ENERGY DISSIPATION IN THIN AIR LAYERS

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by

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ABSTRACT
This paper deals with a theoretical and experimental analysis of the interaction between a thin layer of air and vibrating flexible surfaces. Special attention is paid to the dissipation of energy in the air layer. A model for the pressure distribution in the air is presented which includes the effects of inertia, viscosity, compressibility and thermal conductivity. The motion of the air in the layer is governed by a number of dimensionless parameters, which can be used to illustrate the different flow regimes and to justify simplifications. The model is validated for an oscillating rigid panel. In addition a new finite element model is developed which enables fully coupled acousto-elastic calculations including the dissipation of energy in the air layer. Numerical and experimental results for an airtight box with a flexible coverplate show fair agreement.

1. INTRODUCTION

Thin layers of air are capable of dissipating energy. In standard acoustic calculations the effects of viscosity and thermal conductivity are usually neglected. This is not allowed when the medium is trapped in a narrow gap between vibrating surfaces. This paper deals with a theoretical and experimental investigation to illustrate the aforementioned effects. The results can be used to analyze for instance the damping capabilities of double wall panels, separated by a thin layer of air.

As a first step, an analytical model was developed for the pressure distribution between vibrating rigid surfaces. The model includes the effects of viscosity, inertia, compressibility and thermal conductivity. The model is written in terms of dimensionless parameters. These parameters can be used to justify several simplifications. As a second step, specially designed experiments were carried out in order to validate the analytical model. In the third step a new viscous acoustic finite element was developed, based on the analytical model. This element can be coupled to structural elements. This enables fully coupled acousto-elastic calculations for vibrating surfaces, separated by a thin layer of air, including the effects of viscosity, inertia, compressibility and thermal conductivity. Finally, the results are compared with experimental results for an airtight box with a flexible coverplate.

Several investigations in the literature concern the effects of viscosity and thermal conductivity in thin air layers [1,2,3,4,5]. To the authors' knowledge, however, none of these investigations takes into account the full coupling between the air layer and the structure for complex geometries. It will be demonstrated that for narrow gaps the coupling between the thin layer of air and the structure can be very strong.
2. THEORY

2.1 Introduction

In this chapter the equation governing the pressure distribution in a thin layer of air is presented. For a detailed derivation of this equation the reader is referred to reference [6]. The air is trapped in a narrow gap between a vibrating flexible plate, with dimensions $2l_x \times 2l_y$, and a fixed surface (see fig 1).

![Flexible plate and thin air layer](image)

**Fig. 1. Flexible plate and thin air layer**

The gap width, i.e., the distance between the plate and the fixed surface, for small sinusoidal perturbations can be written as:

$$h(x, y, t) - h_0 \left[1 + h(x, y)e^{i\omega t}\right]$$  \hspace{1cm} (1)

where $h_0$ denotes the mean gap width, $h$ is the dimensionless amplitude of oscillation, $\omega$ is the angular frequency and $t$ is time. The co-ordinates are made dimensionless with the plate dimensions and the gap width:

$$x = \frac{x}{l_x} \ ; \ y = \frac{y}{l_y} \ ; \ z = \frac{z}{h_0}$$  \hspace{1cm} (2)

2.2 Dimensionless parameters

The motion of the air in the gap is governed by the following equations: the Navier Stokes equations, the equation of continuity, the equation of state for an ideal gas and the energy equation. The quantities of interest, velocities, pressure, density and temperature, are all written in a non-dimensional form. The mean pressure, $p_0$, the mean density, $\rho_0$, the undisturbed speed of sound, $c_0$, and the mean temperature, $T_0$, are used for this purpose. In the absence of mean flow one can write:

$$\vec{u} = c_0 u(x, y, z) e^{i\omega t} \ ; \ \vec{p} = p_0 \left[1 + p(x, y, z) e^{i\omega t}\right]$$

$$\vec{v} = c_0 v(x, y, z) e^{i\omega t} \ ; \ \vec{p} = p_0 \left[1 + p(x, y, z) e^{i\omega t}\right]$$

$$\vec{w} = c_0 w(x, y, z) e^{i\omega t} \ ; \ \vec{T} = T_0 \left[1 + T(x, y, z) e^{i\omega t}\right]$$  \hspace{1cm} (3)

These expressions are used to obtain the basic equations in a linearized form. The resulting set of equations contains the parameters specified in (4). In these expressions $\mu$ is the viscosity of the air, $\lambda$ is the thermal conductivity, $C_p$ is the specific heat at constant pressure and $C_v$ is the specific heat at constant volume. The shear wave number, $s$, represents the ratio between inertial forces and viscous forces. It is also a measure for the ratio between the gap width and the unsteady boundary layer thickness. It is sometimes referred to as the Stokes number or the unsteady Reynolds number.
\[ \begin{align*}
\alpha &= h_0 \sqrt{\frac{\rho_0 \omega}{\mu}} : \text{shear wave number} \\
\sigma &= \sqrt{\frac{\mu C_p}{\lambda}} : \text{root of Prandtl number} \\
k &= \frac{\omega h_0}{c_0} : \text{reduced frequency} \\
g &= \frac{h_0}{l_x} : \text{narrowness of the gap} \\
\gamma &= \frac{C_p}{C_v} : \text{ratio of specific heats}
\end{align*} \]

The reduced frequency, \( k \), is the ratio between the gap width and the acoustic wave length.

### 2.3 Narrow gap equation

When the gap width is small in comparison with the acoustic wave length, i.e. \( k \ll 1 \), and the in-plane velocities are large compared to the velocity in the \( z \)-direction, the equations are further simplified. If the surfaces of the plate and the fixed surface are considered to be isothermal, the equations can be combined into one equation in terms of the dimensionless pressure perturbation, the narrow gap equation [6]:

\[ g \frac{\partial^2 p}{\partial x^2} + \frac{k^2 \gamma}{n(\sigma \alpha) B(s)} \frac{\partial^2 p}{\partial y^2} - \frac{k^2 \gamma h(s, y)}{B(s)} = 0 \tag{5} \]

where:

\[ B(s) = 2 \frac{1 - \cosh (\alpha l)}{\alpha l \sinh (\alpha l)} \]

\[ n(\sigma \alpha) = \left[ 1 - \frac{\gamma - 1}{\gamma} B(\sigma \alpha) \right]^{-1} \tag{6} \]

The narrow gap equation does not contain any derivatives with respect to the \( z \)-direction, as a consequence of the assumption that the pressure is constant across the gap width.

### 2.4 Viscosity

The influence of the viscosity depends on the value of the shear wave number. For low shear wave numbers the viscous effects dominate, whereas for high shear wave numbers the inertial effects dominate. The shape of the velocity profile in the gap is given for three shear wave numbers in figure 2. The figure clearly illustrates the transition from viscous dominated flow to inertial dominated flow.

![Fig. 2. Velocity profile in the gap](image-url)
In the narrow gap equation the viscosity affects the value of the function \( B(s) \) (see (6)). For high shear wave numbers this function can be approximated as \( B(s) \approx 1 \). For low shear wave numbers this function reduces to \( B(s) = \frac{1}{12} \). This implies that for high shear wave numbers the narrow gap equation reduces to a modified form of the wave equation. For low shear wave numbers a linearized version of the Reynolds' equation is obtained.

2.5 Thermal conductivity

The theory presented in this chapter takes into account the effects of thermal conductivity in the coefficient \( n(s \sigma) \) defined in (6). This coefficient can be interpreted as a kind of polytropic constant, since it relates pressure and density according to:

\[
\frac{P}{\rho^n} = \text{constant} \tag{7}
\]

The polytropic constant \( n(s \sigma) \), however, is a function of the product \( s \sigma \) which implies that the value depends on the frequency. Also note that the product \( s \sigma \) does not contain \( \mu \), i.e. the polytropic constant does not depend on the viscosity of the air. For high shear wave numbers \( n \) reduces to \( \gamma \): the process can be regarded as adiabatic (isentropic). For low values of the product \( s \sigma \) however \( n \) reduces to 1, which corresponds to isothermal conditions.

2.6 Effective speed of sound

The narrow gap equation resembles the classical wave equation. The link with the classical wave equation is obvious when the narrow gap equation is rewritten to:

\[
\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\omega^2}{c_{\text{eff}}(s)^2} \frac{\partial P}{\partial x^2} = -\frac{\rho_0 \omega^2}{B(s)} h \tag{8}
\]

where:

\[
c_{\text{eff}}(s) = c_0 \sqrt{\frac{n(s \sigma)}{\gamma} B(s)} \tag{9}
\]

A comparison with the wave equation learns that the undisturbed speed of sound is replaced by a complex and frequency dependent quantity: an effective speed of sound. Viscosity affects the function \( B(s) \), while thermal effects are accounted for in the polytropic constant \( n(s \sigma) \). For a given gas, e.g. air, the effective speed of sound reduces to \( c_0 \) for large shear wave numbers. Under these conditions the viscosity can be neglected and the process can be regarded as adiabatic.

3. TRANSLATING RIGID PANEL

In order to validate the narrow gap model, calculations and experiments were carried out for an oscillating rigid panel. Because the surfaces are rigid, any uncertainty with respect to the flexibility of the panel is excluded, so one can concentrate on the narrow gap model. For this purpose a rectangular solar panel is located parallel to a fixed surface. The panel is suspended in springs at the corners and performs a small, normal oscillation. The mean distance between the panel and the fixed surface, \( h_0 \), can be varied.
3.1 Calculations

Because the panel is rigid and remains parallel to the fixed surface the amplitude of oscillation is equal for all points on the panel. The narrow gap equation for this case reduces to:

$$g^2 \frac{\partial^2 p}{\partial x^2} + \frac{g^2}{a^2} \frac{\partial^2 p}{\partial y^2} + \frac{k^2 \gamma}{n(x o)B(x)} p = - \frac{k^2 \gamma}{B(x)} h$$

(10)

The solution for this equation for open ends, \( p=0 \) for \( x=\pm 1 \) and \( y=\pm 1 \), is:

$$p(x,y) = \frac{4 k^2 \gamma \hbar a^2}{\pi g^3 B(x)} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{q D^2} \left[ \frac{\cosh (Dy)}{\cosh (D)} \right] \cos \left( \frac{2 \pi x}{D} \right)$$

(11)

where:

$$D = \sqrt{\left( \frac{q \pi}{2} \right)^2 - \frac{k^2 a^2 \gamma}{g^2 n(x o)B(x)}}$$

(12)

The dimensionless air load on the panel can be obtained by integration:

$$\frac{\bar{R}}{\rho V} \int \int p \, dx \, dy = \frac{32 k^2 \gamma \hbar a^2}{\pi^2 g^3 B(x)} \sum_{n=1}^{\infty} \frac{1}{q^2 D^2} \left[ \frac{\tanh (D)}{D} \right]$$

(13)

The air load will affect the behaviour of the panel. The eigenfrequency and the corresponding damping coefficient of the system can be calculated with expression (13) and the equation of motion for the oscillating panel. In the calculations, the influence of the air on the upper side of the panel is not taken into account. For narrow gaps the influence of the air on the upper side of the panel can be neglected compared to the influence of the air in the gap.

3.2 Experiments

For the experiments a light and relatively stiff solar panel was used. The panel was 0.98 x 0.98 m with a mass of only 2.5 kg. It was suspended in 8 springs, 2 located at the top and bottom of each corner (see fig 3). In the theory it was assumed that the panel is rigid. In the experiments the stiffness of the springs was such that the eigenfrequency of the mass-spring system in vacuum was 9.74 Hz. This frequency is far below the first elastic eigenfrequency of the panel. Furthermore, due to the added mass effect of the air, the eigenfrequency will decrease. The range of interest is therefore 1-10 Hz.

In order to verify the assumption of rigid behaviour a number of accelerometers was mounted on the panel. Measurements indicate that in the aforementioned frequency range the deformation of the panel is negligible. A rectangular rigid plate of 2.20 x 1.80 m was used as a fixed surface. The fixed surface was mounted on a frame parallel to the panel. The gap width for this configuration could be varied between 3 and 50 mm. In the frequency range of interest this implies that the shear wave number varies between 1.9 and 100.
Fig. 3. Experimental setup translating panel

The properties of interest are:

Air: \( \mu = 1.8 \times 10^{-6} \text{ Ns/m}^2 \)

Panel:

\( m = 2.516 \text{ kg} \)
\( \kappa = 1178 \text{ N/m (14)} \)

\( \lambda = 25.6 \times 10^{-3} \text{ W/mK} \)
\( \rho = 1.2 \text{ kg/m}^3 \)
\( c = 340 \text{ m/s} \)

\( I_x = 0.49 \text{ m} \)
\( I_y = 0.49 \text{ m} \)

\( C_p = 1004 \text{ J/kgK} \)
\( C_V = 716 \text{ J/kgK} \)
\( T_0 = 290 \text{ K} \)

where \( m \) denotes the mass of the panel and \( \kappa \) is the stiffness of one spring.

### 3.3 Results

The measured eigenfrequency (phase resonance), \( f_\alpha \), and the dimensionless damping coefficient, \( \xi \), are listed in table I.

<table>
<thead>
<tr>
<th>( h_0 ) (mm)</th>
<th>( f_\alpha ) (Hz)</th>
<th>( \xi ) (%)</th>
<th>( h_0 ) (mm)</th>
<th>( f_\alpha ) (Hz)</th>
<th>( \xi ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>7.574</td>
<td>0.29</td>
<td>8</td>
<td>4.977</td>
<td>5.03</td>
</tr>
<tr>
<td>35</td>
<td>7.254</td>
<td>0.53</td>
<td>6</td>
<td>4.511</td>
<td>8.04</td>
</tr>
<tr>
<td>25</td>
<td>6.801</td>
<td>0.94</td>
<td>4</td>
<td>3.809</td>
<td>18.6</td>
</tr>
<tr>
<td>15</td>
<td>6.025</td>
<td>1.80</td>
<td>3</td>
<td>3.223</td>
<td>44.1</td>
</tr>
<tr>
<td>10</td>
<td>5.402</td>
<td>3.52</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Experimental results translating panel [7]

The experimental results and the analytical results are depicted in figure 4 and figure 5. The eigenfrequency shows a strong decrease with decreasing gap width, whereas the damping shows a strong increase. For a gap of 3 mm, the damping is almost critical. Calculations and experiments show fair agreement. For narrow gaps there is a strong pumping effect: the air has to move back and forth in the gap.

This pumping effect results in a considerable added mass and a substantial damping for the panel. For the panel under consideration, the added mass for a gap of 3 mm is about 20 kg, while the mass of the panel itself is only 2.5 kg. The added mass can be extracted from the shift in eigenfrequency. Because of the high damping values involved, it is important to distinguish between the amplitude resonance frequency and the phase resonance frequency. The amplitude resonance frequency, i.e., the frequency corresponding to a maximum peak value in the transfer function between force and displacement, is affected by the amount of viscous damping. When this frequency would be used to calculate the amount of added mass, a shift in frequency due to damping is wrongly attributed to added mass. The phase resonance frequency is not affected by viscous damping.
4. ACOUSTO-ELASTIC INTERACTION: FINITE ELEMENT FORMULATION

In practical applications the air is often trapped between vibrating flexible surfaces with a more complex geometry. The theory, as described in section 2, can be extended to flexible surfaces. For narrow gaps the mutual interaction between the pressure distribution in the air and the deformation of the surfaces can be very strong, as will be shown in chapter 5. The coupling can result in a significant change in the dynamical behaviour of the system. A correct description of the acousto-elastic interaction is therefore essential. For this purpose a special finite element model was developed which includes the acousto-elastic interaction between a vibrating structure and a thin layer of air for realistic configurations with complex geometries.

4.1 STRUCTURAL MODEL

The dynamical behaviour of the structure, in the absence of structural damping, can be written in the following standard finite element formulation:

\[-\omega^2 [M] [U] + [K] [U] = [F] \]

(15)

where \([M]\) is the mass matrix, \([K]\) is the stiffness matrix, \([U]\) is the vector with the nodal degrees of freedom and \([F]\) is the external nodal force vector.

4.2 ACOUSTIC MODEL

The narrow gap equation forms the basis for the viscous acoustic finite element. In section 2.6 it was shown that the narrow gap equation can be written as:

\[ \frac{\partial^2 \tilde{P}}{\partial x^2} + \frac{\partial^2 \tilde{P}}{\partial y^2} + \frac{\omega^2}{c_{\text{eff}}^2(x)} \tilde{P} = -\frac{\rho_v \omega^2}{h_v B(x)} h(x, y) \]

(16)

where \(c_{\text{eff}}\) is the effective speed of sound. Analogous to standard acoustics, a finite element formulation can be derived from this equation.
This results in the following system:

\[ -\omega^2 \begin{bmatrix} [M_a(s)] & [0] \\ [0] & [K_a] \end{bmatrix} \begin{bmatrix} \{P\} \\ \{F_r\} \end{bmatrix} = \begin{bmatrix} [K_a] \{U\} \\ \{0\} \end{bmatrix} \begin{bmatrix} \{P\} \\ \{0\} \end{bmatrix} \]

(17)

where \([M_a(s)]\) is the acoustic mass matrix, \([K_a]\) is the acoustic stiffness matrix, \(\{P\}\) is the vector with the nodal pressure degrees of freedom and \(\{F_r\}\) is the external nodal force vector. Note that due to the fact that the effective speed of sound is a complex and frequency dependent quantity, the mass matrix is also complex and frequency dependent. A linear 4 nodded and a quadratic 8 nodded element were implemented in the finite element code B2000 [8]. Both elements were equipped with a number of key options. These options can be used to activate or deactivate for instance viscous or thermal effects. This implies that the element can degenerate to a "modified wave equation" element or a "linearized Reynolds" element.

4.3 ACOUSTO-ELASTIC MODEL

There is a mutual interaction between the vibrating structure and the thin layer of air. On the one hand the motion of the structure has to be followed by the air, while on the other hand the pressure in the layer serves as an excitation for the structure. Especially for thin layers and lightweight structures the coupling can be very strong. A fully coupled finite element analysis can be set up by demanding continuity of normal velocity across the interface. One finally obtains a coupled set of equations:

\[ -\omega^2 \begin{bmatrix} [M_a] & [0] \\ [0] & [K_a(s)] \end{bmatrix} \begin{bmatrix} \{U\} \\ \{P\} \end{bmatrix} = \begin{bmatrix} [K_a] \{U\} \\ \{0\} \end{bmatrix} \begin{bmatrix} \{P\} \\ \{0\} \end{bmatrix} \]

(18)

The coupling is established by the matrices \([M_a(s)]\) and \([K_a(s)]\). These matrices are related by:

\[ [M_a(s)] = \frac{\rho c^2}{h_a B(s)} [K_a(s)]^T \]

(19)

The coupled set of equations is complex and asymmetric.

5. AIRTIGHT BOX WITH FLEXIBLE PLATE

5.1 SETUP

Calculations and experiments were carried out for an airtight aluminium box with a flexible coverplate (see fig 6). The aluminium coverplate, 0.49 x 0.245 m, was clamped on all four sides. A rigid bottom plate was located in the box parallel to the plate. A thin layer of air is trapped in the cavity. The bottom plate was mounted on a frame and the gap width, \(h_a\), could be varied between 1 and 50 mm. The coverplate plate was excited by an electrodynamic shaker. The response was measured with a number of accelerometers. A schematic drawing of the experimental setup is given in figure 7. The eigenfrequency, the damping coefficient and the corresponding mode shape for the first and the second mode were measured as a function of the gap width. Special attention was paid to the effects of viscosity and thermal conductivity.
5.2 UNCOUPLED MODES

5.2.1 Structural modes

In order to verify the finite element model of the plate, the structural modes, i.e. the modes of the plate in vacuum, were calculated. The results were compared with analytical values taken from the literature [9]. The plate was modelled with 20 x 20 linear plate elements. The results from the finite element calculations and the analytical results show fair agreement, as can be seen in table II. The properties of the plate are listed in (20).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenfrequency (Hz)</th>
<th>Analytical</th>
<th>B2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>100.6</td>
<td>101.0</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>130.4</td>
<td>131.1</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>183.4</td>
<td>186.9</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>261.7</td>
<td>266.1</td>
<td></td>
</tr>
</tbody>
</table>

Table II. Frequencies structural modes

Plate: \( E = 70 \times 10^9 \text{ N/m}^2 \)

\( \rho_p = 2710 \text{ kg/m}^3 \)

\( v_p = 0.3 \)

\( t_p = 1 \times 10^{-3} \text{ m} \)
5.2.2 Acoustic modes

The acoustic modes are the modes of the air layer when the surfaces surrounding the air layer are assumed rigid. In this case one therefore has to solve the following equation:

\[ g^2 \frac{\partial^2 p}{\partial x^2} + \frac{g^2 \partial^2 p}{a^2 \partial y^2} + \frac{k^2 \gamma}{n(\omega) B(\omega)} p = 0 \]  \hspace{1cm} (21)

with the boundary condition:

\[ \frac{\partial p}{\partial n} = 0 \quad \text{for} \quad x = \pm 1 \quad \text{and} \quad y = \pm 1 \]  \hspace{1cm} (22)

The eigenfrequencies and the mode shapes can be obtained by straightforward separation of variables. This finally results in the following equation in terms of the frequency:

\[ \omega = \pi \sqrt{\frac{g}{k_x} + \frac{r^2}{r_x^2}} \quad q = 0,1,2,... \quad r = 0,1,2,... \]  \hspace{1cm} (23)

The effective speed of sound is a function of the shear wave number. The right hand side of expression (23) therefore depends on the gap width and the frequency. The complex eigenfrequencies can be solved from this equation. For large shear wave numbers expression (23) reduces to the expression from the eigenvalue problem for a rectangular domain when using the wave equation.

As an example, the acoustic modes were calculated for a gap width of 1 mm. In the finite element calculations the air layer is modelled with 20 × 20 linear viscous acoustic finite elements. The properties of the air are specified in expression (14). The results of the calculations are listed in table III. The modes are identified by the number of half wave lengths in the x- and the y-direction respectively. Again, good agreement is obtained between the analytical and numerical results.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenfrequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
</tr>
<tr>
<td>10</td>
<td>317.1 + 32.3 i</td>
</tr>
<tr>
<td>01</td>
<td>651.4 + 44.8 i</td>
</tr>
<tr>
<td>20</td>
<td>651.4 + 44.8 i</td>
</tr>
<tr>
<td>11</td>
<td>730.9 + 47.2 i</td>
</tr>
</tbody>
</table>

Table III: Frequencies acoustic modes

5.3 ACousto-ELastic Modes

The acousto-elastic modes were measured for gap widths from 50 down to 1 mm [10,11]. Calculations were carried out with a mesh of 20 × 20 linear plate elements and 20 × 20 linear viscous acoustic elements. The coupling between the air layer and the plate was established by 20 × 20 linear interface elements. The eigenfrequencies, the damping and the corresponding mode shapes were calculated for a number of gap widths. The measured and the calculated frequencies are depicted in figure 8. The corresponding dimensionless damping coefficients are depicted in figure 9.
The calculations and the measurements show that the eigenfrequency is strongly affected by the gap width. The first mode first slightly increases and then decreases with decreasing gap width, while the second mode shows a steady decrease. The damping increases with decreasing gap width for both modes. The agreement between calculations and experiments is satisfying.

The acousto-elastic mode shapes of the system change as a function of the gap width. The real part of the calculated mode shapes for the first and the second mode are depicted in figure 10 and figure 11. The mode shapes are not very complex, i.e. the difference in phase angle between the points is small. The shape of the first mode is affected by the gap width, while the shape of the second mode remains unchanged. A physical explanation for these phenomena will be given in the next section.
5.4 PHYSICAL INTERPRETATION

In figure 8 it can be seen that the eigenfrequency of the first mode is higher than the eigenfrequency of the second mode. The name first mode is used for this mode because it originates from the first mode of the plate in vacuum, i.e., the 11 structural mode. The second mode corresponds to the second mode of the plate in vacuum, i.e., the 21 structural mode. When the gap width is decreased both frequencies cross at a certain gap width [12]. The eigenfrequency of the first mode increases with decreasing gap width, while the eigenfrequency of the second mode decreases. For this setup the cross over takes place at a gap width of approximately 100 mm. When the gap width is then further decreased, the curves of figure 8 are obtained. The shift in frequency and the change in mode shape can be explained with the added mass and the added stiffness effect. The damping is related to the effect of viscosity.

5.4.1 Added mass

The second mode is an asymmetric mode. This mode induces a strong pumping effect in the air layer. Due to the periodic motion of the plate the air is forced to move back and forth in the gap. The plate experiences the motion of the air as an added mass. When the gap width decreases, the pumping effect increases and therefore the added mass increases. This explains why the eigenfrequency of the second mode decreases with decreasing gap width. For gap widths below 10 mm the shape of the first mode has changed in such a way that also this mode induces a pumping of the air. Therefore its eigenfrequency decreases.

5.4.2 Added stiffness

The first mode shows a slight increase with decreasing gap width. This can be explained with the added stiffness effect. For the first mode, the deformation of the coverplate is accompanied by a significant change in cavity volume. This leads to a change in pressure in the air layer. The pressure disturbance increases with decreasing gap width. In the situation of interest, the acoustic wave length is large compared to the dimensions of the box. The plate now experiences this increase in pressure disturbance as an added stiffness. Therefore the eigenfrequency of the first mode increases with decreasing gap width. Due to the added stiffness effect the mode shape also changes. Especially for narrow gaps the modes tend to a shape which does not affect the cavity volume. For gap widths below 10 mm the shape of the first mode has changed in such a way that the net change in cavity volume tends to zero. The added stiffness effect vanishes and the mode shape now induces a pumping effect: the eigenfrequency decreases with decreasing gap width.

5.4.3 Damping

The damping is caused by the effects of viscosity in the air layer. The ratio between the inertial forces and the viscous forces is given by the shear wave number. The shear wave number as a function of the gap width is depicted in figure 12.

![Fig. 12. Shear wave number as a function of \( h \)]
Figure 12 shows that for narrow gaps the effects of viscosity have to be taken into account. The shear wave number is of the order of unity, which implies that the viscous and the inertial effects are of the same order of magnitude. The damping of the plate is also related to the pumping effect. When the air has to move back and forth in the gap, viscous losses are introduced. The second mode is an asymmetric mode that induces a strong pumping effect. This contributes to the relatively high damping of this mode.

6. CONCLUSIONS

A new model was developed and experimentally validated for the interaction between vibrating flexible surfaces and a thin layer of air. This enables fully coupled acoust-elastic calculations, including the dissipation of energy due to viscous or thermal effects in the air layer. The motion of the air in the layer is governed by a number of dimensionless parameters. The most important parameter is the shear wave number, which represents the ratio between the inertial forces and the viscous forces. The results from experiments and calculations show fair agreement. Calculations and experiments illustrate that for low shear wave numbers the effects of viscosity have to be taken into account.

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