We present a hybrid analytic-numeric method to calculate the transmission and reflection of light that is flus into a bounded complicated optical structure surrounded by air. The solution is obtained by numerical calculations inside a square containing the structure and by analytical calculations outside the square. For solving the 2D Helmholtz equation we formulate Transparent-Influx Boundary Conditions (TIBC's) on the boundaries of the square; these are incorporated into a variational formulation of the Helmholtz equation to obtain a FEM-implementation for the interior calculations.

**Modelling problem and general approach**

On the plane, when restricting to TE-polarization, the propagation of time-harmonic light is described by the scalar Helmholtz Equation (HE) for the principal component $E$ of the electric field perpendicular to the plane, $\Delta E + k^2 E = 0$, where $(x, y) = \omega(n(x, y))$, with $\omega$, the free-space wavenumber, $n(x, y)$ is the refractive index of the media characterizing the geometry of the device, and $k = \frac{2\pi}{\lambda}$ is the vacuum wavelength.

Our approach for solving the Helmholtz problem is as follows:

- Formulate Transparent-Influx Boundary Conditions (TIBC's) on the boundaries of the square; these are incorporated into a variational formulation of the Helmholtz equation to obtain a FEM-implementation for the interior calculations.

**Numerical implementation**

We formulate a discretization of the TIBC's as follows:

- **An incoming field is assumed from the left** $E^i = f$ and $\partial_x E^i = 0$ at $\Omega^i$.
- **Interface conditions** $E_0 = E^i$ and $\partial_x E_0 = E^e$ at $\partial \Omega$.
- **Solution in the exterior** $E^e = E^i + E^m$.

**Transparent-influx boundary conditions (TIBC's)**

- **Dirichlet-to-Neumann operators** (DN): given a Dirichlet data $g$ at $\partial \Omega$,
  
  $D^i(g) = \partial_z E_0|_{\partial \Omega}$, $E$ outgoing solution (HE) with $E_{\Omega^i} = g$
  
  $D^e(g) = \partial_z E_0|_{\partial \Omega}$, $E$ incoming solution (HE) with $E_{\partial \Omega} = g$

  **Analytical expressions for the DN operators can be obtained via plane wave decomposition**

- **For the square with influx through the western side, we find as boundary conditions:**
  
  $\partial_z E_0 - D^e(E_0)|_{\partial \Omega} = D^e(f) - D^e(f)|_{\partial \Omega}$ with $f = E^i|_{\partial \Omega}$

**Numerical results**

**Interior domain**

Parameters: computational window of $[0, 10] \mu \text{m} \times [0, 10] \mu \text{m}$, $\lambda = 1 \mu \text{m}$.

- **Error analysis:** propagation of a Gaussian beam in free space

**Exterior: hybrid analytic-numeric method**

The Hybrid Window (HW): Interior domain $\Omega$ (FEM solution) + a region on which the solution is extended in the exterior.

**Conclusions**

- We have developed well posed boundary conditions (TIBC's) for 2D Helmholtz problems in optics.
- The boundary conditions depend only on the behavior of the solution in the exterior domain.
- Implementation of those boundary conditions can be done in any numerical method.
- The boundary conditions are formulated here for harmonic problems (TE-modes). Extension to TM-case is immediate.

**References**