Multivariable Frequency Response Functions Estimation for Industrial Robots

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Introduction
The accuracy of industrial robots limits its applicability for high demanding processes, like robotised laser welding. We are working on a nonlinear flexible model of the robot manipulator to predict these inaccuracies. This poster presents the experimental results on estimating the Multivariable Frequency Response Functions (MFRF) of the Stäubli RX90 robot depicted in figure 1. Future work will be the parametrisation of the frequency response functions based on physical models.

Closed loop robot system
For stability and safety reasons, the experiments will be carried out in closed loop; see figure 2. The robot controller will drive the robot such that the joint angles $e^{(m)}$ and velocities $\dot{e}^{(m)}$ are in agreement with the reference trajectory $r$ and $\dot{r}$. The driving torques $\tau^{(m)}$ are perturbed with feedforward torques $\tau^{(ff)}$ having a frequency spectrum above the bandwidth of the closed loop system. The outputs of the system are the joint angles $e^{(m)}$ and velocities $\dot{e}^{(m)}$. Furthermore a 3D acceleration sensor measures the accelerations of the tip in horizontal $s^{(h)}$ and vertical $s^{(v)}$ direction.

![Figure 2: Closed loop system](image)

Experiment Design
The feedforward signal $\tau^{(ff)}$ is a multi sine containing 100 frequencies in the range from 10 to 100 Hz. Crest factor optimisation of this signal has improved the signal to noise ratio of the measurements by a factor 2. The reference trajectory $\dot{r}$ for the joint velocity is a square wave. In figure 3 the resulting velocity of joint three is given. The figure shows that the square wave is effective in preventing unwanted velocity reversals, leading to complicated friction behavior.

![Figure 3: Resulting velocity joint 3](image)

MFRF estimation
Let $U(\omega_k)$ be the Discreet Fourier Transform (DFT) of the input signals consisting of the driving torques $\tau^{(m)}$ at frequency $\omega_k$ and let $Y(\omega_k)$ be the output vector consisting of the DFT of the joint accelerations $\ddot{e}^{(m)}$ and the accelerations $s^{(h,v)}$. For periodic signals the following linear mapping holds

$$Y(\omega_k) = G(\omega_k)U(\omega_k),$$

where $G$ is the MFRF. To be able to extract $G$ from data $m$ different (independent) experiments are needed, were $m$ is the number of inputs. The data vectors from different experiments can than be collected into matrices $U$ and $Y$, where each column corresponds to one experiment. An estimate of $G$ can be formed as

$$\hat{G}(\omega_k) = Y(\omega_k)U^{-1}(\omega_k),$$

provided matrix $U$ has full rank.

Results
The estimated MFRF from $\tau^{(m)}$ to $s^{(h)}$, $\dot{s}^{(v)}$ and $s^{(h)}$ are given in figure 4. The figure shows that although the excitation is only in the vertical plane (see figure 1), the resulting motion is a complicated 3D motion.

![Figure 4: Frequency Response Functions in (rad/Nms²) and (m/Nms²)](image)