Enhancing Trust and Stability in Purchasing Consortia: Fair Allocation of Gains

Fredo Schotanus

Summary

Purchasing consortia are becoming more and more common practice. However, many consortia end prematurely or do not flourish. Important reasons indicated for these problems are in(directly) related to the fair allocation of gains. The purpose of this paper is to build further on solutions to allocation problems, aiming to enhance purchasing consortia trust and stability. In this paper several existing allocation concepts are described and adapted to purchasing consortia. Also, a new allocation concept is introduced and compared to existing concepts. Recommendations are given concerning which concept to use in which situation. We conclude by emphasizing the importance for purchasing consortia to choose an allocation concept.

Keywords: Purchasing consortium; Gains allocation; Co-operation; Cooperative game theory

Introduction

Value-added pricing, lower transaction costs, reduced risks and workload, improved quality and service, better access to resources, stronger negotiation positions, lower supply risks, sharing experiences and information, and learning from others are all theoretical advantages related to groups of organisations purchasing products and/or services collectively (purchasing consortia). These advantages should outweigh related disadvantages such as setup and maintenance costs, anti-trust (legal) and commitment issues, disclosure of sensitive information, supplier resistance, and the ‘fear of parasites’ in a large number of cases (Pye 1996; Doucette 1997; Hendrick 1997; Ireland 2002; Aylesworth 2003; Virolainen 2003). However, premature endings of existing purchasing consortia still occur regularly and other purchasing consortia do not flourish.

Purchasing consortia have received relatively little attention in purchasing management research. In addition, purchasing consortia research so far has focused primarily on inductive explanations of practice and qualitative deductive reasoning. The use of quantitative deductive reasoning has been limited until now.

The lack of purchasing consortia research attention seems unjustified, with purchasing consortia being more and more well-established in the public sector and gaining popularity in the private sector (Macie 1995; Sickinger 1996; Hendrick 1997; Major 1997; Zentes 2000).

One specific purchasing consortia issue receiving minor research attention is the fair allocation of gains. It is disturbing that important reasons indicated from practice for purchasing consortia problems - anti-trust, no commitment and ‘fear of parasites’ - are related to these gains allocation problems. Therefore, the main question in this paper is how to divide purchasing consortia gains among consortium members?

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The purpose of this paper is (1) to build further on solutions to gains allocation problems of purchasing consortia (Heijboer 2003) and (2) to contribute to the quantitative deductive development of purchasing management.

The model

To analyse the effects of different allocation concepts, we model purchasing consortia by taking into account price reduction due to economies of scale. As mentioned in the introduction several other issues also play an important role in (the success of) establishing and managing consortia. However, here we focus on the price issue.

For the price per item \( p(q) \) we assume a decreasing volume discount is given, with more items being purchased. Of course there is a minimum price \( p_0 \), so \( p(q) \) is a convex function. In addition, we assume the total purchasing volume \( q \cdot p(q) \) to be increasing with the number of items being bought. These assumptions hold for many practical situations (Dolan 1987).

We refer to this model as a Cooperative Purchasing-game or CP-game \((N,q,p)\) (Heijboer 2003). \( N \) is the number of organisations, \( q \) is the number of items each organisation wants to purchase and \( p \) is the price per item. The total gains function \( v(S) \) of each coalition (purchasing consortium) \( S \) is defined as the gains it generates by buying items together compared to the situation where each of the organisations has to buy these items on its own:

\[
v(S) = \sum_{i \in S} (q \cdot p(q)) - \sum_{i \in S} q_i \cdot p(\sum_{i \in S} q_i)
\]

Gains allocation concepts

In practice, simple rules are usually used when distributing gains among consortium members. The concepts used in practice as Same Price are compared in this paper to concepts (see Appendix A for descriptions) from cooperative game theory.

Cooperative game theory

Game theory is a mathematical research field that deals with multilateral decision making. Each decision maker (player) has his own interests and has a number of possible actions open to him. By his actions, each player affects the outcomes for the other players. In cooperative game theory it is assumed that gains can be made when all players cooperate. One of the problems that are addressed in this theory is how to divide these gains in a fair way among all players.

The Compromise Value

We analyse the game theoretical concept the Compromise Value (CV) in some more detail. The CV \( \tau_i(v) \) is based on the maximum \( M_i(v) \) and minimum \( m_i(v) \) amount of the total gains that each organisation \( i \) can reasonably claim (Driessen 1985; Borm 1992) and:

\[
\tau_i(v) = \alpha M_i(v) + (1-\alpha) m_i(v) \text{ with } \alpha \in [0,1] \text{ unique such that } \sum_{i \in N} \tau_i(v) = v(N)
\]

The maximum claim \( M_i(v) \) for organisation \( i \) equals the total gains of the coalition minus the value the other organisations can establish without organisation \( i \): \( M_i(v) = v(N) - v(N \setminus \{i\}) \).
The minimum claim \( m_i(v) \) can be determined by looking at each sub-coalition that organisation \( i \) could belong to. In each of these coalitions organisation \( i \) will give the other organisations their maximum claims and see what is left for organisation \( i \). The maximum leftover is the claim \( m_i(v) = \max_{S \subseteq S} \left( v(S) - \sum_{j \in S, j \neq i} M_j \right) \).

However, this is not always the minimum claim for consortium members. We state that the minimum claim for an organisation should be extended with the gains this organisation creates for itself by joining the consortium. This is illustrated by the following case.

**Case introduction**

Consider three organisations purchasing cooperatively 60 items without any additional costs. The price \( p \) for the items as a function of the quantity \( q \) that will be ordered is known as \( p(q_i) = 959 \cdot \left(1 + \frac{1}{\sqrt{q_i}}\right) \). This case can be modelled into a CP-game as is shown in Table 1.

**Table 1 - CP-game for three organisations**

<table>
<thead>
<tr>
<th>Coalition ( S )</th>
<th>Quantity</th>
<th>Price per item</th>
<th>Total</th>
<th>( v(S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>35</td>
<td>1.121</td>
<td>39.246</td>
<td>0</td>
</tr>
<tr>
<td>{2}</td>
<td>10</td>
<td>1.262</td>
<td>12.625</td>
<td>0</td>
</tr>
<tr>
<td>{3}</td>
<td>15</td>
<td>1.207</td>
<td>18.102</td>
<td>0</td>
</tr>
<tr>
<td>{1,2}</td>
<td>45</td>
<td>1.102</td>
<td>49.597</td>
<td>2.273</td>
</tr>
<tr>
<td>{1,3}</td>
<td>50</td>
<td>1.095</td>
<td>54.741</td>
<td>2.607</td>
</tr>
<tr>
<td>{2,3}</td>
<td>25</td>
<td>1.151</td>
<td>28.775</td>
<td>1.952</td>
</tr>
<tr>
<td>{1,2,3}=N</td>
<td>60</td>
<td>1.083</td>
<td>64.980</td>
<td>4.993</td>
</tr>
</tbody>
</table>

**Case gains of organisation 2 and 3**

The total gains organisation 2 creates for the consortium are equal to its maximum claim \( M_2 = v(N) - v(\{1,2\}) = 4.993 - 2.607 = 2.386 \). The maximum claim \( M_2 \) can be divided into three types of gains as is shown in Table 2.

**Table 2 - Gains of organisation 2**

<table>
<thead>
<tr>
<th>Gains</th>
<th>Description</th>
<th>Calculation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Gains for and by 2</td>
<td>quantity of {2} \cdot (price(consortium without {2})-price({1,2,3}))</td>
<td>=10 \cdot (1.095-1.083)</td>
<td>118</td>
</tr>
<tr>
<td>(2) Gains by 2 for ( N \setminus {2} )</td>
<td>quantity of {1,3} \cdot (price(consortium without {2})-price({1,2,3}))</td>
<td>=50 \cdot (1.095-1.083)</td>
<td>591</td>
</tr>
<tr>
<td>(3) Gains for 2 by ( N \setminus {2} )</td>
<td>quantity of {2} \cdot (price({2})-price(consortium without {2}))</td>
<td>=10 \cdot (1.262-1.095)</td>
<td>1.677</td>
</tr>
<tr>
<td>Total Gains</td>
<td>maximum claim of 2</td>
<td>=M_2</td>
<td>=2.386</td>
</tr>
</tbody>
</table>
As stated before, the minimum claim of the CV of an organisation should be extended with the gains this organisation creates for itself by joining the consortium. In this case the extension of the minimum claim of organisation 2 is € 118. The calculations for the gains of organisation 3 can be made in the same way as for organisation 2.

Case gains of organisation 1

For organisation 1 an exception occurs (see Table 3), because the volume of this organisation \( q_2 = 35 \) exceeds the volumes of organisations 2 and 3 \( q_{2+3} = 25 \). This influences the calculations for the gains (1) and (3): instead of price(consortium without \{1\}), price(\{1\}) should be used, as price(\{1\}) is lower than price(consortium without \{1\}).

Table 3 - Gains of organisation 1

<table>
<thead>
<tr>
<th>Gains for and by 1</th>
<th>Description</th>
<th>Calculation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Gains for and by 1</td>
<td>quantity of ( {1} \cdot (\text{price}({1})-\text{price}({1,2,3})) )</td>
<td>( 35 \cdot (1.121-1.083) = 1.341 )</td>
<td></td>
</tr>
<tr>
<td>(2) Gains by 1 for ( \text{N \setminus {1} )</td>
<td>quantity of ( {2,3} \cdot (\text{price (consortium without 1)}-\text{price}({1,2,3})) )</td>
<td>( 25 \cdot (1.151-1.083) = 1.700 )</td>
<td></td>
</tr>
<tr>
<td>(3) Gains for 1 by ( \text{N \setminus {1} )</td>
<td>quantity of ( {1} \cdot (\text{price}({1})-\text{price}({1})) )</td>
<td>( 35 \cdot (1.121-1.121) = 0 )</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>maximum claim of 1</td>
<td>( = M_1 = 3.041 )</td>
<td></td>
</tr>
</tbody>
</table>

The Adapted Compromise Value

Integrating the gains for and by organisation \( i \) into \( m_i(v) \) gives a new minimum claim for the allocation concept the Adapted Compromise Value (ACV):

\[
m_i(v) = \max_{S \subseteq \mathcal{S}} \left\{ q_i \cdot (\min_{S \subseteq \mathcal{S}} \left\{ p(\sum_{j \in S} q_j), p(q_i) \right\} - p(\sum_{j \in S} q_j)) \right\}
\]

Allocations of case gains compared

Table 4 gives an overview of several allocation concepts (see Appendix A for descriptions) applied to the case. The bold values represent those values that the concerning organisations logically would accept. A remarkable outcome of this case is that using the Same Price concept leads to a situation where the largest organisation receives the smallest amount of gains.
Table 4 - Allocations of case gains compared

<table>
<thead>
<tr>
<th>Allocation concept</th>
<th>Organisation 1 (35 items)</th>
<th>Organisation 2 (10 items)</th>
<th>Organisation 3 (15 items)</th>
<th>Total (60 items)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Equal amount</td>
<td>1.664</td>
<td>1.664</td>
<td>1.664</td>
<td>4.993</td>
</tr>
<tr>
<td>2. Proportional</td>
<td>2.912</td>
<td>832</td>
<td>1.248</td>
<td>4.993</td>
</tr>
<tr>
<td>3. Equal percentage</td>
<td>2.800</td>
<td>901</td>
<td>1.292</td>
<td>4.993</td>
</tr>
<tr>
<td>4. None / same price</td>
<td>1.341</td>
<td>1.795</td>
<td>1.857</td>
<td>4.993</td>
</tr>
<tr>
<td>5. SER (decreasing)</td>
<td>2.398</td>
<td>1.108</td>
<td>1.486</td>
<td>4.993</td>
</tr>
<tr>
<td>6. SER (increasing)</td>
<td>2.154</td>
<td>1.282</td>
<td>1.557</td>
<td>4.993</td>
</tr>
<tr>
<td>7. Nucleolus</td>
<td>1.990</td>
<td>1.335</td>
<td>1.668</td>
<td>4.993</td>
</tr>
<tr>
<td>8. Shapley Value</td>
<td>1.827</td>
<td>1.500</td>
<td>1.666</td>
<td>4.993</td>
</tr>
<tr>
<td>9. Compromise value</td>
<td>1.864</td>
<td>1.462</td>
<td>1.667</td>
<td>4.993</td>
</tr>
<tr>
<td>10. Adapted compromise value</td>
<td>2.203</td>
<td>1.269</td>
<td>1.521</td>
<td>4.993</td>
</tr>
</tbody>
</table>

Properties of fairness compared

Table 5 gives an overview of which concepts satisfy which properties of fairness for CP-games (see Appendix B for descriptions). Proofs have been omitted here.

Table 5 - Properties of concepts for CP-games without costs (✓ = satisfied, ✗ = not satisfied in general)

<table>
<thead>
<tr>
<th>Allocation concept</th>
<th>EFF</th>
<th>SYM</th>
<th>DUM</th>
<th>CDUM</th>
<th>ADD</th>
<th>IND</th>
<th>FR</th>
<th>STA</th>
<th>MON</th>
<th>time</th>
<th>ease</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Equal amount</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2. Proportional</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3. Equal percentage</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4. Same Price</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>5. SER (decreasing)</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>6. SER (increasing)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>7. Nucleolus</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>8. Shapley Value</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>9. Compromise value</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>10. Adapted compromise value</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 5 shows among other things that using the Same Price allocation concept can lead to situations where the largest organisation receives the smallest part of the total gains (FR property). Also, the situation could occur that an organisation increases its purchases through the consortium, but in return receives a smaller amount of gains (MON property). This could slow down potential growth of the purchasing consortium.

Using the Equal Amount concept, the Proportional concept, the Equal Percentage concept, or SER (decreasing) can lead to situations where organisations could want to split up the consortium to increase their individual gains (STA property).

Unlike the concepts mostly used in practice, the game theoretical concepts and SER (increasing) satisfy most properties associated with fairness. Thus, these concepts can be...
considered as fairer and more stable alternatives. The Shapley Value can be considered as the fairest alternative because it also satisfies the ADD property. A disadvantage of this concept is that calculating a solution for $N \geq 10$ takes a large amount of calculation time.

The CV and the ACV can be explained and applied relatively easily, this in contrary to SER (increasing), the Nucleolus and the Shapley Value. Although not proven in Table 5, the ACV seems fairer than the CV because the ACV minimum claim seems a fair extension of the CV minimum claim.

**Recommendations**

Table 6 gives an overview of which allocation concepts to use in different situations. The rows represent the financial risks the participating organisations take by buying through the consortium. These risks depend among other things on the type of products and on the consortium volume of the organisations compared to the total volume of the organisations.

In the columns distinction is made between the private and the public sector and between consortia with all organisations purchasing an (almost) equal or unequal volume.

<table>
<thead>
<tr>
<th>Financial risk</th>
<th>Private sector</th>
<th>Public sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equal volumes</td>
<td>Unequal volumes</td>
</tr>
<tr>
<td>Low</td>
<td>none (same price)</td>
<td>N&lt;10: shapley value N≥10: ACV</td>
</tr>
</tbody>
</table>

Table 6 is based upon the following assumptions:

(1) The Same Price allocation concept should be used in situations where the participating organisation’s volumes are (almost) equal. In these situations all concepts provide (almost) the same results. Thus, the choice to use the easiest concept is obvious.

(2) If the organisations’ volumes are unequal and financial risks are low, consortia in the public sector should use the Same Price concept because of its ease. In the private sector we expect consortia to be less stable than in the public sector, mainly because of a more competitive attitude. Therefore, if volumes are unequal, private consortia should always use the Shapley Value or the ACV. The Shapley Value should be used for small consortia. When $N$ becomes larger than 10 the Shapley Value calculation time becomes inadmissible and the ACV should be used. Anyhow, both concepts are fairer than the Same Price concept and more capable in preventing coalition instability, even though these concepts are more difficult to calculate.

(3) The higher the financial risks are, the higher the interests of organisations become, the higher the chances of consortium instability become, even in the public sector. Therefore, if the organisations’ volumes are unequal and financial risks are high, the Shapley Value or the ACV concept should be used to prevent coalition instability.
Conclusion

In this paper we build further on the link between purchasing consortia and cooperative game theory. Several (new) allocation concepts are analysed and compared. We recommend using the Same Price allocation concept in situations with equal organisations in a consortium or when financial risks are low in the public sector. When organisations are unequal and financial risks are high, the Adapted Compromise Value or the Shapley Value should be used. When choosing another allocation concept in these situations, it is highly important that this is an intentional choice and that the consortium members are aware that problems could arise (like ADD, FR, STA and MON related problems).

Choosing a gains allocation concept will provide financial clarity to organisations in a purchasing consortium. It helps in negotiations, by reducing the fear of other organisations benefiting parasitically and it can help by enhancing the stability of a consortium. By enhancing trust and stability, commitment to the consortium can improve, which for a purchasing consortium is highly important for success (Doucette 1997; Heijboer 2003).

To increase the relevance and applicability further research will incorporate (1) determining purchasing consortium setup, maintenance, and compensation costs, (2) taking more discounts into account than only volume discounts as benefits of cooperation, and (3) determining the extent to which the concepts in Table 6 satisfy the properties of fairness by testing the concepts in real life situations and by scenario analysis.

Appendix A: Gains allocation concepts

(1) Equal amount and (2) Equal percentage

\[ \text{Equal}_1(v) = \frac{v(N)}{n} \quad \text{and} \quad \text{Equalperc}_i(v) = \frac{q_i \cdot p_i}{\sum_{i \in N} (q_i \cdot p_i)} \cdot v(N) \]

(3) Average cost pricing and (4) Same price

\[ \text{ACP}_i(v) = \frac{q_i \cdot p(\sum_{i \in N} q_i)}{\sum_{i \in N} q_i \cdot p(\sum_{i \in N} q_i)} \cdot v(N) = \frac{q_i}{\sum_{i \in N} q_i} \cdot v(N) \quad \text{and} \quad \text{Sameprice}_i(v) = q_i \cdot (p(q_i) - p(\sum_{i \in N} q_i)) \]

(5) Serial cost sharing (decreasing rule)

Given a list of demands \( q_1, \ldots, q_n \) order them first in decreasing order: \( q_n \leq \ldots \leq q_2 \leq q_1 \). Now organisation 1 receives \( \text{SERD}_1 = \frac{v(N_1)}{n} \) and \( v(N_0) = n \cdot q_0 \cdot p(q_0) - n \cdot q_0 \cdot p(n \cdot q_0) \). To calculate \( \text{SERD}_n \) to \( \text{SERD}_1 \) we use the following demands:

\[ N_0: \{n \cdot q_0\} \quad \text{and} \quad N_i: \{(n-i+1) \cdot q_i, q_{i+1}, \ldots, q_n\} \quad \text{and} \quad \text{SERD}_i = \frac{v(N_i) - \sum_{j=0}^{i-1} \text{SERD}_j}{n-i+1} \quad \text{(Frutos 1998)} \]

(6) Serial cost sharing (increasing rule)

Given a list of demands \( q_1, \ldots, q_n \) order them first in increasing order: \( q_1 \leq q_2 \leq \ldots \leq q_n \). Now organisation 1 receives \( \text{SERI}_1 = \frac{v(N_1)}{n} \) and \( v(N_0) = n \cdot q_1 \cdot p(q_1) - n \cdot q_1 \cdot p(n \cdot q_1) \). The agents

\[ v(N_i) - \sum_{j=0}^{i-1} \text{SERI}_j \]

\[ n-i+1 \quad \text{(Frutos 1998)} \]

\[ b \text{ Allocation software is available for download at} \quad \text{http://www.sms.utwente.nl/utips} \]
2,...,n then (1) receive \( \frac{v(N_i)}{n} \) to cover their demand up to level \( q_i \), and (2) divide the surplus of their incremental demands \( q^*_i = q_i - q_1 \), \( i \geq 2 \). To calculate \( \text{SERI}_1 \) to \( \text{SERI}_n \) we use the following demands:

\[ N_i: \{n \cdot q_i\} \text{ and } N_i: \{i-n\cdot q_i \}, \text{ and } \text{SERI}_i = \frac{v(N_i) - \sum_{j=0}^{i-1} \text{SERI}_j}{n-i+1} \text{ (Moulin H. 1992)} \]

(7) The nucleolus
The nucleolus minimizes the maximum dissatisfaction level of all coalitions (Schmeidler 1969; Borm P. 2001). As a measure for dissatisfaction, the excess \( E(S,x) \) of coalition \( S \) is introduced: \( E(S,x) = v(S) - \sum_{i \in S} x_i \). The nucleolus \( n(v) \) for CP-games can be calculated by using an adapted version of the Aumann-Maschler rule:

\[
n(v)_i = AM_i(V(N),M_i(v)) = \begin{cases} CE(A(V(N)),\frac{1}{2} \cdot M_i(v)) & \sum_{i \in N} M_i(v) \geq 2 \cdot v(N) \\
M_i(v) - CE(A(\sum_{i \in N} M_i(v) - V(N)),\frac{1}{2} \cdot M_i(v)) & \sum_{i \in N} M_i(v) < 2 \cdot v(N) \end{cases}
\]

\[ M_i(v) = v(N) - v(N \setminus \{i\}) \text{ and } CE(A(V(N),M_i(v)) = (\min\{\alpha, M_i(v)\})_{i \in N} \text{ with } \alpha \text{ such that } \sum_{i \in N} \min\{\alpha, M_i(v)\} = V(N) \]

(8) The Shapley value
\[
\Phi(v) = \frac{1}{|N|!} \sum_{\sigma \in \Pi(N)} m^\sigma v, \text{ } m^\sigma = v(\{1,...,\sigma(k)\}) - v(\{1,...,\sigma(k-1)\}) \text{ with } \sigma \in \Pi(N),
\]

\[ \Pi(N) = \{\sigma : [1,...,|N|] \to N | \sigma \text{ bijective}\} \text{ (Shapley 1953)} \]

(9) The compromise value
\[
\tau_i(v) = \alpha M_i(v) + (1-\alpha) m_i(v) \text{ with } \alpha \in [0,1] \text{ unique such that } \sum_{i \in N} \tau_i(v) = v(N) \text{ and } M_i(v) = v(N) - v(N \setminus \{i\}) \text{ and } m_i(v) = \max_{S \subseteq S} \left\{ v(S) - \sum_{j \in S, i \not\in j} M_j(v) \right\} \text{ (Borm 1992)} \]

(10) The adapted compromise value
\[
\tau_i(v) = \alpha M_i(v) + (1-\alpha) m_i(v) \text{ with } \alpha \in [0,1] \text{ unique such that } \sum_{i \in N} \tau_i(v) = v(N) \text{ and } M_i(v) = v(N) - v(N \setminus \{i\}) \text{ and } m_i(v) = \max_{S \subseteq S} \left\{ q_i \cdot (\min_{S \subseteq S} \left\{ p\left( \sum_{j \in S, i \not\in j} q_j \right), p(q_i) \right\} - p(\sum_{j \in S} q_j)) \right\} \}
\]

Appendix B: Properties of allocation concepts (Driessen 1991; Heijboer 2003)

(1) EFF: efficiency. All gains are allocated back to the organisations: \( \sum_{i \in N} f_i(v) = v(N) \)
(2) **SYM:** symmetry. If for two organisations $i$ and $j$ can be interchanged without changing any $v(S)$ then $f_i(v) = f_j(v)$. It means that equal organisations should get equal pay-offs.

(3) **DUM:** dummy. If $v(S \cup \{i\}) - v(S) = v(\{i\})$ for all $S \subseteq N \setminus \{i\}$ then $f_i(v) = v(\{i\})$. It means that an organisation, which does not contribute anything, should not get anything.

(4) **CDUM:** converse dummy. If $V(N) > V(N \setminus \{i\})$ then $f_i(v) > 0$. CDUM implies that any organisation contributing positively to the consortium profits should receive some gains.

(5) **ADD:** additivity. For two games $v$ and $w$ with solutions $f(v)$ and $f(w)$ it holds that $f(v + w) = f(v) + f(w)$. A purchasing consortium could be used for multiple (types of) items at the same time. Each item could be treated as a separate game with a separate gain allocation. The gains from all items could also be added up and be allocated at once. It seems fair that when the same allocation concept would be used for each item separately or for all of them together the total amount allocated to each organisation should be the same. This is just another way of saying that ADD has to hold.

(6) **IND:** individual rationality. Not only EFF is satisfied, but also for all organisations $i$ it holds that $f_i(v) \geq v(\{i\})$. It means that for each organisation the pay-off of cooperation is equal or higher than the pay-off of working alone.

(7) **STA:** stability. EFF is satisfied and for all coalitions $S$ it holds that $\sum_{i \in S} f_i(v) \geq v(S)$. It means that for each organisation the pay-off of cooperation in the grand coalition is equal or higher than the pay-off of working alone or in any other sub coalition. Note that STA implies that organisation $i$ cannot receive a larger pay-off than $M_i(v)$ or a smaller pay-off than the adapted $m_i(v)$.

(8) **FR:** fair ranking. If for two organisations $i$ and $j$ $q_i' \geq q_j$ then $f_i(v) \geq f_j(v)$. It means that FR is satisfied if an organisation with an equal or larger quantity of items to be purchased through a consortium (higher leverage) receives an equal or larger share of the gains.

(9) **MON:** monotonicity. If for one organisation $i$ $q_i' \geq q_i$ then $f_i'(v) \geq f_i(v)$. Satisfying this property means that if the quantity of items to be purchased by one organisation stays equal or becomes larger than in a former situation, this organisation should receive an equal or larger amount of gains. In other words, $f_i$ is nondecreasing in $q_i$.

(10) **TIME:** calculation time. Calculation time can increase tremendously with an increasing number of purchasing consortium members.

(11) **EASE:** ease of calculating, applying and understanding the allocation concept solution.

**References**


Heijboer, G. (2003). Mathematical and statistical analysis of initial purchasing decisions. Enschede, University of Twente.