ANALYSIS OF MICROMACHINED CAPACITIVE INCREMENTAL POSITION SENSOR

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Abstract — This article presents an analysis for two related concepts of a capacitive incremental position sensor. In Incremental Capacitance Measurement Mode the periodic change in capacitance is measured to determine the relative displacement between two periodic geometries S1 and S2 with gap-distance of ~ 1 µm. In Constant Capacitance Measurement Mode the distance between S1 and S2 is controlled to keep the capacitance between S1 and S2 constant. Analysis and 2D- Finite Element simulations show that SNR for CCMM can be >300x over ICMM and with a lower non-linearity in the position sensor signal, CCMM will perform better in accurate quadrature position detection.

Key Words: Capacitive position sensor, micromachining, nanopositioning

I INTRODUCTION

We investigate a capacitive incremental position sensor integrated with micromachined microactuators for nano-position control. The aim is to develop a position sensor with nanometer accuracy over a large displacement range e.g. +/-50-µm or more. Micropositioners with the capability of nanometer position accuracy over a large displacement range, have a high potential in applications such as probe-based data storage [1], scanning probe microscopy [2], biomedical analysis [3],[4], optical mirror manipulation [5], microtooling and robotics [6].

Earlier, two different concepts for a capacitive incremental position sensor have been presented and demonstrated through experiments on a micromachined microsystem depicted in Figure 1. All structural parts in Figure 1 are made by surface-micromachining of the 5µm-thick Boron-doped poly-silicon layer. The structure is released with a freeze-drying process after etching the sacrificial oxide layer underneath the poly-layer [7],[8],[9].

This article examines through analysis and 2D-Finite Element analysis the differences between the two concepts in terms of signal-to-noise ratio (SNR) and non-linearity.

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The concepts are the following (Figure 2):
In the ICMM concept the periodic change in capacitance between geometries S1 and S2 is measured to determine the relative displacement x between S1 and S2. With additional microactuators the gap y0 is made smaller only once, prior to the measurement operation.
In the CCMM concept the periodic geometry S2 is displaced in y-direction at every x-position to keep the capacitance $C(x,y)$ equal to a set point value i.e. $C(x, y(x)) = C_{set}$.

An incremental sensing principle can provide a very large dynamic measurement range. This principle is based on quadrature detection with two periodic sensor signals. These are shifted by a quarter period ($P_x/4$) to unambiguously determine the position within each period while counting the number of periods. With perfect sine or triangular sensor signals it is possible to determine a position within each quadrant using interpolation, arctan calculation or look-up tables [10].

![Figure 1. Example of a micromachined capacitive incremental position sensor.](image-url)
III. ANALYSIS

III.1 ESTIMATION SIGNAL-TO-NOISE RATIO FOR ICMM AND CCMM

In this section the ratio in SNR for ICMM and CCMM is estimated for a geometry combination given in Figure 2. For the ‘fingers’ of geometry S2 we assume probe-like dimensions. This means for ICMM that the ratio of period size \( P_x \) over the width of the fingers \( w \) is large (i.e. \( P_x / w \gg 1 \)) and for CCMM an additional assumption is that finger length \( \ell / A \) is much larger than the amplitude of geometry S1 i.e. \( \ell / A \gg 2A \) (Figure 2).

For position detection in ICMM-mode only the change in capacitance \( \Delta C \) versus position \( (x) \) is relevant:

\[
\Delta C(x) = \frac{C_0}{y_0 + A(1 - \cos kx)}
\]  
(Eq.1)

With the term \( C_0 \) in [Fm] including the structure height \( h \) and \( k = 2\pi / P_x \) in [\( \mu \text{m} \)^{-1}].

Taking the amplitude of \( \Delta C \) gives for the SNR of ICMM with noise contribution \( \delta C \):

\[
S / N = \frac{\Delta C}{\delta C} = \frac{C_0 \cdot A}{y_0 \cdot (y_0 + 2A)} \frac{1}{\delta C}
\]  
(Eq.2)

In CCMM the geometry S2 is moveable in \( y \)-direction by an additional comb-drive actuator with control voltage \( U_c \). For an electrostatic actuator the displacement \( y \) is proportional to the square of the applied voltage \( U \). Reducing the gap between S1 and S2 will increase the capacitance. The capacitance between S1 and S2 becomes:

\[
C(x, U_c) = C_p + \frac{C_0}{y_0 + A(1 - \cos kx)} - KU_c^2
\]  
(Eq.3)

For position detection the control-voltage \( U_c(C_{set}, x) \) is used. For an arbitrary position \( 0 < (x) < P_x, U_c \) is such that \( C(x, U_c) + C_p = C_{set} \).

Thus, we can write for the voltage \( U_c \):

\[
U_c^2 = \frac{1}{K} \left[ \frac{y_0 + A(1 - \cos kx)}{C_0} - \left( \frac{C_{set} - C_p}{C_0} \right)^{-1} \right]
\]  
(Eq.4)

At arbitrary position \( x = P_x/4 \), the voltage \( U_c = U_{set} \) for which the capacitance \( C(P_x/4, U_{set}) = C_{set} \). The voltage change \( \Delta U_c(x) \) is the difference between \( U_c(x) \) and \( U_{set} \). A disturbance of the capacitance with \( \delta C \) due to noise, or changes in humidity or temperature will lead to a change in voltage \( \Delta U_c \) with \( \delta u \) to compensate the change \( \delta C \).

\[
U_c^2 = (U_{set} + \Delta U_c(x) + \delta u)^2 = \frac{1}{K} \left[ \frac{y_0 + A(1 - \cos kx)}{C_0} - \left( \frac{C_{set} - C_p - \delta C}{C_0} \right)^{-1} \right]
\]  
(Eq.5)

Using

\[
U_c^2 = (\Delta U_c(x) + \delta u)^2 \ll 2 \cdot (\Delta U_c(x) + \delta u) \cdot U_{set},
\]

the expression for the SNR for CCMM becomes (SNR_{ICMM}):

\[
S / N_{CCMM} = \frac{\Delta U_c(x)}{\delta C} = \frac{[A]}{(y_0 + A - KU_{set}^2)} \frac{C_0}{\delta C}
\]  
(Eq.6)

The estimation of the improvement in SNR for CCMM in comparison with ICMM is determined by the ratio \( \text{SNR}_{ICMM} / \text{SNR}_{CCMM} \):

\[
\frac{S / N_{ICMM}}{S / N_{CCMM}} = \frac{y_0 \cdot (y_0 + 2A)}{(y_0 + A - KU_{set}^2)^2}
\]  
(Eq.7)

For amplitude \( A = 4 \, [\mu \text{m}] \), \( y_0 = 2 \, [\mu \text{m}] \), and a controlled gap between geometry S1 and S2 of \( g = y_0 + A - KU_{set}^2 = 0.25 \, [\mu \text{m}] \), gives an improvement in SNR of \( \sim 320x \) for CCMM over ICMM.

The following conclusions are drawn from this analysis:
• From the expression in (Eq.7) it can be concluded that the Signal-to-Noise Ratio (SNR) is increased when
1. The amplitude $A$ of the periodic geometry $S_1$ is large, and the term $C_0$ which includes the structure height $h$ is large
2. $KU_{set}^2 \sim y_0 + A$, i.e. gap-size is small
• The shape of geometry $S_1$ is recovered well in the control voltage $U_c(x)$ and ideally, geometry $S_1$ would be pure sine or triangular shaped [10].

III.2 EVALUATION OF NON-LINEARITY WITH 2D-FINITE ELEMENT AND FFT ANALYSIS
This section examines the conclusions of the previous section in further detail using 2D-FE simulations and Fourier analysis for both ICMM and CCMM. The non-linearity due to higher-order harmonics is determined for both simulated capacitance function $C(x)$ for ICMM and position function $y(x)$ for CCMM. The simulation method presented earlier in [8] has been further developed and automated using a Matlab script file for FEMLAB software [11]. For the geometry given in Figure 2 the resulting capacitance functions for ICMM for two gap sizes $g_1 = 0.5 \mu$m and $g_2 = 0.2 \mu$m, corresponding to a maximum capacitance of $C_1 = 200$ aF and $C_2 = 300$ aF are given in Figure 3. This is the capacitance per segment, or per period $P_x$ for a structure height of 5 µm.

The first harmonic in space-domain $f_1(x) = C_{k0}(x) = A_1 \cos(k_0 x)$. A residual function $r(x)$ is defined as the difference in space domain between the function $f(x)$ and the first harmonic $f_1(x)$, i.e. $r(x) = f(x) - f_1(x)$. In other words this is the non-linearity due to higher order harmonics. With the inverse FFT for all residual higher order frequency components of the capacitance functions $C(x)$ the amplitude of the function $r(x)$ is determined. The ratio with the amplitude of the first order harmonic $f_1(x)$ is given in Table 1.

These results are compared with simulation results for the CCMM concept. For every displacement of the rounded finger in x-direction in Figure 2 a displacement in y-direction is calculated in order to keep the calculated capacitance $C(x,y)$ equal to the setpoint $C_{set}$. If $dC_x = C(x_i, y_j) - C_{set} < 1\%$ position $y_j$ is recorded and a new position $x_{i+1}$ is taken. Apart from rounded also rectangular-shaped fingers are used for the simulation and comparison.

### Table 1. (ICMM) amplitudes and ratios for FFT components of simulated $C(x)$ for different gaps $g$

<table>
<thead>
<tr>
<th>$g$ [nm] / $C$ [aF]</th>
<th>$f_{k0}(x)$ [aF] $(k_0=0.2, \pi)$</th>
<th>$r(x_n)/f_{k0}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 / 200</td>
<td>36.9136</td>
<td>0.3619</td>
</tr>
<tr>
<td>200 / 300</td>
<td>72.1783</td>
<td>0.6240</td>
</tr>
</tbody>
</table>

### Table 2. (CCMM) amplitudes and ratios for FFT components of simulated $y(x)$ for different setpoints $C_{set}$

<table>
<thead>
<tr>
<th>$C_{set}$ [aF]</th>
<th>$f_{k0}(x)$ [µm]</th>
<th>$r(x_n)/f_{k0}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1.348</td>
<td>0.0615</td>
</tr>
<tr>
<td>250</td>
<td>1.8186</td>
<td>0.0888</td>
</tr>
<tr>
<td>300</td>
<td>2.0919</td>
<td>0.1104</td>
</tr>
<tr>
<td>250R</td>
<td>1.7143</td>
<td>0.1576</td>
</tr>
</tbody>
</table>

![Figure 3. Simulated $C(x)$ for ICMM for 2 gaps for the geometry in Figure 2, period size $P_x = 10 \mu$m](image1)

![Figure 4. Simulated $y(x)$ for different setpoints $C_{set}$. The value $y(x=0)$ is the minimum gap $g$.](image2)
In Figure 4 the displacement in y-direction of geometry S2 is given for every displacement in x-direction for which \( C(x, y(x)) = C_{\text{setpoint}} \).

The \( y = 0 \) – line corresponds to the top of the sine geometry and to the minimum gap-size between geometry S1 and S2 in Figure 2.

Table 2 shows the ratio \( r(x)/f_1(x) \) of the amplitude of residual function \( r(x_0) \) of the higher order frequency components in the simulated \( y(x) \) function and the first order harmonic for the geometry with rounded finger and sine pattern. The last row in Table 2 shows the result for a setpoint capacitance \( C_{\text{set}} = 250 \text{ aF/period} \) and a rectangular instead of a rounded finger shape.

For a setpoint capacitance of \( C_{\text{set}} = 300 \text{ aF/period} \) and \( r(x_0) \) is about \( 11\% \) of the first order harmonic \( f_1(x_0) \). This is for ICMM given in Table 1 about \( 62\% \). In other words, the non-linearity for CCMM is less than for ICMM. This is also predicted by the analysis in section III.1. Therefore, the ideal geometry combination that produces a pure sine-function \( C(x) \) for the ICMM concept will not be the same ideal geometry combination that produces a sine-function \( y(x) \) for the CCMM concept. However, for an increase in setpoint capacitance \( C_{\text{set}} \) for CCMM also the ratio between higher order frequency components and first order harmonic increases. This is very probably because the ratio between period size and rounded fingers is not exactly a situation with probe-like dimensions i.e. \( P_x/w \) is too small. Taking rounded fingers with a finger length \( L_f > 2A \) (Figure 2) gives a better result than rectangular fingers.

The 2D-FE simulation results sustain the conclusion of the SNR analysis in section III.1 in terms of expected shape of \( y(x) \) for the geometry combination of a sine with a finger-like pattern with probe-like dimensions. The predictions for CCMM in terms of SNR improvement have not been examined. However, for all calculated positions for CCMM the capacitance can be \( C(x, y(x)) = C_{\text{set}} = 300 \text{ aF} \) while for ICMM the capacitance \( C(x) = 300 \text{ aF} \) only around \( x = 0 \) and \( C(x) = 125 \text{ aF} \) around \( x = P_x/2 \) for the given geometry and ratio of feature sizes. Therefore, in terms of signal amplitude of the capacitance measurement, the SNR can be expected to be higher for CCMM than for ICMM at given noise-levels.

**IV CONCLUSIONS**

This article examines the differences between two related concepts ICMM and CCMM for a capacitive incremental position sensor, through analysis and 2D-FE simulations. The change in capacitance between two periodic geometries S1 and S2 is measured to determine the relative displacement between S1 and S2.

The SNR for CCMM can be \( >300x \) larger than for ICMM. If geometry S1 is pure sine and geometry S2 has rounded fingers with probe-like dimensions, the position sensing signal for CCMM will approximate a pure sine better than for ICMM. A pure sine position sensing signal is desired for quadrature position detection with nm-accuracy over infinite motion range.

**REFERENCES**