Direct Josephson coupling between superconducting flux qubits

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We have demonstrated strong antiferromagnetic coupling between two three-junction flux qubits based on a shared Josephson junction, and therefore not limited by the small inductances of the qubit loops. The coupling sign and magnitude were measured by coupling the system to a high-quality superconducting tank circuit. Design modifications allowing to continuously tune the coupling strength and/or make the coupling ferromagnetic are discussed.

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Quantum superposition of macroscopic states was conclusively demonstrated in superconducting Josephson structures in 2000 (Ref. 1). Such structures are natural candidates for the role of qubits (quantum bits), the constituent elements of quantum computers. Successful operation of a quantum computer would be the ultimate confirmation of the validity of quantum mechanics on the macroscopic scale, which makes it difficult to couple 3JJ qubits, making progress towards meeting the requirements, and discuss the ways of its further improvement. The 3JJ qubit consists of a superconducting loop with small inductance L interrupted by three Josephson junctions. The two different directions of persistent current in the loop form the qubit’s basis states. The 3JJ design enables classical bistability even for L → 0, resulting in a weak coupling to environmental magnetic-flux noises. As a result, quantum behavior with long decoherence times was observed in this type of qubit by several groups.3–5

However, their small L makes it difficult to couple 3JJ qubits inductively; generally, J is smaller than the single-qubit level splitting. We therefore implement the proposals6,7 to directly link two qubits through a shared junction (Fig. 1). The resulting coupling not only is strong, but can also be varied independently of other design parameters by choosing the shared junction’s size.

To calculate J, we neglect the inductances so that the potential term in the Hamiltonian contains only the Josephson energy \( U_j = -\Sigma_j e_j \cos \phi_j \), and use flux quantization, \( \phi_1 + \phi_2 + \phi_3 + \phi_0 = 2\pi (1 + f_a) \) and \( \phi_2 + \phi_3 + \phi_0 - \phi_0 = 2\pi (1 + f_b) \), to eliminate \( \phi_{1,5} = \Phi_{e,b}/\Phi_0 = 1/2 \). In the simplest case \( f_{a,b} = 0 \), where \( U_{AF} = \pm \alpha E_0 \) and \( \phi_{AF} = \pm \cos(1/2\alpha) \), the AF yields an antiferromagnetic (AF), with parallel and antiparallel loop currents, respectively:

\[
\phi_{AF}^{FM} = 0, \\
\phi_{AF}^{FM} = \pm \frac{\hbar I_p}{e E_0} + O((E/E_0)^2),
\]

where \( I_p = (2e/h)E_0(1-1/4\alpha^2) \), and \( \alpha < 1 \). Inserting these into \( U_j \), one finds that the AF states have the lower energy by

\[
\Delta U = 2J \approx \frac{\hbar^2 I_p^2}{2e^2 E_0^2} + O((E/E_0)^2),
\]

so that the effective mutual inductance \( \hbar^2/4e^2E_0 \) is just the standard Josephson inductance of the coupling junction. For an explanation, note that flipping the signs of \( \phi_{AF}^{FM} \) yields an antiferromagnetic coupling, and reducing flux bias \( f_{a,b} \) weakens it.

FIG. 1. Schematics of a 7JJ device: two 3JJ qubits with direct Josephson coupling and reduced flux bias \( f_{a,b} \).
AF configuration with $\phi_0=0$, and with the same energy as the FM configuration. Such a state of course is nonstationary (since charge must be conserved), and adjustment of the phases will lower $U_J$, with the stationary AF state (2) realizing the global minimum. More intuitively: the nonzero $\phi_0^{\text{AF}}$ reduces the effective frustration in the individual qubits, which is maximal for $f_{ab}=0$ [cf. above (1)]; the attendant reduction in qubit energy overcomes (by a factor of 2) the increase in Josephson energy in the coupling junction itself.

Thus the direct Josephson coupling of 3JJ qubits has the same sign as their inductive coupling (the latter corresponding to the natural north–south alignment of their magnetic moments), but is not restricted by the geometric inductances. For $E_0 \to \infty$, $J$ in (3) disappears as it should, since then we have two qubits sharing a common leg without a junction. In our approximation, this is equivalent to two adjacent qubits, with only inductive coupling. In reality, the shared leg’s kinetic inductance will make a small contribution in (3); this is negligible for our samples, but it will also contribute to a finite $J(E_0 \to \infty)$.

We determined $J$ using an impedance measurement technique, applied previously to 3JJ qubits and extended to multiple qubits in Ref. 12. The qubits are placed inside a tank circuit with known self-inductance $L_T$ and quality factor $Q_T$, driven by a dc bias plus a small ac current at its resonance frequency $\omega_T$ (Fig. 2). The tank’s voltage–current phase angle $\theta$ is given by $\tan \theta = -Q_T/L_T \chi'$. Here, $\chi'$ is the qubits’ contribution to the tank susceptibility, related to the curvature of their energy bands: for qubit–tank mutual inductances $M_{\alpha T}=M_{b T}=M$, one has

$$\tan \theta = \frac{Q_T}{L_T} M^2 \frac{d^2 E_{\text{tot}}}{d(\Phi^2)} ,$$

where $\Phi^2$ denotes a symmetric change of flux bias in both qubit loops. At temperature $T=0$, $E_{\text{tot}}$ is the qubits’ ground-state energy; at finite $T$, the derivative simply becomes a Boltzmann average over the levels. The band curvature is large near anticrossings, so that $\tan \theta(f_a,f_b)$ contains important information about the level structure.

One obtains $J$ from such a plot as follows. The standard four-state Hamiltonian for two coupled qubits is

$$H = -\varepsilon_{ij} \sigma^z_{ai} \Delta_{ij} \sigma^z_{bj} - \varepsilon_{ij} \sigma^x_{ai} \sigma^x_{bj} - J \sigma^z_{ai} \sigma^z_{bj} ,$$

where $\sigma^x, \sigma^z$ are Pauli matrices, $\Delta_j$ is the tunneling amplitude, and $\varepsilon_{ij}=I_p \Phi_{0,j}$ is the energy bias $(j=a,b)$. For low $T$ and small $\Delta$, the location of the peaks in $\tan \theta$ (due to anticrossings) follows simply from the classical stability diagram, showing which flux states minimize (5) for $\Delta_{ab}=0$ as $\varepsilon_{ab}$ are varied. For instance, the $|0\rangle \longleftrightarrow |1\rangle$ transition ($|1\rangle$):

$$\sigma^z_a = -\sigma^z_b = 1,$$

occurs at $-\varepsilon_{ab}+J = -\varepsilon_a - \varepsilon_b + J \Rightarrow \varepsilon_a = J$. Therefore, the peak-to-peak distance in $\varepsilon_a$ or $\varepsilon_b$ equals $2J$.

For our samples, we first fabricate niobium (Nb) pancake coils and dc flux-bias lines on 4-in. oxidized silicon wafers, and then the qubits inside the coils by aluminum (Al) shadow evaporation on 12×12 mm² chips.

The Nb process starts with sputtering and dry etching of the 200-nm-thick coil windings with 1-μm width, 1-μm line spacing, and typically 30 turns. The patterning uses e-beam lithography and a CF₄ reactive-ion etching process. Then, a silicon oxide isolation layer and the second 300-nm-thick Nb film are deposited for the central coil electrode and the 2-μm-wide dc lines; photolithography is used for all required resist masks of these layers. Finally, 400 nm silicon oxide is deposited for protection and isolation.

The Al process uses e-beam lithography to prepare the double-layer resist mask for the qubits with a 150-nm linewidth. The two Al layers are deposited in situ by e-beam evaporation with different angles of incidence at a rate of 1.8 nm/s. The surface of the first AI film is oxidized with pure oxygen at a pressure of 10⁻² mbar. The qubits are completed after the final lift off.

Results are shown in Fig. 3(a). As explained below (5), one has $J=I_p \Phi_0 f_a f_b = I_p \Phi_0 \delta f_a \delta f_b$. One can find $I_p \Phi_0$ in two different ways, which agree to within ~20%. First, we used $I_p = I_a \sqrt{1-1/4\alpha^2}$ [cf. below (2)]. Here, $I_a$ is the critical current of a junction fabricated on the same chip and with the same area of 650×150 mm² as junctions 1/3/4/6, enclosed in a superconducting loop and measured by the conventional rf-SQUID technique (SQUID: superconducting quantum interference device); $\alpha=0.75$ by design. The second way is to fit the shape of the peaks in $\tan \theta(f_a,f_b)$ [Fig. 3(b)], using the spectrum of (5) to evaluate (4), which yields $\Delta_j$ and $I_p$ [11,12,13]. The required tank parameters were extracted from its resonance characteristic; the mutual inductances follow accurately from the tank current needed to induce a quantum of flux in the qubits; for sample 1, $L_T=136$ nH, $Q_T=664$, $\omega_T/2\pi=19.925$ MHz, and $M_{\alpha T}=M_{b T}=66.5$ pH. The $|0\rangle \leftrightarrow |1\rangle$ anticrossing does not show up in the figure because there is no net flux change, hence no contribution to the qubit susceptibility; one can also say that the level curvature is maximal in the direction perpendicular to the symmetric one stipulated in (4).

The results of the fit are summarized in Table I for two measured two-qubit samples, with different sizes of junction 0. Note how, say, $\Delta_a < \Delta_b$ for sample 1 makes the $a$-anticrossing sharper, resulting in a deeper color for the corresponding peak (vertical bands in Fig. 3). Since $E_0/E = 3.1$ and 1.5, respectively, the perturbative analysis leading to (3) does not apply quantitatively (the latter would have required large coupling junctions, which proved difficult to
make with sufficient homogeneity). Instead, a theoretical prediction is made for $\delta f(E_0/E, \alpha)$, by calculating the classical stability diagram directly from $U_J(\phi_0, \phi_1, \phi_2, \phi_3, \phi_0)$. Incidentally, this has the advantage that the critical-current density drops out of the comparison (entries for $\delta f$ in the table), which therefore shows greater accuracy than we can claim for $J$ itself.

Data taken at a higher $T$ support the effective Hamiltonian (5) for our 7JJ system beyond the ground state. Namely, for, say, $f_a \approx 0.035$, $f_b \approx 0.042$, an $|\downarrow\uparrow\rangle \leftrightarrow |\uparrow\uparrow\rangle$ anticrossing persists between excited states of sample 1. At finite $T$, it should contribute in (4), with rapidly decreasing Boltzmann weight as $f_a$ is reduced. This is precisely what is seen in Fig. 4(a); the fit in Fig. 4(b) shows detailed agreement with the theory. In both Figs. 3 and 4, the discrepancy between effective and mixing-chamber temperatures is well within the range expected due to heating through external leads etc.; we observed no significant deviations from an equilibrium distribution.

The remarkable $J \sim 1$ K significantly exceeds both the tunnel splitting and the inductive coupling (estimated to be $\sim 20$ mK). It can be flux tuned by using a standard compound junction (dc SQUID) for the coupling. Instead, one can also apply a bias current $I_b$ through junction 0 (Refs. 16 and 17). The corresponding generalization of Eq. (3) is

$$J = \frac{\hbar I_p^2}{2 e \sqrt{I_{c0}^2 - I_b^2}} + \mathcal{O}[(E/E_0)^2]. \quad (6)$$

Thus, $J$ can only be increased, albeit significantly. Hence, this mechanism does not allow, e.g., changing the coupling sign and tunable decoupling of qubits. These are desirable for most quantum algorithms; but existing proposals for flux qubits rely on, and therefore are limited by, mutual inductances.$^{18}$ A bias line will introduce some noise. For reference, we give the coupling linewidth due to low-frequency fluctuations in $I_b$ with spectral density $S_b(\omega)$:

$$\Delta I = \frac{\hbar I_p^2 S_b(0)}{4 e^2 (I_{c0}^2 - I_b^2)^3}. \quad (7)$$

To this end, we present a comparison of the measured samples.

TABLE I. Coupling-junction areas $S_a$, tunneling amplitudes $\Delta_j$, persistent currents $I_j$, peak locations $\delta f_j$, and coupling energies $J$ for the measured samples.

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>$S_a$</th>
<th>$\Delta_a$</th>
<th>$\Delta_b$</th>
<th>$I_{pa}$</th>
<th>$I_{pb}$</th>
<th>$\delta f_{exp}$</th>
<th>$\delta f_{th}$</th>
<th>$J$ (K)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.30</td>
<td>80</td>
<td>90</td>
<td>120</td>
<td>110</td>
<td>0.037-0.041</td>
<td>0.0360</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>30</td>
<td>30</td>
<td>150</td>
<td>120</td>
<td>0.068</td>
<td>0.0675</td>
<td>1.2</td>
</tr>
</tbody>
</table>
FIG. 5. Two 3JJ qubits with ferromagnetic Josephson coupling. (a) Twisted design. (b) Overlapping design.

Other variations are presented in Fig. 5. In Fig. 5(a), the relative twist between the qubit loops interchanges the role of the AF and FM configurations, so that the latter have the lowest energy. In particular, the strength of the direct FM Josephson coupling can overcome any residual AF inductive interactions to the expected accuracy. We also proposed design modifications allowing tunable and/or ferromagnetic coupling. Future experimental work should also consider linear qubit arrays, to which the design of Fig. 1 is readily generalized. 

Indeed, the remarks below (3) show the derivation to be analogous to the inductive case; see A. Maassen van den Brink, Phys. Rev. B 71, 064503 (2005).


For a nonzero leading-order \( \phi_0 = \arcsin (I_0 / \phi_0) \), the qubits are effectively at degeneracy if \( 2 \pi f_a - \phi_0 = 2 \pi f_b + \phi_0 = 0 \). See above (1) and note 8.

By considering “black box” devices in the outer arms, \( U_j = U_j(2 \pi f + f_b - \phi_0) + U_j(2 \pi f + f_a + \phi_0) - E_0 \cos \phi_0 \), one readily verifies that the latter result also holds if the two qubits are asymmetric, biased, or of a different type. Defining the proper phase-dependent Josephson inductance at the working point, also (6) for the biased case is seen to be of this form.

01230 and the other is 04560; by choosing 1:2 area ratios, both qubits can be brought close to degeneracy with a homogeneous field, for \( \Phi_a^b = \frac{1}{2} \Phi_a^b, \frac{1}{2} \Phi_b \). One obtains FM coupling without a twisted layout, but with strongly asymmetric qubits. Alternatively, the right loop in Fig. 5(b) can be kept small, and used only for small bias fluxes; this is the Josephson-inductance-based counterpart of Fig. 1(b) in Ref. 19. The two discussed modifications can be combined: by current biasing the junctions 0 in Fig. 5, one obtains tunable FM coupling.

In conclusion, we have demonstrated direct antiferromagnetic Josephson coupling between two individually controllable three-junction flux qubits. The coupling strength can be on the order of a Kelvin, and agrees with theoretical predictions to the expected accuracy. We also proposed design modifications allowing tunable and/or ferromagnetic coupling. Future experimental work should also consider linear qubit arrays, to which the design of Fig. 1 is readily generalized. 

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