Scale resolved intermittency in turbulence

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(Received 15 April 1993; accepted 19 October 1993)

The deviations \( \delta \xi_m \) ("intermittency corrections") from classical ("K41") scaling \( \xi_m = m/3 \) of the \( m \)th moments \( \langle |u(p)|^m \rangle \) in high Reynolds number turbulence are calculated, extending a method to approximately solve the Navier–Stokes equation described earlier. It is suggested to introduce the notion of scale resolved intermittency corrections \( \delta \xi_m(p) \), because these \( \delta \xi_m(p) \) are found to be large in the viscous subrange, moderate in the nonuniversal stirring subrange but, surprisingly, extremely small if not zero in the inertial subrange. If ISR intermittency corrections persisted in experiment up to the large Reynolds number limit, it would show by calculation that this could be due to the opening of phase space for larger wave vectors. In the higher order velocity moments \( \langle |u(p)|^m \rangle \) the crossover between inertial and viscous subrange is \( (10q_m/2)^{-1} \), thus the inertial subrange is smaller for higher order moments.

I. INTRODUCTION

Experimentally turbulent flow has long been known to be intermittent. A signal is called intermittent, if there are relatively calm periods which are irregularly interrupted by strong turbulent bursts either in time or in space. Correspondingly, the probability density function (PDF) develops tails of large fluctuations and a center peak due to the abundance of calm periods, i.e., the PDF becomes of stretched exponential type instead of being Gaussian. This also means that the \( p \)-scaling exponents \( \xi_m \) of the velocity Fourier components \( u(p) \),

\[
\langle |u(p)|^m \rangle \propto p^{-\delta \xi_m},
\]

do not vary linearly with \( m \), namely as \( m/3 \), as was originally suggested by dimensional analysis of the universal, inertial subrange of fully developed turbulent flow. Any deviations \( \delta \xi_m - \xi_m = m/3 \) are called intermittency corrections. Note, that we use discrete Fourier representation throughout.

Phenomenological intermittency models describe the measured intermittency corrections \( \delta \xi_m \) more or less successfully. For a detailed discussion see, e.g., Refs. 4 and 5. But from our point of view an understanding of intermittency has to come from the Navier–Stokes equation.

As full simulations for high Reynolds number (\( Re \approx 10^5 \)) turbulence are out of range even for near future computers, one is thrown on approximations of the Navier–Stokes dynamics.

The main idea of such an approximation has been introduced by us in Refs. 5 and 6. Meanwhile we have considerably improved our approach and in this paper we employ it to determine the intermittency corrections \( \delta \xi_m \). For completeness, we briefly repeat how our approximation scheme works.

II. REDUCED WAVE-VECTOR SET APPROXIMATION

It starts from the common Fourier series in terms of plane waves \( \exp(ip\cdot x), p = (p_i), p_i = n_iL^{-1}, n_i = 0, \pm 1, \pm 2, \ldots \). The periodicity volume is \( (2\pi L)^3 \), \( L \) is the outer length scale. To deal feasibly with the many scales present in turbulent flow, we only admit a geometrically scaling subset \( K \) of wave vectors in the Fourier sum, \( K = \cup_{i=1}^N K_i \), thus \( u_i(x,t) = \sum_{n \in K_i} u_n(p) \exp(ip\cdot x) \). Therefore, we have called our approximation scheme "Fourier–Weierstrass decomposition." For \( K_0 = \{ n \}, n = 1, \ldots, N \) contains appropriately chosen wave vectors, which already have quite different lengths but dynamically interact to a good degree. The \( K_l = \{ p_{n}^l \} = 2^lK_0, n = 1, \ldots, N \), \( l = 1, \ldots, l_{\text{max}} \), are scaled replica of \( K_0 \) which represent smaller and smaller eddies. Here, \( l_{\text{max}} \) is chosen large enough to guarantee that the amplitudes \( u(p_{n}^l, t) \) of the smallest eddies are practically zero. Of course, \( l_{\text{max}} \) depends on the viscosity \( \nu \) and thus on \( Re \).

We solve the Navier–Stokes equation for incompressible flow [i.e., \( \mathbf{p} \cdot u = 0 \)] in the subspace defined by the wave-vector set \( K \),

\[
\dot{u}_i(p) = -iM_{ijk}(p)\sum_{q_1,q_2} u_j(q_1)u_k(q_2) - \nu^2 u_i(p) + f_i(p).
\]

The set \( K_0 \) is chosen in a way that as many triadic Navier–Stokes interactions \( p = q_1 + q_2 \) as possible are admitted. The degree of the nonlocality in \( p \) space of any triadic interaction can be characterized by the quantity \( s = \max(p_{n_1}, q_1) / \min(p_{n_1}, q_1) \). We allow for \( s \) up to 5.74. To force the flow permanently, we choose

\[
f(p,t) = \frac{\mathbf{e} \cdot u(p,t)}{\sum_{q \in K_i} |u(q,t)|^2} \quad \text{for } p \in K_{in},
\]

\[
f(p,t) = 0 \quad \text{for } p \notin K_{in},
\]
accounted for by a competition effect between turbulent energy transfer downscale and viscous dissipation. The main features of fully developed turbulent eddy size. We used $N=26$, $I_{\text{max}}=10$ and $N=80$, $I_{\text{max}}=12$, respectively. The main features of fully developed turbulence as chaotic signals, scaling, turbulent diffusion, etc., are well described within our approximation. In Refs. 6 and 7 we achieved $Re=2 \times 10^6$ and many more scales than in full simulations can be taken into account. In Fig. 1 we offer our results.

The spectra $\langle |u(p)|^2 \rangle$ and $\langle |u(p)|^6 \rangle$ calculated with $N=86$ wave vectors in $K_0$ and with $Re=125 000$ are shown in Fig. 2(a). For comparison the same spectra are also given for $N=38$ as used in our previous work [Fig. 2(b)]. As expected the scatter becomes less with increasing $N$. We fit the spectra with the three parameter functions $\langle |u(p)|^m \rangle = \epsilon_m p^{-\delta_m} \exp(-p/p_{D,m})$. (4)

In Table II the fit parameters $\delta_m$ and $p_{D,m}$ are listed. The ansatz (4) is theoretically known to hold for $m=2$. We find that it also holds for $m>2$ with $p_{D,m} = 2 p_{D,2}/m$ as one can expect, if in the VSR the higher order moments factorize. For the dissipative cutoff $p_D:=p_{D,2}$ we obtain $p_D = (11 \eta)^{-1}$, where $\eta = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov length.

\begin{table}[h]
\centering
\begin{tabular}{ccc}
\hline
$N$ & Number of interacting triads & $s_{\text{max}}$ & $b$
\hline
26 (Ref. 6) & 39 & 1.92 & 300
38 (Refs. 5 and 6) & 102 & 1.92 & 170
50 & 273 & 3.46 & 80
74 & 741 & 5.00 & 70
74 & 129 & 5.74 & 70
74 & 858 & 3.46 & 70
80 & 783 & 5.74 & 70
86 & 966 & 5.74 & 65
\hline
\end{tabular}
\caption{Characteristic parameters of several different wave sets $K_0$. $N$ denotes the number of wave vectors in $K_0$. In the second column the number of triadic interactions $p=q_1+q_2$ between the wave vectors of any one set $K_i$ is given. $l$ not too large or small to avoid edge effects. $s_{\text{max}}$ is the maximal nonlocality (definition see text) of the triadic interactions. $b$ is the dimensionless constant in the structure function $D(r) = \langle |u(x+r)-u(x)|^2 \rangle = b(r^{2/3})$, re ISR. The experimental value $b=8.4$. The larger values in our approximation can well be understood, the decrease of $b$ with the increase of $N$ is in keeping with that explanation.}
\end{table}
length. This well agrees with the long-known (experimentally15 and theoretically11,12) crossover between the viscous subrange VSR and the inertial subrange ISR in the structure function $D_{c2}(r) = \langle |u(x+r) - u(x)|^2 \rangle$ at $r$ about $10\eta$. According to our finding $p\delta_{c,m}/2m$ is the dissipative cutoff, which agrees very well with $2p_D/m$, as shown in the last row.

| $m$ | $\langle |u|^2 \rangle$ | $\langle |u|^4 \rangle$ | $\langle |u|^6 \rangle$ | $\langle |u|^8 \rangle$ | $\langle |u|^{10} \rangle$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1   | 0.682           | 1.021           | 1.359           | 2.034           | 2.707           |
| 2   | 0.668           | 1.000           | 1.331           | 1.993           | 2.651           |
| 3   | 0.602           | 0.990           | 1.301           | 1.889           | 2.550           |
| 4   | 0.522           | 0.970           | 1.261           | 1.823           | 2.500           |
| 5   | 0.460           | 0.950           | 1.221           | 1.773           | 2.450           |
| 6   | 0.408           | 0.930           | 1.181           | 1.723           | 2.400           |
| 7   | 0.364           | 0.910           | 1.141           | 1.673           | 2.350           |
| 8   | 0.324           | 0.890           | 1.101           | 1.623           | 2.300           |
| 9   | 0.288           | 0.870           | 1.061           | 1.573           | 2.250           |
| 10  | 0.256           | 0.850           | 1.021           | 1.523           | 2.200           |

FIG. 2. Spectra $\langle |u(p)|^{m} \rangle$ for $m=2$ (△) and $m=6$ (+). (a) $N=86$ wave vectors in $K_0$, $v=8\times10^{-6}$, $\epsilon_{max}=5.74$, $R_D=9030$, averaging time from 100 to 180. The input set $K_0$ of the forcing (3) consists of the 12 shortest wave vectors. The line are the fits (4). (b) $N=38$, $v=8\times10^{-6}$, averaging time 150 large eddy turnovers, same forcing as in (a).

TABLE II. Results from the fit (4) to the spectra $\langle |u(p)|^{m} \rangle$ obtained with $N=86$ wave vectors in $K_0$ for moments up to $m=10$, $v=8\times10^{-6}$. The average is over 80 large eddy turnover times (skipping the first 100 turnovers). We fitted the $p$ range $[0,10000]$. In general, $\xi_3$ is rather near, but not exactly equal to 1 as it should be according to Kolmogorov’s structure equation. We therefore calculate the intermittency corrections from renormalized exponents $\xi_m/\xi_3$, namely, $\delta_m = \xi_m/\xi_3 - m/3$. For comparison, the values for Kolmogorov’s log normal model, $\delta_m = -\frac{\mu(m-3)}{18}$, are also given, which are well known to fit the data for $m<10$ with $\mu=\nu=0.20$. $\delta_m(ran-\beta) = -\log[1-x + x(1/2)^{-m/3}]$ are the intermittency corrections due to the random $\beta$ model ($x=0.125$) (Ref. 38). $\rho_D(m)$ is the dissipative cutoff, which agrees very well with $2p_D/m$, as shown in the last row.

FIG. 3. $\delta_m(p)$ for $m=2, 4, 6, 8, 10$, bottom to top. Same data as in Fig. 2(a). The shaded ranges on the right show the Kolmogorov values $\delta_m = -\frac{\mu(m-3)}{18}$ for $\mu=0.20$ through $\mu=0.30$, because in Ref. 39 $\mu=0.25\pm0.05$ is given as “best estimate.” In (a) the fit range is $[p/\sqrt{10},p/10]$, in (b) the larger local range $[p/\sqrt{20},p/20]$ is chosen.

The intermittency corrections $\delta_m(p)$ from our overall fit (4) are much smaller than the experimental ones around. At the other hand we observe here, as in Ref. 5, that there is much intermittency in the signals, at least for small scales. We therefore determined the exponents $\xi_m$ in (4) by fitting restricted $p$ ranges only. We suggest to intro-duce a local $\xi_m(p)$. These are defined by local fits of the type (4), using for each wave vector $p$ the moments in the local $p$ decades [$p/\sqrt{10},p/10$]. The cutoff wave vectors are kept fixed at their global values $p_D=(11\eta)^{-1}, p_D(m)=2p_D/m$. Also, as before (see caption of Table II) we devide the local $\xi_m(p)$ by $\xi_3(p)$. As already mentioned $\delta_m(p)$ is also about $\xi_3(p)-m/3$. One could also also take the deviation $\delta_m(p)$ of the $\xi_m(p)$ from the linear behavior as a measure of intermittency. It holds $\delta_m(p) = -m\xi_3(p)/3 = \xi_3(p)\delta_m(p)$ in the ISR it is $\xi_3(p) \approx 1$, so at smaller $p$, namely, approximately at $(10\eta/2)^{-1}$. The ISR for higher order moments is thus definitely smaller. This does not necessarily mean that the ISR for higher order structure functions $U^n(r)$ is also smaller, because they are not simply connected with $\langle |u(p)|^{m} \rangle$ via a Fourier transform as in the case $m=2$. In passing by we re-mark that by properly renorming the wave vector $p$ and the spectral intensity, the spectra (4) can be shown to be universal for all Reynolds numbers both in experiment and in full simulations and in our approximate Navier-Stokes solution.17
both definitions for the intermittency corrections essentially agree, but in the VSR we find \( \zeta_3(p) > 1 \), so \( \delta \zeta_m(p) > \delta \zeta_m(p) \).

The astonishing results are shown in Fig. 3(a). There are large intermittency corrections \( \delta \zeta_m(p) \) for the small scales (large \( p \), VSR), only moderate intermittency corrections for the large scales (small \( p \), stirring subrange SSR), but hardly any deviations for \( p \) in the ISR.

The small-scale intermittency is well understood\(^{15,16}\) and was extensively discussed in Ref. 5. It is best seen in small-scale quantities as, for example, in the energy dissipation rate \( \epsilon(x,t) \) or in the vorticity. Here we observe in addition that the intermittency corrections \( \delta \zeta_m(p) \) in the VSR remarkably well agree with the \( r \)-scaling exponents \( \mu(m/3) \), defined by \( \langle \epsilon_r^{m/3} \rangle \propto r^{-\mu(m/3)} \), which we had already calculated in Ref. 5. Here \( \epsilon_r \) is defined as time average \( \epsilon_r(t) = \langle (1/r) \int_{t-r}^{t} \epsilon(t') dt' \rangle \), as in the analysis of experimental data.\(^4\) To connect this observation with Kolmogorov’s refined similarity hypothesis (RSH) \( \nu_r \propto (\epsilon_r)^{1/3} \), where \( \epsilon_r \) is defined in correspondence to \( \epsilon_r \) and \( \nu_r(\infty) = u(x+r) - u(x) \), two assumptions have to be made: (1) Taylor’s hypothesis, connecting time and space differences via \( \tau = r/U \), where \( U \) is the mean velocity; (2) \( p \)- and \( r \)-scaling exponents are the same, i.e., \( \langle |u(p)|^m \rangle \propto p^{-c_m} \) is supposed to imply \( \nu_r \propto r^{c_m} \). With these two assumptions our observation means that the RSH is fulfilled in the VSR, but, on the other hand, not in the ISR. This also is in agreement with Kraichnan’s,\(^{17}\) Frisch’s,\(^{18}\) and our\(^{19}\) objections against the RSH in the ISR, arguing that for \( r \) in the ISR \( \nu_r \) is an ISR quantity, whereas \( \epsilon_r \) still mainly is a VSR quantity. Thus a relation like the RSH should only be expected, if \( r \) is in the VSR and both \( \nu_r \) and \( \epsilon_r \) are VSR quantities.

Our result is also consistent with latest full numerical simulations,\(^{20}\) which find the RSH fulfilled. Note that in these simulations \( r \) is always in or at least near the VSR since \( Re \) is still small. And last not least our finding also agrees with the observation of Chen et al.\(^{21}\) that the RSH is less and less fulfilled the larger \( r \) becomes. For further comparison with experiment, see below.

Before we interpret the behavior of \( \delta \zeta_m(p) \) in the stirring subrange SSR and in the ISR, we checked how our findings depend on various changes of our Navier–Stokes approximation: (i) To be sure that the SSR-intermittency does not depend on the kind of forcing (3), we compared with the alternative forcing \( f(p) \propto u(p) \), again \( p \in K_n \). We also took a random forcing, but the results did not change noticeably. (ii) We varied the set \( K_n \) and allowed for more or fewer modes which are stirred, but again there was no change. (iii) We varied the type of wave vectors in \( K_0 \) and their number \( N \) as well as the maximal nonlocality \( s_{\text{max}} \) of the contributing triadic interactions (see Table 1). Again, no sizeable change. In particular, the intermittency corrections did not increase with increasing nonlocality of the triadic interactions as we speculated in Ref. 5. (iv) Different values of \( \delta \zeta_m(p) \) were only obtained when the flow field was not yet statistically stationary, see Fig. 4. In Fig. 4(a) we averaged over seven large eddy turnover times only. The total rate of dissipated energy \( \epsilon_{\text{diss}} \) increases rather slowly, while \( \delta \zeta_m(p) \) varies from run to run we refer to Fig. 5. The deviations \( \delta \zeta_m(p) \) for \( p \) in the ISR is very small, but still seem to be significant. (vi) We decreased the degree of locality of the \( \zeta_m(p) \) by fitting the larger range \([p/\sqrt{20},p/\sqrt{20}] \), see Fig. 3(b). Again no qualitative change; \( \delta \zeta_m(p) \) now tends to become even smaller in the ISR. (vii) We artificially extended the ISR by putting \( \nu = 0 \) and extracting the energy from the smallest eddies by using a phenomenological eddy viscosity as employed in Ref. 6. Now, as expected, \( \delta \zeta_m(p) \approx 0 \) also for the large \( p \), i.e., the small-scale intermittency really originates from the competition between transport downscales and the viscous damping. (viii) One might speculate that intermittency corrections in the ISR would show up if our Fourier–Weicrstrass ansatz would not only be wave number but also space resolving, as in Ref. 23. But when doing this we found that the intermittency corrections observed in Ref. 23 vanish if the number \( N \) of wave vectors in \( K_0 \) is increased.\(^5\) One should note that we include in fact some degree of position space localization since any Fourier rep-
different runs. For two runs the averaging time is about 30 large eddy

**Fig. 5.** (a) $\delta F_2(p)$ and (b) $\delta F_3(p)$ for $N=80$, $v=5 \times 10^{-4}$, $s_- = 5.74$ for
different runs. For two runs the averaging time is about 30 large eddy
turnovers, for another two runs it is about 70. The (weighted) means and
representation with many modes already allows for localization
in space.

To have another check, we also calculated the scale dependent velocity flatness $F(p) = \langle |u(p)|^4 \rangle / \langle |u(p)|^2 \rangle^2 \propto p^{-\xi_2 - 2\xi_4}$. If there is intermittency, then $2\xi_2 > \xi_4$, thus $F(p)$ has to increase with $p$. In fact we find such an increase of $F(p)$ in the SSR from $F(p=3) \approx 2.7$ (definitely $< 3$, a result achieved also in various full numerical simulations and experiments, see, e.g., 14 for a recent reference) to the value $F(p) \approx 3.0$ valid for a Gaussian distribution. For $p$ in the ISR $F(p=3)$ stays constant. Approaching the VSR by further increased $p$, the flatness now strongly grows.3 This can be understood as being due to the small-scale intermittency, as we extensively reported in Ref. 5. This behavior of $F(p)$ agrees well with the above described findings for $\delta F_2(p)$.

The same is true for the scale resolved Siggia–Kerr invariants $F_j$, $j=1, 2, 3, 4$, introduced in Ref. 24 and numerically calculated in Ref. 25. They are normalized fourth-order moments of the velocity derivatives, defined as follows: $s_{ij} = (\partial_j \mu_i + \partial_i \mu_j)/2$ is the strain, $\omega = \text{curl} u$ the vorticity. Define $F_1 = \langle \omega^2 \rangle / \langle \omega \rangle^2$, $F_2 = 3 \langle \omega^2 \rangle / \langle \omega \rangle^2$, $F_3 = 3 \langle \omega \delta_j \delta_k \delta_l \delta_m \rangle / \langle \omega \rangle^4$, and $F_4 = 3 \langle \omega \delta_j \rangle^2 / \langle \omega \rangle^2$. We calculated all $F_j$ for each scale, $F_j(p)$. Gaussian behavior means $F_1 = F_2 = F_4 = 3$ and $F_3 = 1$, see Ref. 25. We find that the $F_j(p)$, $j=1, 2, 4$, essentially behave as $F(p)$, whereas $F_3(p) \approx 0.8-0.9$, clearly smaller than 1 for $p$ in the ISR, and $F_3(p) \approx 1$ in the

Finally, we report how the flatness of the velocity derivative $F_{1,1} = \langle (\partial_j \mu_i) \rangle / \langle (\partial_j \mu_i) \rangle^2$ behaves as a function of $Re$. For the large Reynolds numbers which we consider we find $F_{1,1} \approx 3.15$, independent of $Re$. This again means we find no intermittency whereas models which are constructed to describe intermittency obtain an increase of $F_{1,1}$ with the $Re$ number in terms of the ISR intermittency exponent $\mu(2)$. $F_{1,1} \propto Re^{\mu(2)/4}$, see, e.g. 19,27 Our non-scale-resolved Siggia–Kerr invariants $F_{1,2,4}$ are in the range of $F_{1,1}$, whereas $F_3 \approx 1.01$.

Two conclusions of our findings are possible.

First, the very small if not missing intermittency might be due to our wave-number restriction. Even in the present, considerably improved ansatz the larger wave vectors are still too sparse (cf. Fig. 1). If this is indeed responsible for the nearly missing ISR intermittency, this ISR intermittency would have been identified by our calculation as an effect of the opening of the phase space for larger wave vectors. Consequently, there should be no intermittency in 2-D turbulence, where the energy cascade is inverse—and in fact, Smith and Yakhot28 do not find intermittency in numerical 2-D turbulence.

The second possible conclusion is that there indeed might be no intermittency in the pure ISR in the limit of large $Re$ without physical boundaries (remember, we have periodic b.c. and a stirring by volume forces, though nonstochastic). Of course, if so that must be due to the particular form of the nonlinearity, namely the $u \cdot \text{grad} u$ term in the Navier–Stokes equation. It provides energy transport both downscale and upscale which, as our solutions show, fluctuates wildly and with large amplitudes around a rather small mean value of downscale transport. This nearly symmetric down- and upscale transport is perturbed on the large scales (i.e., in the SSR) due to the finite size of the system, i.e., we have broken energy transport symmetry in the SSR. The largest eddies do not get energy by turbulent transfer downscale but only deliver turbulent energy to smaller scales. The symmetry of transport is also broken for small scales by the competition with the viscous dissipation. May be that the symmetry breaking of the energy transport causes the large- and the small-scale intermittency. Note that Galileian invariance is only broken by the boundaries, i.e., by the finite size of the system. Both dissipation and our forcing scheme keep it.

This second possible conclusion is in agreement with a recent theory developed by Castaing et al.29 This theory predicts that $\delta \omega_m = 0$ in the limit of large $Re$, and that in this limit $F_{1,1}$ is independent of $Re$, which we also find, as mentioned above. The value of the $F_{1,1}$ limit, if it exists, is probably larger than what we find. Vincent and Meneguzzi30 calculated $F_{1,1} = 5.9$ already for a Taylor–Reynolds number $Re_2 \approx 100$. Castaing et al.29 find from their data analysis, that the flatness $F(p)$ increases as $\log F(p) \propto (\eta p)^\beta$ with $\beta \approx 1/\log(Re_2/75)$. Our finding $F(p) \approx 3$ in the ISR (for $Re_2 \approx 10^2$, $Re_2 \approx 9000$) agrees with the large $Re$ limit of this expression. But note that in ex-
experiment it is still $\beta \approx 0.24$ even for $Re_a = 2720$ (Ref. 29). Also the measured intermittency corrections $\delta_{v_m}$ at least for $m > 6$ are not 0 even for $Re_1 = 2720$ (Refs. 29 and 31). However it could well be that in experiment the intermittency corrections might be overestimated, because they will tend to increase if the averaging time is not large enough and the flow is not yet statistically stationary (see Fig. 4 and the remarks in Ref. 22).

Our finding, that there might be three ranges for high $Re$ turbulence—namely, the SSR with moderate intermittency, the ISR with practically no intermittency, and the VSR with strong intermittency—might also be supported by some experimental data around. In Fig. 6 the spectrum $\langle |u(p)|^2 \rangle$ is shown, taken from Gagne's wind tunnel measurements\textsuperscript{32} with the very high Reynolds number $Re_3 = 2720$. While in the ISR $\xi_2 = 0.67$, i.e., $\delta_{v_2} = 0$, seems to be a good fit, for the large scales (SSR) the exponent $\xi_2 = 0.70$ is more appropriate. In the VSR the exponential damping according to (4) is not separated, so that $\xi_2$ cannot reliably be identified in that range. But note, that in the same experiment higher moments and the scale resolved flatness $F(p)$, which is more sensitive to intermittency corrections, show intermittency also in the ISR.\textsuperscript{29,32}

A similar interpretation seems possible by inspection of Praskovsky's\textsuperscript{33} data for $\langle |v_r|^6 \rangle$ measured at the also very high $Re_1 = 3200$, see Fig. 7. In the middle of the ISR we clearly have $\delta_{v_6}(p) = 0$, whereas in the VSR it takes the value $\delta_{v_6} = 0.31$. This is precisely what one expects from the RSH, namely $\delta_{v_6} \approx \mu \approx 0.30$. In the SSR the intermittency correction is $\delta_{v_6}(p) = 0.27$, which is considerably larger than what we found in this range. May be this is due to the plumes, swirls, or other structures\textsuperscript{34} which detach from the boundary in real flow and might increase the intermittency in the SSR.

Finally, we remark that also the quasi-Lagrangian perturbation analysis of the Navier-Stokes equation, performed by Belinicher and vonWeizsäcker,\textsuperscript{35} leads to $\delta_{v_m}(p) = 0$ for $p$ in the ISR in the large $Re$ limit.

To summarize, it cannot yet be ultimately decided which of the discussed conclusions of our numerical data will turn out to be robust. Either there in fact is ISR intermittency also for $Re \rightarrow \infty$ as an effect of phase space opening for large wave vectors (which we miss by construction of our approximation scheme), or there is indeed no ISR intermittency in the limit of large $Re$.\textsuperscript{29,35} To decide this alternative, it would be very helpful to at least allow some opening of phase space, e.g., to increase the number of wave vectors per level as log $k$ as already done in a 2-D approximate solution of the Navier–Stokes equation.\textsuperscript{36} If then intermittency does not show up again, we clearly have to favor the conclusion that there is no ISR intermittency in the large $Re$ limit as our results demonstrate.

ACKNOWLEDGMENTS

We heartily thank Bernard Castaing for very enlightening discussions and the referees for helpful comments. D. L. thanks the Aspen Center of Physics for its hospitality. We also heartily thank Itamar Procaccia and Reuven Zeitak for their hospitality during our stays at the Weizmann-Institute, Rehovot, Israel. Partial support by the German–Israel-Foundation (GIF) is gratefully acknowledged. The HLRZ Jülich supplied us with computer time.


