) is the shear wave number, \( \sigma \) is the square root of the Prandtl number, \( R_j \) is the radius of the tube, \( \mu \) is the dynamic viscosity, \( C_p \) is the specific heat at constant pressure and \( \lambda \) is the thermal conductivity.

Due to inlet effects at the entrance of the tube, the physical length of the resonator has to be slightly corrected by adding an end correction. For a tube with the open end in an infinite baffle, see figure 8(a), this end correction \( d_j \) is given by \[ 3 \]

\[
d_j = \frac{8\pi}{3R_j}. \quad (5)
\]

**Optimisation**

From equation (2) it can be seen that the impedance at the entrance of the resonator and subsequently also the absorption coefficient of the panel are influenced by the lengths of the resonators. The radii of the resonators determine the shear wave number and thus also the propagation constant, as well as porosity and the end correction. In order to find the combination of resonator dimensions that gives the best possible approximation of a desired absorption curve for a specified frequency range, an optimisation algorithm has been implemented. The objective of the optimisation procedure is to minimise the difference between the desired and the calculated absorption curves. The input parameters of the optimisation algorithm are: the frequency range in which the noise has to be reduced, the desired absorption curve, the characteristic area of the panel, and the number of resonators in the panel. The output parameters of the optimisation process are the dimensions of the resonators, in this case the lengths and the radii. In order to reduce the total number of variables, the panel can be subdivided into a number of identical patches, so-called characteristic areas. In this way only a small part has to be optimised, which can later be used to "tile" the panel, see figure 2.

The objective function is minimised using the standard MATLAB routine \texttt{fminbnd}; a routine that minimises a function of one variable on a fixed interval. In this case there is one objective written as function of the tube length and one as function of the tube radius. Because each variable is optimised separately, the same procedure has to be repeated several times. This is done by alternately optimising the length and the radius of each tube. When the optimisation of all variables has been completed, the process
is repeated until the result has converged. A schematic overview of this procedure is shown in figure 3.

![Figure 3: Optimisation procedure.](image)

**Results**

In order to demonstrate the results that can be obtained with the optimisation algorithm, an optimisation is performed for a case covering the frequency range of 1000-2000 Hz\(^1\). The objective is to find the dimensions of the resonators that will yield maximum absorption (i.e. \(\alpha = 1\)) for the entire frequency range. The optimisation is carried out for 20 resonators in a characteristic area of 0.002 \(\text{m}^2\). The characteristic area is chosen here equal to the cross-sectional area of the impedance tube that is used for the experimental validation. The predicted resonator dimensions that result from the optimisation are listed in table 1\(^2\). Figure 6 shows the optimised absorption curve. It is seen that almost maximum absorption is obtained for the entire frequency range. The average absorption coefficient is 0.98.

**EXPERIMENTAL VALIDATION**

**Impedance tube technique**

The absorption curve of the analytically predicted optimal configuration is validated by means of the impedance tube measurements. In an impedance tube a one-dimensional random sound field is generated by a speaker, see figure 4. The sound pressure levels in the tube, \(p_1\) and \(p_2\), are measured with two pressure transducers and determine the transfer function \(H_{21}\) according to

\[
H_{21} = \frac{p_2}{p_1}.
\]

With this transfer function, the reflection coefficient at the surface of the sample can be calculated by using the theory for one-dimensional wave propagation as described in

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\(^1\) It should be mentioned that the frequency range and the resulting resonator dimensions that are used here only serve for validation purposes. For the actual trim panel, resonators can be optimised for other frequency ranges, and different dimensions and/or other geometries are possible.

\(^2\) For the calculations the following standard air conditions have been used: \(c_0 = 343.3 \text{ m/s}, \rho_0 = 1.22 \text{ kg/m}^3, \mu = 18.2 \cdot 10^{-6} \text{ Ns/m}^2, \gamma = 1.4\) and \(\sigma = 0.845\).
the previous section. The absorption coefficients $\alpha$ are finally calculated with equation (1).

![Impedance tube setup](image)

**Figure 4: Impedance tube setup.**

**Figure 5: Sample.**

### Results and discussion

Figure 5 shows the sample that has been used for the experimental validation. The sample has a radius of 50 mm and a length of 87.4 mm. The dimensions of the resonators obtained with the optimisation procedure are listed in table 1. Figure 7 shows both the calculated absorption curve and the measured absorption curve that results from the impedance tube measurements. Due to the limited tolerances that could be achieved with the production of the sample, some of the resonator dimensions appeared not to be in agreement with the prescribed ones. In order to be able to make an adequate comparison with the theory, these dimensions (depicted as actual dimensions in table 1) have been adjusted in the analytical model and the resulting absorption curve is also shown in figure 7. It is seen that the absorption level that is obtained with the optimised configuration is very high; the sample has an average absorption coefficient of 0.88. When a comparison is made between the analytically predicted and the measured absorption curves it is seen that there are, however, still some discrepancies.

![Optimised absorption curve](image)

**Figure 6: Optimised absorption curve for a frequency range of 1000-2000 Hz.**

![Experimental results](image)

**Figure 7: Experimental results compared with analytical results.**

One of the possible reasons for these discrepancies can be explained by the fact that the end corrections that are used in the optimisation (see section 2) are valid for a resonator in a baffle [3], whereas the measurements are performed in an impedance tube, where different boundary conditions are present. In his book, Bies [1] gives a formula for the end correction of a tube centrally located in another tube, see figure.
8(b). To the author’s knowledge, however, no analytical relation is known for a tube that is not centrally located in another tube. Figure 9 shows that by arbitrarily tuning the end corrections, a better agreement can be obtained between the analytical absorption curve and the one that is measured in the impedance tube. Further research is, however, needed to find an exact relation for the end corrections of the different positioned tubes.

Figure 8: Different situations for end corrections.

Another possible reason for the discrepancies between theory and experiment is that there might be cross-talk between adjacent resonators with similar resonant frequencies. In order to examine the influence of the position of the resonators on the absorption characteristics, a second sample has been made and tested. Sample 2 contains the same amount of resonators with exactly the same dimensions as sample 1, only the position of the resonators in the surface is different, see figure 11. In order to prevent the possible cross-talk, sample 2 has been designed in such a way that resonators with similar resonant frequencies have been placed as far as possible from each other. Figure 10 shows the measured absorption curves of both samples, as well as the analytical results.

Figure 9: Effect of tuning end corrections; calculated and measured absorption curves.

Figure 10: Experimental results of sample 1 and sample 2 compared with analytical results.

The first thing that can be noticed is that the absorption curves of the samples are different. This means that the position of the resonators in the surface influences the
absorption characteristics, which can both be caused by changing end corrections and the occurrence of cross-talk. The second thing that can be seen is that sample 2 shows a better agreement with the analytical results than sample 1. Since in sample 2, resonators with similar resonant frequencies have been placed as far as possible from each other, it seems that the relatively large discrepancies between theory and experiment of sample 1 are at least partly caused by cross-talk. Another observation is that the average absorption level of the second sample is higher, 0.95 of sample 2 versus 0.88 of sample 1. Because the average absorption level has been improved for the second sample, it seems that cross-talk has a negative effect on the absorption characteristics.

<table>
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<th>Predicted</th>
<th>Actual</th>
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<tbody>
<tr>
<td>Radius</td>
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Table 1: Predicted and actual resonator dimensions [mm].

CONCLUSIONS

The results of the case presented in this paper show that a desired absorption curve can be accurately approximated by optimising the dimensions of quarter-wave resonators in a panel. Experimental validation of the analytically predicted optimal configurations shows that high levels of absorption can be realised. There are, however, still some discrepancies between the analytical and experimental results, which are partly caused by the fact that the end corrections that are used in the optimisation are defined for a
baffled situation, whereas at the impedance measurements different boundary conditions are present. By tuning the end corrections, it is possible to find a better agreement between analytical and experimental data. Additional experiments have shown that the occurrence of cross-talk between resonators with similar resonant frequencies is another reason for the discrepancies between the analytical and the experimental results. Cross-talk seems to have a negative effect on the absorption characteristics of a panel and can be prevented by placing resonators with similar resonant frequencies as far as possible from each other.

ACKNOWLEDGEMENTS

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REFERENCES


APPENDIX A: NOMENCLATURE

\( A_j \) cross-sectional area of resonator \( j \)
\( A_w \) area of the panel
\( c_0 \) speed of sound
\( C_p \) specific heat at constant pressure
\( d \) end correction
\( i = \sqrt{-1} \) imaginary unit
\( J_n(x) \) Bessel function of the first kind of order \( n \)
\( H_{21} \) transfer function of two pressures
\( k = \omega / c_0 \) wave number
\( L \) effective resonator length
\( N \) number of resonators
\( n \) polytropic constant
\( p \) pressure perturbation
\( R \) resonator radius
\( \mathcal{R} \) reflection coefficient
\( s \) shear wave number
\( Z_j \) impedance of resonator \( j \)
\( Z_w \) impedance of the panel
\( \alpha \) absorption coefficient
\( \Gamma \) propagation constant
\( \gamma \) ratio of specific heats
\( \lambda \) thermal conductivity
\( \mu \) dynamic viscosity
\( \rho_0 \) mean density
\( \sigma \) square root of the
\( \Omega \) Prandtl number
\( \omega \) porosity
\( \varphi \) angular frequency