V-I Curves of a 100-kVA Class High-\(T_c\) Resonator Coil

Oleg A. Shevchenko, Jan Jaap Rabbers, Arno Godeke, Bennie ten Haken and Herman H. J. ten Kate

Abstract—Potentially coils made with BSCCO-2223/Ag tape and operating in liquid nitrogen at 50-60 Hz are interesting for power related applications. The electrical characteristics of such a coil depend on the radial magnetic field in the windings. One way to suppress the radial field is by placing small iron pieces around coil edges. V-I curves of the 100-kVA coil are calculated and measured for two cases: coil edges without and with iron pieces. The coil is a solenoid with inner and outer diameters and a height of 470, 474 and 614 mm respectively. A numerical model adapted to the coil geometry is used to calculate the conductor losses. The loss voltage is also evaluated by measuring the coil temperature. The total loss of the coil is measured electromagnetically. Calculated and measured losses are in agreement. Finally, the loss V-I curves of the coil are simulated in the frequency range of 0 to 400 Hz. The results are applied to the design of a 1-MVA resonator coil.

Index Terms—High temperature superconductors, AC losses, voltage- current curves, BSCCO-2223 tapes and coils.

I. INTRODUCTION

High-\(T_c\) superconducting coils operating in liquid nitrogen may be used in resonant circuits as shown in Fig. 1 to generate high voltage at 50 - 60 Hz with a relatively small power supply [1]. The development of high-\(T_c\) coils is still empirical. In order to further reduce the empirical part, a large coil is characterized by means of a numerical model using short sample data and magnetic field profiles. This paper focuses on the calculated loss V-I curves of a 100-kVA coil class validated by the experiment.

II. BS\(CO\) COIL CONDUCTOR

The present long lengths, rolled multi-filamentary tapes are DC conductors. A good AC conductor with many fine and decoupled superconducting filaments has still to be developed. The specification of the tape used for the coil is listed in Table I. The bare tape has 55 non-twisted filament embedded in silver matrix (core) and outer Ag-sheath; the tape length in the coil is 880 m.

![Diagram of a high-voltage resonant circuit](image)

**Fig. 1** High-voltage resonant circuit: inductor \(L = 24\) mH, capacitor \(C = 440\) \(\mu\)F, resistor \(R = 6\) m\(\Omega\), power supply \(V = 10\) V, transport current \(I_T = 100\) A.

A. Basic Equations of the Numerical Model

Consider a long tape with a given transport current \(I_T\) in a uniform transverse applied magnetic field \(B_r\). At any moment of time, far from the tape ends, the distribution of the current is the same for any electric cross-section \(C\) along the length. A line current \(dI\) within fraction \(dS\) of the tape electric cross-section \(C\) obeys Kirchhoff laws [1]:

\[
\oint_C M dI = E_a + E_r - E_\rho ; \int_C dl = I_T .
\]

with \(M\) the position-dependent inductance of the line current per unit length, \(dI = \partial / \partial t(\partial t dI) = f dS\) and \(t\) - time. Equally:

\[
J = \nabla \times \mu^{-1} \nabla \times (E_\rho - E_a - E_\tau) \quad \text{with} \quad \mu \text{ permeability.}
\]

The source electric fields \(E_a\) and \(E_\rho\) are due to the magnetic field change \((\nabla \times E_a = -B_r)\) and the transport current source, respectively; \(E_\tau = \rho \cdot J\) is Ohm’s law with the specific resistivity \(\rho\) defined by the direct \(E-J\) characteristic.

The static longitudinal specific resistivity of a tape with non-twisted filaments is defined as follows:

\[
\rho^{-1} = \rho_s^{-1} + (1 - \gamma) \rho_m^{-1}, \quad \rho_s = \frac{E_\rho}{J_s} \left( \frac{J}{J_s} \right)^{n-1}
\]

where \(\gamma\) is the superconductor - to - total filling factor, \(\rho_s\) the specific resistivity of a superconductor defined by the power law; \(J_s, N\) the critical current density, and the power index respectively and their magnetic field and temperature dependence is given in [1]. The total losses per cycle of duration \(T\) and the loss voltage both per unit length are respectively [2]:

\[
Q_{tot} = \int \int \frac{J E_\rho dS dt}{\tau} \quad \text{and} \quad E_{\rho res} = \frac{Q_{tot}}{\tau I_T}.
\]
Fig. 2. The circular coil (1/4 of the actual cross-section is depicted, the symmetry lines are shown as broken). The arrows show the usual locations of the maximum magnetic field and the mirror line indicates the position of the ferromagnetic c-cups.

TABLE II
SPECIFICATION OF THE HIGH-Tc COIL

<table>
<thead>
<tr>
<th>Case</th>
<th>a, mm</th>
<th>A, mm</th>
<th>b, mm</th>
<th>g, mm</th>
<th>turns</th>
<th>L, mHenry</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>235</td>
<td>237</td>
<td>307</td>
<td>∞</td>
<td>296</td>
<td>23.5</td>
</tr>
<tr>
<td>B</td>
<td>235</td>
<td>237</td>
<td>307</td>
<td>0.3</td>
<td>296</td>
<td>24.8</td>
</tr>
</tbody>
</table>

B. Coil Conductor
The conductor with its transport current is exposed to an external magnetic field created by other turns of the coil. In a coil as depicted in Fig. 2, the magnetic field changes significantly both along and across the conductor tape. To obtain an approximate solution, the averaging of the actual magnetic field over the tape electrical cross-sections is used.

Representative short samples of the BSCCO tape were studied previously. Magnetic field and temperature dependent direct V-I curves serve as input for the network model. Comparison of the calculated and measured losses of a short sample for characteristic combinations of frequency, temperature, transport current, magnetic field amplitude and the direction has confirmed the validity of the model [2].

III. HIGH-Tc COIL
The dimensions of the solenoid are presented in Table II. BSCCO tape is wet-wound in 4 layers on a G-10 coil former using STYCAST and thin glass-cloth. The 440-m long coil conductor is two adjacent and insulated BSCCO tapes electrically connected in parallel at the ends. The details are given in [3], [4].

A. Magnetic Arrangements
In this paper four arrangements of the same coil are considered: cases A and B: a single coil without [3] and with iron c-cups around the coil edges respectively, see Table II; cases C and D: same as A and B respectively, but as the outer sub-coil of the 1-MVA coil [4]. In all cases the magnetic field along the tape varies with a certain periodicity. The periodic patterns are depicted in Figs. 3 and 4. The c-cups are iron pieces [1], [4], which work as magnetic mirrors and mimic coaxial coils of the same dimensions spaced by the gap 2g from the real coil, see Fig. 2.

B. Magnetic Field Profiles
The coil studied is a thin solenoid with a large diameter. For this reason the distribution of the radial magnetic field versus the tape length is the same for any layer. The periodic patterns of both field components are depicted in Figs. 3 and 4 for 1/4 of the coil at a transport current of 50A. The maximum and the averaged over the tape length values are listed in Table III.

Case A. Within a layer the radial magnetic field of the adjacent turns partly cancels out and B, gradually increases towards the coil edge as it is depicted in Fig. 3. There a relatively small amount of the tape is exposed to a relatively high B, as shown in Fig. 3 and in Table TII. Namely this area limits the current of a coil. The axial field in the windings is shown in Fig. 4 for each layer starting from the innermost.

Case B. Iron c-cups around the coil edges reduce the radial field B, as it is depicted in Fig. 3 and listed in Table III. The axial magnetic field is almost the same as in Case A, Fig. 4.

Case C. The total current of the 1-MVA system is higher and also the radial field of the (sub-) coil is higher, see Fig. 3 and
TABLE III
RADIAL MAGNETIC FIELD OF THE COIL

<table>
<thead>
<tr>
<th>Case</th>
<th>$B_0$, mT Max. value</th>
<th>$B_0$, mT Avg. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>55</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Table III. The axial field in the windings is not shown.

Case D. Iron c-cups reduce the radial field $B_0$, as is shown in Fig. 3 and in Table III. The dashed lines in Fig. 4 show the axial field.

IV. EXPERIMENT

Three independent methods are used to study the loss $V-I$ curves of the coil: a numerical, a thermal and an electro-magnetic method. The measurements at 77 K are performed for most of the coil arrangements using the LC – circuit of Fig. 1. Cases A and C are treated here in less detail, as the higher radial field decreases the electric characteristics of the coil and makes the cases rather unpractical.

A. Numerical Method

To adopt the network model explained in [2] in order to calculate the losses of coils, the following assumptions are made. Both transport current and magnetic field of the coil oscillate harmonically with the same frequency and phase. At every point in the coil winding, the amplitude of the external magnetic field $B_0$ is coupled to the amplitude of the transport current $I_t$ via the field constant. The loss voltage is calculated using equation (3):

$$V_{pl} = \int_0^l E_{pl} dl$$,

where $l$ and $l_t$ are the length and the total length of the tape respectively.

B. Thermal Method

The conductor losses of the coil are measured using miniature Si-diodes as the thermometers placed in thermal contact with the windings and insulated from the liquid nitrogen. The thermometers are calibrated during DC testing using the coil itself as a heater with the same transport current flowing in opposite directions through the adjacent conductor tapes [3] and Fig. 5.

C. Electro-Magnetic Method

The transport current through the coil is measured with a zero-flux meter. The voltage across the coil is measured using global voltage taps. The taps are soldered to each tape individually and exclude the influence of the end joints. The inductive (out of phase) part of the voltage is compensated by subtracting the analogue signal directly proportional to the derivative of the transport current through the coil. The in-phase part of the remaining voltage represents the total electro-magnetic loss of the coil and of the objects coupled to it. The laboratory environment (iron in the floor etc.) was moderately filled with ferro-magnetic objects during the experiment. As a consequence, the conductor losses of the high-$T_c$ coil appeared to be lower than the losses in the coupled objects outside the cryostat.

V. RESULTS

A tape self-field critical current $I_{0B}$ of 51 A and $N_0$ of 17 (at the field criterion $E_0 = 10^4$ V/m) are used throughout the analysis in all cases. The lines in Figs. 5 - 7 depict the calculated coil $V-I$ curves based on the short sample data [2] and the magnetic field profiles. Direct $V-I$ curves are for both conductor tapes. Direct Current

Case A. $V-I$ curves of the coil were presented in [3] and are replotted in Fig. 5 as the reference. The calculated $V-I$ curve is in good agreement with the measurements. This proves that the tape is not damaged during the winding and the operation. The coil critical current is 67 A (at the criterion $V_0 = 44$ mV, most of the voltage builds up at the coil edges).

Case B. Measured and calculated $V-I$ curves of the coil are depicted in Fig. 5. The critical current of the coil is about 90 A, 30% higher than in case A. The model does not account for a residual radial field caused by the gaps between adjacent turns. This explains in part the observed discrepancy. Another reason is that the tapes originally have a slightly different critical current. In agreement with calculations, the thermometers show a far lower and uniform temperature rise of the winding than in case A.

Case D. Compared to case B, the radial field has increased. This compensates the gain and as a result the $V-I$ curve shifts back to that of case A, Fig. 5. The critical current of the coil is about 66 A. The calculations predict that the voltage along the tape, see equation (4), builds up more uniformly than in case A.

Case C. The $V-I$ curve is depicted in Fig. 5 and gives the critical current of the coil equal to 47 A.

B. Static Voltage - Current Curves

At the frequency limit $f \rightarrow 0$ the loss $V-I$ curve of the coil approaches the static $V-I$ curve [1]. The static $V-I$ curve (free of magnetization currents) is frequency independent, but depends on the shape of the transport current. For a sinusoidal current $I_t$, using the data from Fig. 5, the static $V-I$ curves of the coil are calculated. An example of the curve for both conductor tapes in case B is shown in Figs. 6-7 as line F.
C. Sinusoidal Transport Current

Amplitude. The results are summarised in Figs. 6 and 7. Lines $K_B$ and $L$ are simulated and account only for the coil conductor ($K_B$ – the total losses and $L$ – the transport losses [2]). Line $G$ is measured with the coil placed on the laboratory floor and line $H$ is obtained with the coil 1 m above the floor. The behaviour suggests that part of the losses might be outside the cryostat in the coupled objects (ferro-magnetic structural elements of the building in particular). Therefore, line $H$ gives the upper limit of the total electro-magnetic losses for this environment.

At the design current $I_0$ of 50 – 60 $A_{rms}$ all three methods agree well, $V_{RI} = 0.6 V_{rms}$. This also proves that the dissipation in the c-cups is far lower than the coil conductor losses. As a reference the simulated total conductor loss voltage for case $A$ at 49 Hz is also plotted, line $K_A$. Due to the c-cups the loss voltage in case $B$ is reduced by almost 4 times. The thermometers show far lower and more uniform temperature rise of the winding (in average 0.5 and 0.1 K for cases $A$ and $B$ respectively at $I_0 = 50 A_{rms}$).

Frequency. Using a numerical method the loss V-I curves of the coil conductor are simulated in the frequency range of 0 to 400 Hz. The loss voltage measured thermally at 16, 49 and 97 Hz is depicted in Fig. 7 and it ensures that the agreement of the numerical and thermal methods at 49 Hz is not incidental. Analysis of the data presented in Fig. 7 reveals that over the range covered, the losses per cycle of the coil conductor are frequency-dependent. A similar effect is found for the magnetisation loss of the same tape. The dependency is somewhat more pronounced at higher transport currents. While within a narrow range 50 to 60 Hz the effect is a few percent only, over the range 1 to 400 Hz it reaches ~50%.

Quality. The ratio of the impedance to the resistance is the electric quality factor $Q$ of a coil. Based on the data obtained for case $B$ at 50 Hz it is found that $Q$ is 700 at $I_0 = 55 A_{rms}$. To illustrate the potential of high-$T_c$ coil, V-I curves are shown of identical coils made with copper and operating at 300 K (line $M$) and with silver and operating at 77 K (line $N$). To enable the comparison, the loss voltage of the copper coil is divided by the cooling penalty that is equal to 20. The loss voltage of the high-$T_c$

VI. CONCLUSIONS

The behaviour of an accurately made large high-$T_c$ coil operating at 77 Kelvin is studied numerically and verified by experiment. Calculated and measured direct V-I curves are in fair agreement, which ensures that the sc. tape is not damaged. Loss V-I curves of the coil operating in liquid nitrogen are calculated numerically and validated by the measurements. Iron c-cups placed around the coil edges simultaneously increase the critical current, reduce the losses and improve the thermal state of a superconducting coil.

REFERENCES