Memorandum No. 1319

Cyclic graphs

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April 1996

ISSN 0169-2690

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Abstract

A subclass of the class of circulant graphs is considered. It is shown that in this subclass, isomorphism is equivalent to Adam-isomorphism. Various results are obtained for the chromatic number, line-transitivity and the diameter.

Keywords: Circulant, Adam isomorphism, line transitive, diameter.

1992 Mathematics Subject Classification: 05C99

1 \ Introduction and summary

We study a subset of the class of circulant graphs. A circulant graph can be defined as follows. Let \(n\) be a natural number and let \(S = \{k_1, \ldots, k_r\}\) with \(1 \leq k_1 < k_2 < \cdots < k_r \leq \frac{n}{2}\). Then the point set of the circulant graph \(G(n, S)\) is \(\{0, 1, \ldots, n-1\}\) and the set of neighbors of the point \(p\) is \(\{(p \pm k_j) \mod n | j = 1, \ldots, r\}\).

Circulant graphs have been extensively studied, see e.g. Elspach and Turner [1], Davis [2]. The special case we consider, the cyclic graphs \(C(n, k)\) have point set \(\{0, 1, \ldots, n-1\}\) and lines \(\{i, i+1\} \mod n\) and \(\{i, i+k\} \mod n (i = 1, \ldots, n)\) where \(k\) is an integer with \(2 \leq k \leq n-2\). So \(C(n, k) \cong G(n, S)\) with \(S = \{1, \min\{k, n-k\}\}\).

The graphs \(C(n, k)\) are point-transitive, 3-regular if \(n = 2k\) and 4-regular otherwise.

In section 2 we identify some well-known graphs of the form \(C(n, k)\) and we consider isomorphism between its members.

In section 3 we consider the chromatic number, in section 4 line-transitivity, and in section 5 the diameter of \(C(n, k)\).
2 Special graphs and isomorphism

In Table 1 below we list some members of the $C(n,k)$-family.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$k$</th>
<th>graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>$K_4$</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>$K_5$</td>
</tr>
<tr>
<td>$2m$</td>
<td>2</td>
<td>$m$-sided antiprism ($m \geq 3$)</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>$K_{3,3}$</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>$K_{4,4}$</td>
</tr>
<tr>
<td>$2m$</td>
<td>$m$</td>
<td>Möbius-ladders ($m \geq 3$)</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>$K_{5,5} \setminus$ perfect matching</td>
</tr>
</tbody>
</table>

Table 1

Note that $C(5,2) \simeq C(5,3)$. More generally, $C(n,k) \simeq C(n,n-k)$ and also if $kk' \equiv \pm 1 \pmod{n}$, then $C(n,k) \simeq C(n,k')$. The notion of Ádám-isomorphism for circulant graphs (see e.g. Boesch and Tindell [3]) reduces to just these two cases for cyclic graphs. For circulant graphs, pairs of isomorphic graphs are known that are not Ádám-isomorphic. The first example $G(16,\{1,2,7\}) \simeq G(16,\{2,3,5\})$ was found by Elspas and Turner [1]. However the following result says that for cyclic graphs, Ádám isomorphism is equivalent to isomorphism. It is convenient to use the term “sides” for lines of the form $\{i,i+1\}$, and “chords” for lines of the form $\{i,i+k\}$.

**Theorem 2.1** If $k' \neq k$, $k' \neq n-k$, and $kk' \neq \pm 1 \pmod{n}$, then $C(n,k) \not\simeq C(n,k')$.

**Proof.** For $n \leq 8$ the result can be verified directly. Let $n > 8$. If an isomorphism maps all sides to sides, then $k' = k$ or $k' = n-k$. And if an isomorphism maps all sides to chords, then $kk' \equiv \pm 1 \pmod{n}$.

Let $\phi$ be an isomorphism that maps neither all sides to sides nor all sides to chords. Then without loss of generality, the side $\{1,2\}$ is mapped to a side and the side $\{2,3\}$ to a chord; see Figure 1.

![Fig. 1](image-url)
The predecessor of $\phi(3)$ in the image graph is the image $\phi(p)$ of a point $p \neq 2$. Now $p$ is adjacent to both 1 and 3, hence either $k = 3$ or $k = \frac{n-2}{2}$; see Figure 2.

![Fig. 2](image)

Since $k = k'$ is excluded, at least one of $k$ and $k'$ is 3. We assume w.l.o.g. that $k = 3, k' = \frac{n-2}{2}$.

In $C(n, 3)$ we now have three possibilities for the sides $\{0, 1\}$ and $\{2, 3\}$.

1. $\{0, 1\}$ is mapped to a side,
2. $\{0, 1\}$ and $\{2, 3\}$ are mapped to intersecting chords,
3. $\{0, 1\}$ and $\{2, 3\}$ are mapped to parallel chords.

In case 1, $\phi(3)$ is adjacent to $\phi(4)$ and $\phi(6)$, hence $\phi(1)$ is also adjacent to $\phi(4)$ and $\phi(6)$; see Figure 3. It follows that 1 is adjacent to 4 and 6, so $n = 8$: a contradiction.

![Fig. 3](image)

In cases 2 and 3 the argument is exactly the same. Figure 4 gives the situation in case 2. 

\[\square\]
In view of $C(n-k) \simeq C(n,n-k)$ we restrict our attention to pairs $(n,k)$ with $n \geq 2k$.

3 The chromatic number

We denote the chromatic number of $C(n,k)$ by $x(n,k)$. For the colors we use red, white, blue, abbreviated $R, W, B$.

Proposition 3.1

a) $n$ even and $k$ odd $\iff x(n,k) = 2$.

b) $n = 5 \iff x(n,k) = 5$.

c) If $3 \mid n$ then $x(n,2) = 3$,
   if $3 \nmid n$ and $n \neq 5$ then $x(n,2) = 4$.

d) If $k$ even and $n = 2k \geq 8$, then $x(n,k) = 3$.

Proof. Trivial. \qed

We now present several sub-families of graphs, each with a 3-coloring.

Proposition 3.2

If $n$ and $k$ are odd and $n \geq 3k$, then $x(n,k) = 3$.

Proof. Let $n$ and $k$ be odd and $n \geq 3k$. Color the points $1, 2, \ldots, k$ alternatingly $B$ and $R$, the points $k+1, \ldots, 2k$ alternatingly $R$ and $W$, and the rest alternatingly $W$ and $B$. This is a proper coloring as is easily verified. \qed

Proposition 3.3

$x(n,k) \leq 3$ if $n \equiv 0, k \equiv 1$ or $2$,
   or $n \equiv 2, k \equiv 1$, $n \geq 8$,
   or $n \equiv 1, k \equiv 1$, $n \geq 2k + 4$,

where all congruences are modulo 3.
Proof. If \( n \equiv 0 \) and \( k \equiv 1 \) or 2, one simply repeats the pattern RWB. It works locally since \( 3 \nmid k \), and globally since \( 3 \mid n \).

If \( n \equiv 2, k \equiv 1 \), the pattern RWB can again be used, now followed by a “tail” of two points which are colored RW. The case \( n = 5 \) has to be excluded.

If \( n \equiv 1, k \equiv 1 \), then write \( n = n_1 + n_2 \) where \( n_i \equiv 2 \) and \( n_1 - n_2 = 0 \) or 3. Then apply the pattern RWB and a tail RW to the points \( 1, 2, \ldots, n_1 \), and the same for the remaining points. \( \square \)

**Proposition 3.4**

\[
x(n, k) \leq 3 \quad \text{if} \quad n \equiv 0, \ k \equiv \pm 1,
\]
\[
or \quad n \equiv 2, \ k \equiv 1,
\]

where all congruences are modulo 5.

**Proof.** Repeat the pattern RWBRW; cut it off when \( n \equiv 2 \). \( \square \)

**Proposition 3.5**

\[
x(n, k) \leq 3 \quad \text{if} \quad n \equiv 0, \ k \equiv \pm 1,
\]
\[
or \quad n \equiv 2, \ k \equiv 1,
\]
\[
or \quad n \equiv 5, \ k \equiv -1
\]

where all congruences are modulo 7.

**Proof.** Repeat the pattern RWBRWRW; cut it off when \( n \neq 0 \). \( \square \)

**Proposition 3.6**

\[
x(n, k) \leq 3 \quad \text{if} \quad n \equiv 0, \ k \equiv 1, 3, 6, \text{ or } 8,
\]
\[
n \equiv 2, \ k \equiv 1, 3, \text{ or } 8,
\]
\[
n \equiv 4, \ k \equiv 1 \text{ or } 3,
\]
\[
n \equiv 5, \ k \equiv 6 \text{ or } 8,
\]
\[
n \equiv 7, \ k \equiv 1, 6, \text{ or } 8,
\]

where all congruences are modulo 9.

**Proof.** Repeat the pattern RWRWBWBRB; cut off when \( n \neq 0 \). \( \square \)

The cases covered by Propositions 3.1a,3.2, . . . ,3.6 do by no means form a set of covering congruences. The smallest case not covered is \( C(13, 5) \).

**Proposition 3.7** \( x(13, 5) = 4 \).

**Proof.** Suppose a 3-coloring of \( C(13, 5) \) exists. We distinguish two cases.
1. There exists an \( i \) such that the points \( i \) and \( i + 3 \) have the same color. Without loss of generality assume that the points 0, 1, 2, 3 have the colors \( R, W, B, R \), respectively, and that point 8 has the color \( W \). Then the colors of the points 7, 6, 5, 4, 9 are successively forced, and point 10 then has 3 differently colored neighbors.

2. For all \( i \), the points \( i \) and \( i + 3 \) have different colors. Then without loss of generality the points 0, 5, 10 have the colors \( R, W, B \), respectively. Each following point in the sequence \( 10 + 5i (\text{mod } 13) \) then has a forced color, and at the last point, a fourth color is necessary.

In both cases we have a contradiction, hence \( x(13, 5) = 4 \).

\[ \square \]

4 Line-transitivity

It is easily seen that of the “known” graphs in Table 1, \( C(n, k) \) is line-transitive for the pairs \( (n, k) = (4, 2), (5, 2), (6, 2), (6, 3), (8, 3) \), and \( (10, 3) \). But there are others.

Proposition 4.1

If \( k^2 \equiv \pm 1(\text{mod } n) \), then \( C(n, k) \) is line-transitive.

Proof. \( f(i) = ki(\text{mod } n) \) is an automorphism.

Proposition 4.2

If \( n = 2k + 2 \), then \( C(n, k) \) is line-transitive.

Proof. If \( k \) is odd, say \( k = 2v + 1 \), then \( k^2 = 4v^2 + 4v + 1 \equiv 1(\text{mod } n) \), so these cases are covered by Proposition 4.1. If \( k \) is even, consider the map \( f \) defined by

\[
 f(i) = \begin{cases} 
 ki & \text{for } 0 \leq i \leq k, \\
 k + k + 1 & \text{for } k + 1 \leq i \leq 2k + 1.
\end{cases}
\]

We claim that \( f \) is an automorphism.

First consider a pair of the form \( \{i, i + 1\} \). There are 3 cases.

1. \( 0 \leq i < i + 1 \leq k \). \( f(i) = ki, f(i + 1) = (k + 1)i \), and the difference is \( k \).

2. \( i = k \). \( f(i) = f(k) = k^2, f(i + 1) = k(k + 1) + k + 1 \), and the difference is \( 2k + 1 \equiv -1(\text{mod } n) \).

3. \( i = 2k + 1 \). \( f(i) = k(2k + 1) + k + 1, f(i + 1) = f(0) = 0 \), and the difference is \( 2k^2 + 2k + 1 \equiv 1(\text{mod } n) \).

Next consider a pair of the form \( \{i, i + k\} \). There are 4 cases.
4. $i = 0$, $f(i) = 0$, $f(i + k) = f(k) = k^2$, and the difference is $k^2 = \frac{k}{2}(2k + 2) - k \equiv -k \pmod{n}$.

5. $1 \leq i \leq k$, hence $k + 1 \leq i + k \leq 2k$. $f(i) = ki$, $f(i + k) = k(i + k) + k + 1$, and the difference is $k^2 + k + 1 \equiv 1 \pmod{n}$.

6. $i = k + 1$. $f(i) = k(k + 1) + k + 1$, $f(i + k) = k(2k + 1) + k + 1$, and the difference is $k^2 \equiv -k \pmod{n}$. (See 4).

7. $k + 2 \leq i \leq 2k + 1$. $f(i) = ki + k + 1$, $f(i + k) = k(i + k)$; the difference is $k^2 - k - 1 \equiv 1 \pmod{n}$.

Note that in the cases 4,5,6,7 we must have $k$ even.

In all 7 cases, a line is mapped to a line.

We now present some cyclic graphs that are not line-transitive. One way to prove results of this kind is as follows. A priori there are 2 kinds of lines: sides and chords. Now if a side belongs to more triangles, say, than a chord, then obviously the graph is not line-transitive. If triangles do not work, we can take some other graph. In the sequel we use a somewhat more convenient way of counting: if all subgraphs of a specified form together contain more sides than chords, then the graph is not line-transitive (assuming $n \neq 2k$).

**Proposition 4.3**

If $n > k(k + 1)$, then $C(n, k)$ is not line-transitive.

**Proof.** Consider subgraphs of the form $C_{k+1}$. Let $i_0, i_1, \ldots, i_k$ with subscripts mod$(k + 1)$ be the points of some $C_{k+1}$. Then $\forall j : v_j := i_{j+1} - i_j \in \{1, -1, k, -k\}$. Also

$$(*) \quad (\ast) \sum_{0}^{k} v_j \equiv 0 \pmod{n}. $$

From the condition $n > k(k + 1)$ it follows that $\sum_{0}^{k} v_j = 0$ is the only way to satisfy $(\ast)$. Let $v_j$ assume the values $\pm k \ t$ times. If $t$ is odd then the terms $\pm k$ cannot cancel, and hence $t = 1$. Since the number of sides (terms $\pm 1$) is larger than the number of chords (terms $\pm k$) in each $C_{k+1}$, it follows that $C(n, k)$ is not line-transitive when $n > k(k + 1)$ and $t$ odd.

If $t$ is even then the terms $\pm k$ can cancel, leading to other ways of forming $C_{k+1}$’s. These may have a surplus of chords, which might compensate the abundance of $1$’s in the previous type of $C_{k+1}$. But to each $C_{k+1}$ with a surplus of $\pm k$’s, a different $C_{k+1}$ corresponds which annihilates this effect. E.g. when $k = 5$, we have $5 + 5 + 1 - 5 - 5 - 1 = 0$, but also, interchanging $1$’s and $5$’s: $1 + 1 + 5 - 1 - 1 - 5 = 0$. Hence, also when $t$ is even the number of sides is larger than the number of chords in all $C_{k+1}$’s together. Again, $C(n, k)$ is not line-transitive. \qed
Proposition 4.4

For all pairs $(\lambda, k)$ with $\lambda = 2, k \geq 4$ or $\lambda \geq 3, k \geq 3$, $C(\lambda k, k)$ is not line-transitive.

Proof. For $\lambda = 2, k \geq 4$, consider subgraphs of the form $C_4$. These are all of the form $k+1-k-1$, with equal numbers of sides and chords. However, in these graphs (Möbius ladders) the number of chords is only half the number of sides.

For $\lambda = 3, k \geq 3$, consider subgraphs of the form $C_3$. These are all of the form $k+k+k \equiv 0$.

For $\lambda = 4, k = 3$, consider $C_4'$. There are three types; see the table below.

<table>
<thead>
<tr>
<th>type of $C_4$</th>
<th>number</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1-1-1</td>
<td>12</td>
<td>36</td>
<td>12</td>
</tr>
<tr>
<td>3-1-3+1</td>
<td>12</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>3+3+3+3</td>
<td>3</td>
<td>0</td>
<td>12</td>
</tr>
</tbody>
</table>

Under the heading A we find the total numbers of sides per type, under the heading B the total numbers of chords. Since the column totals of A and B are different, $C(12, 3)$ is not line-transitive.

For $\lambda = 4, k \geq 4$ we consider $C_4'$ again, and finally for $\lambda \geq 5, k \geq 3$ we consider $C_4'$. The details for these two cases are left to the reader. \[\square\]

5 Diameter

We denote the diameter of $C(n, k)$ by $d(n, k)$. The diameter has a very irregular behaviour, e.g. $d(n, k)$ is not monotone in $n$ for fixed $k$, and even $\min_k d(n, k)$ is not monotone in $n$. But sharp upper and lower bounds will be given.

Proposition 5.1

\[d(n, k) \leq d(n, 2) = \left\lfloor \frac{n+2}{4} \right\rfloor.\]

Proof. $d(n, 2) = \left\lfloor \frac{n+2}{4} \right\rfloor$ follows easily from the fact that from 0 the points $2i-1$ and $2i$ are at distance $i$ for $0 \leq i \leq \frac{1}{2} \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{n+2}{4} \right\rfloor$.

To prove the inequality, it will suffice to prove that in $C(n, k)$ every point $v$, where $1 \leq v \leq \left\lfloor \frac{n}{2} \right\rfloor$, can be reached from 0 in $d(n, 2)$ steps or less. For $k = 3$, this can be verified directly.

Let $k \geq 4$ and let $v \leq \left\lfloor \frac{n}{2} \right\rfloor$. Choose $r$ such that $rk \leq v < (r+1)k$. Then $v$ can be reached in at most $r+k-1$ steps. If $r \geq 4$, then $(r-2)(k-2) \geq 4$ hence $r+k-1 \leq \frac{rk}{2} - 5 \leq \frac{v}{2} - 5 < \left\lfloor \frac{n+2}{4} \right\rfloor$.

For the remaining cases, i.e. when $v \leq 4k-1$ and $k \geq 4$, the inequality can be easily verified. \[\square\]
Proposition 5.2 \[ \min_k d(n, k) \geq \frac{\sqrt{2n-1}-1}{2} \] with equality if (and only if) the right hand side is an integer.

Proof. To prove the inequality, start at point 0. If \( p \) is a point at distance \( L \) from 0, then \( p = \alpha + \beta k \) (mod \( n \)) with \( \alpha, \beta \in \mathbb{Z} \) and \( |\alpha| + |\beta| = L \). For the value of \( \alpha \) there are \( 2L + 1 \) possibilities. If \( |\alpha| = L \), then \( \beta \) does not occur in the expression \( \alpha + \beta k \), so here we have 2 possibilities. If \( |\alpha| < L \), there are 2 possibilities for \( \beta \), yielding \( 4L - 2 \) possibilities for the pair \((\alpha, \beta)\). Hence the total number of possibilities for the pair \((\alpha, \beta)\) is \( 4L \). It follows that

\[(*) \quad (\ast)n \leq 1 + 4 + 8 + 12 + \ldots + 4d(n, k),\]

which is equivalent to \( d(n, k) \geq (\sqrt{2n-1} - 1)/2 \).

If \((\sqrt{2n-1} - 1)/2\) is an integer, say \( D \), then \( n = 2D^2 + 2D + 1 \). Take \( k = 2D + 1 \). It is sufficient to show that each point in \( C(n, k) \) is at distance \( \leq D \) from 0, or alternatively that for all \( x \), there exist \( \alpha, \beta \in \mathbb{Z} \) such that \( |\alpha| + |\beta| \leq D \) and \( x \equiv (\alpha + \beta(2D + 1)) \mod n \).

We have seen above that there are \( 1 + 4(1 + 2 + \ldots + D) = 2D^2 + 2D + 1 \) possibilities for the pair \((\alpha, \beta)\). Is is sufficient to show that all residue classes are different.

Suppose \( \lambda + \mu(2D + 1) \equiv \alpha + \beta(2D + 1) \) with \( |\alpha| + |\beta| \leq D, |\lambda| + |\mu| \leq D \). Then \( \lambda - \alpha + (\mu - \beta)(2D + 1) \) is a multiple of \( 2D^2 + 2D + 1 \). Taking absolute values, it is clear that this multiple can only be 0 or \( \pm(2D^2 + 2D + 1) \).

If it is 0, then \( \lambda = \alpha, \mu = \beta \), and we are through. So suppose

\[ \lambda - \alpha + (\mu - \beta)(2D + 1) = 2D^2 + 2D + 1. \]

(the minus sign runs similarly.)

Then \( \lambda - \alpha + (\mu - \beta)(2D + 1) = D(2D + 1) + D + 1, \) hence \( D + 1 + \alpha - \lambda \) is a multiple of \( 2D + 1 \). This multiple can only be 0 or \( \pm(2D + 1) \). If \( D + 1 + \alpha - \lambda = 0 \), then \( \mu - \beta = D \), hence \( \lambda = \alpha + D - 1, \mu = \beta + D \). But \( |\lambda| + |\mu| \leq D \) and we have a contradiction. If \( D + 1 + \alpha - \lambda = 2D + 1 \) then \( \alpha - \lambda = D \), hence \( \mu - \beta = D + 1 \), and again we have a contradiction. The case \( D + 1 + \alpha - \lambda = -(2D + 1) \) is similar.

\[ \square \]

Proposition 5.3 If \( d(n, k) \leq \frac{n}{k} \), then \( d(n + 2k, k) = 1 + d(n, k) \).

Proof. Clearly \( d(n, k) < d(n + 2k, k) \); in a shortest path from 0 to \( k + i \), \( i = 1, \ldots, n - 1 \), in \( C(n + 2k, k) \), leave out first chord to obtain a path from 0 to \( i \) in \( C(n, k) \).

Also since \( d(n, k) \leq \frac{n}{k} \), every shortest path from 0 to \( i \) in \( C(n, k), i = 1, \ldots, n - 1 \) can be made to a path from 0 to \( i + k \) in \( C(n + 2k, k) \) by adding one chord.

Clearly the points 1, \ldots, \( k \) and \( n + k, \ldots, n + 2k - 1 \) can be reached in (either \( d(n, k) \) or) \( d(n, k) + 1 \) steps.

\[ \square \]

Corollary If \( n \geq k^2 \), then \( d(n + 2k, k) = 1 + d(n, k) \).

The condition \( n \geq k^2 \) may seem strong, but it cannot be omitted. The smallest counterexample is \( d(19, 8) = 3, d(35, 8) = 5 \). The smallest counterexample with difference 3 is \( d(31, 14) = 5, d(59, 14) = 8 \).
Proposition 5.4 For every $M > 0$ there are $n$ and $k$ such that $d(n+2k, k) > d(n, k)+M$.

Proof. Take $n = 2k + 3$, $k = 6M + 2$. In $C(n, k)$ the points $3\lambda$ can be reached from 0 in $2\lambda$ steps by using chords ($\lambda = 1, 2, \ldots, M$) and hence the point $3M+1$ in $2M+1$ steps. Again by using just chords, the point $k - 3\lambda$ can be reached in $2\lambda + 1$ steps ($\lambda = 1, 2, \ldots, M$). It follows that $d(n, k) \leq 2M + 1$.

Now consider $C(n + 2k, k)$. To advance 3 places by using chords only, 4 chords are required. Hence here it is in no case optimal to use more than 3 chords in a path. To get from 0 to $k + \frac{1}{2}k + 1$, one may use either 2 or 3 chords. In the first case, the number of sides required is $\frac{1}{2}k - 1$, in the second case it is $\frac{1}{2}k - 2$. Hence the distance from 0 to $k + \frac{1}{2}k + 1$ is $\frac{1}{2}k + 1 = 3M + 2$. It follows that $d(n + 2k, k) \geq 3M + 2 > d(n, k) + M$. □

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