A Polynomial-Time Algorithm for the Computation of the Iteration-Period Bound in Recursive Data-Flow Graphs

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Abstract—Rate-optimal scheduling of iterative data-flow graphs requires the computation of the iteration period bound. According to the formal definition, the total computational delay in each directed loop in the graph has to be calculated in order to determine that bound. As the number of loops cannot be expressed as a polynomial function of the number of nodes in the graph, this definition cannot be the basis of an efficient algorithm. This paper presents a polynomial-time algorithm for the computation of the iteration period bound based on longest path matrices and their multiplications.

I. INTRODUCTION

Parallel processing is unavoidable for the implementation of fast computation-intensive DSP algorithms. As a consequence tools to detect parallelism and appropriate scheduling techniques have to be developed. Especially, techniques for nonpreemptive deterministic scheduling of DSP algorithms without data-dependent conditions have received some attention (see, for example, [1]-[6]).

The scheduling problem is often modeled by mapping the algorithm specifications onto a data-flow graph (DFG). DFG’s model the way computational operations depend on data produced by other operations, where data may be delayed by a multiple of the iteration period before being transferred (see, e.g., [3], [5], [7], or [8]). A DFG can be recursive or nonrecursive.

Nonrecursive DFG’s can be executed arbitrarily fast by supplying the required computing power, combined with the transformation of the original network into a suitable essentially equivalent form [9] where the original parallelism has been increased, e.g., by introducing pipelining.

However, this is not the case for recursive DFG’s, where there exists a limit imposed by the topology of the graph and the speed of the hardware units. Once this limit is reached, the addition of more computing power results only in a decrease of the processor utilization.

That limit is given by the maximum speed of execution of the so-called critical loop [1], [10] which determines the minimum sampling period, or iteration period bound [3], T_{0_{\text{crit}}} A critical loop is that directed loop for which the summation of the processing delays of each operation in the loop divided by the number of delay elements in the loop, n_i, is maximal. This quotient is T_{0_{\text{crit}}}.

Expressed in another way:

$$T_{0_{\text{crit}}} = \max_{\text{all directed loops}} \left\{ \frac{\sum_{c_i \text{ duration}}}{n_i} \right\}$$

(1)

where c_i duration is the duration of operation c_i.

A number of rate-optimal (maximum speed) scheduling algorithms have been proposed (see, e.g., [1], [3], and [6]). All of them require the computation of the iteration period bound, T_{0_{\text{crit}}}.

The computation of T_{0_{\text{crit}}} by inspection of all directed loops, as given by (1), does not lead to an efficient algorithm, as will be shown later. This paper proposes a polynomial-time algorithm for the computation of the iteration period bound T_{0_{\text{crit}}}.

II. THE DFG

For the purpose of this paper a DFG is defined as a directed graph: DFG(V, E). V = C U D U I U O constitutes the set of vertices (C, D, I, and O are pairwise disjoint). C is the set of vertices associated with computational nodes (or operations), D is the set of delay elements, and I and O are the inputs and outputs, respectively. The delay associated to a delay element is equal to T_{0_{\text{crit}}}, the iteration period. A delay of multiple iteration periods is assumed to be modeled here with single delay elements connected in series. An edge e_i e_j E can connect any pair of vertices in V with the constraint that no edge is incident to an input and no edge is incident from an output.

A directed path in the graph starts at a certain vertex, goes through zero or more vertices and finishes in another vertex following the edges in the graph according to their orientations. When the first and the last vertex of a directed path coincide, the path is called a directed loop. In the rest of the text a directed path will be called a path and a directed loop will be called a loop. A path or loop is simple when the path or loop passes at most once through each vertex in the DFG [11]. When a path or loop is not simple, it has to contain internal loops.

We assume that a DFG is always connected. The topology of a DFG obeys the constraint that any loop passes at least through one delay element.

The symbols c, d, and e will be used to refer to the cardinality of the sets C, D, and E, respectively. The c computational nodes are: c_1, c_2, ..., c_c. The d delay elements are: d_1, d_2, ..., d_d.

III. THE COMPUTATION OF THE ITERATION PERIOD BOUND

In the formal definition of T_{0_{\text{crit}}} (see (1)), all possible simple loops have to be considered. This does not lead to an efficient algorithm. Consider, e.g., DFG’s that have at least one directed simple path consisting of zero or more computational nodes between every pair of delay elements. This type of DFG will be called path complete (see Fig. 1 for an example of a path-complete DFG). In path-complete DFG’s, the minimal number of simple loops passing through k delay elements is (d^k/k(d-k)!)). Hence the minimal number of distinct loops is

$$\sum_{k=1}^{d} \frac{d^k}{k(d-k)!}.$$  

Clearly, as the factorial function grows even faster than an exponential function, the number of loops cannot be expressed as a polynomial function of d. So, an algorithm based on an exhaustive enumeration of all simple loops cannot operate in polynomial time.

This paper presents a polynomial-time algorithm to compute T_{0_{\text{crit}}}. It consists of two parts.

1) The computation of the first-order longest path matrix L^1, a modification of the path matrix introduced in [12].
2) The computation of L^k, the longest path matrix of order k, for finding the loops that go through k delay elements.

3.1. The Longest Path Matrix of Order k

The longest path matrix of order k (k \geq 1), L^k, is a d \times d matrix, and each of its elements l_{ij}^k contains the length of the longest path from delay element d_i to delay element d_j that passes through exactly k - 1 delay elements. The length of a path is the
Fig. 1. Example of a path-complete DFG with three delay elements; inputs and outputs have not been included in the picture.

sum of the durations of the operations in the path. When such a directed path from \( d_i \) to \( d_j \) does not exist, \( I_{ij}^k \) is defined to be \(-1\).

The diagonal elements of \( L^k \) represent the length of the longest paths starting and finishing at the same delay element and going through \( k - 1 \) other delay elements. This means that they represent loops passing through \( k \) delay elements.

From now on, a path going through \( k - 1 \) delay elements will be called a path of order \( k \), since such a path is an element of the longest path matrix of order \( k \). If the path is a loop, such a loop will be called a loop of order \( k \). Note that for \( k > 1 \) a longest path of order \( k \) is not necessarily simple.

3.2. The Computation of the First-Order Longest Path Matrix

The first-order longest path matrix \( L^1 \) can be computed by inspecting the DFG. An algorithm that performs this computation has been displayed in Figs. 2 and 3. Any vertex \( v \) is supposed to have a field \( v.\text{max-length} \) where some accounting can be done. The description is in pseudocode; the intention is to transfer the principal ideas instead of being precise.

In the main program, all delay elements are used one by one to compute the longest paths starting at them and finishing in any delay element. A preparatory step is to “inactivate” all edges in the DFG that cannot be reached from the concerned delay element. The longest paths are then found in a way similar to critical-path detection in cyclic graphs.

For each delay element, the edges in the DFG are made inactive only once, so the time complexity of the algorithm to compute \( L^1 \) is \( O(de) \).}

3.3. The Computation of Longest Path Matrices of Higher Order

Higher order longest path matrices can be obtained by the following recursive rule:

\[ L^{k+1} = L^1 L^k, \quad k > 0. \]

The rule implies an operation called \( lp\)-multiplication (longest-path matrix multiplication). This multiplication combines all first-order longest paths with all longest paths of order \( k \), to get longest paths of order \( k + 1 \).

The longest path of order \( k + 1 \) from \( d_s \) to \( d_t \) can obviously be found by inspecting all first-order longest paths from \( d_s \) to all elements \( d_i \), together with all longest paths of order \( k \) from \( d_i \) to \( d_t \) and selecting the combination through that \( d_i \) that gives rise to the longest total path.\footnote{This idea is essentially the same one as the matrix multiplication method mentioned in [13] for the all-pairs shortest path problem.}

Fig. 2. The algorithm to compute the longest path matrix. Part I:

The procedures.

\[
\text{procedure longest-path starts;}
\]

for all \( 1 = i = \ldots \) do ?

then if \( v.\text{max-length} < 0 \)

else if \( v.\text{max-length} < 0 + v.\text{max-length} + \text{relation} \)

then \( v.\text{max-length} = \ldots \)

\( k \)

\( \text{if all edges incident to } v \text{ are inactive} \)

then \( \text{do } \ldots \)

then longest-path ends;

end of longest-path starts;

Fig. 3. The algorithm to compute the longest path matrix. Part II:

The main program.

\[ k \text{ is defined as follows:} \]

\[ T^k_{\text{max}} = \max_{d_i \in \{d_1, \ldots, d_d\}} l^k_{i}. \]

As the critical loop is a simple loop and therefore passes through at most \( d \) delay elements, \( L^d \) is the highest order longest path matrix that must be computed. Hence, the iteration period bound is the maximal value of all \( T^k_{\text{max}} \):

\[ T_{\text{max}} = \max_{k \in \{1, \ldots, d\}} T^k_{\text{max}}. \]

The loops found by means of the longest path matrix of order \( k \) may or may not be simple; however, they are always the longest possible ones of the given order. It will be shown now that the participation of nonsimple loops in the computations does not lead to an incorrect value of \( T^k_{\text{max}} \).

Suppose that a loop of order \( k \) consists of two subloops of orders \( m \) and \( n \), \( m + n = k \). Let the length of the loop be \( l \) and the length of the subloops be \( l_m \) and \( l_n \); \( l = l_m + l_n - l_i \). The following inequality should be proved to be correct:

\[ l / k \leq \max \left( l_m / m, l_n / n \right). \]

Without loss of generality, it can be stated that \( l_m / m \geq l_n / n \), or

\[ ml_n \leq m. \]

Adding \( ml_m \) to both sides and undertaking some simple manipula-
Fig. 4. The lp-multiplication algorithm.


tions shows the correctness of the inequality to be proved:

\[ m l_m + m l_n \leq m l_m + n l_m \]

\[ m (l_m + l_n) \leq (m + n) l_m \]

\[ m l_k \leq k l_m \]

\[ \frac{l_k}{k} \leq \frac{l_m}{m} \]

In a similar way the result can be generalized for loops built from an arbitrary number of simple subloops. So, the algorithm computes \( T_{\text{max}} \) correctly.

3.4. Time Complexity

The presented method gives a polynomial-time algorithm for the computation of the critical loop. \( \text{lp}-\text{multiplication} \) for getting \( L^k = 1 \) from \( L^1 \) and \( L^k \) requires \( O(d^k) \) steps; so computing \( L^2 \) can be done in \( O(d^2) \) time. The time complexity for computing \( L \) was \( O(\text{de}) \), so the total time complexity of the critical loop detection algorithm becomes \( O(\text{de} + d^2) \).

3.5. An Example

In this subsection the algorithm of this paper will be illustrated using the DFG of Fig. 5 as an example. It represents an all-pole lattice filter.

The matrix \( L^1 \) has the following value:

\[ L^1 = \begin{bmatrix}
8 & 12 & 16 & 16 \\
4 & 8 & 12 & 12 \\
-1 & 4 & 8 & 8 \\
-1 & -1 & 4 & 4
\end{bmatrix} \]

So, \( T_{\text{max}}^1 = 8 \) TU. \( L^2 \) is computed below:

\[ L^2 = \begin{bmatrix}
8 & 12 & 16 & 16 \\
4 & 8 & 12 & 12 \\
-1 & 4 & 8 & 8 \\
-1 & -1 & 4 & 4
\end{bmatrix} \cdot \begin{bmatrix}
16 & 20 & 24 & 24 \\
12 & 16 & 20 & 20 \\
8 & 12 & 16 & 16 \\
-1 & 8 & 12 & 12
\end{bmatrix} \]

\[ = \begin{bmatrix}
32 & 36 & 40 & 40 \\
28 & 32 & 36 & 36 \\
24 & 28 & 32 & 32 \\
20 & 24 & 28 & 28
\end{bmatrix} \]

In a similar way, we find:

\[ L^3 = \begin{bmatrix}
24 & 28 & 32 & 32 \\
20 & 24 & 28 & 28 \\
16 & 20 & 24 & 24 \\
12 & 16 & 20 & 20
\end{bmatrix} \quad L^4 = \begin{bmatrix}
32 & 36 & 40 & 40 \\
28 & 32 & 36 & 36 \\
24 & 28 & 32 & 32 \\
20 & 24 & 28 & 28
\end{bmatrix} \]

It follows:

\[ T_{\text{max}}^2 = T_{\text{max}}^3 = T_{\text{max}}^4 = 8 \text{ TU.} \]

So, \( T_{\text{max}} = 8 \) TU.

IV. Final Remarks

Formally, all directed loops of a DFG have to be investigated to compute the iteration period bound, \( T_{\text{max}} \). The number of loops cannot be expressed as a polynomial function of the number of nodes. This paper shows that \( T_{\text{max}} \) still can be computed in polynomial time.

The algorithm presented here has a time complexity of \( O(d^2 + \text{de}) \). A more sophisticated algorithm exists with time complexity \( O(d^3 \log d + \text{de}) \). This algorithm is obtained by adapting the algorithm for the minimal cost-to-time ratio cycle problem presented in [13] to the problem of computing \( T_{\text{max}} \) [12].

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REFERENCES

A Simple Configuration for Realizing Voltage-Controlled Impedances

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Abstract—In earlier literature, op-amp JFET-based configurations have been advanced for the realization of voltage-controlled resistances (VCR’s) having wide dynamic range and low distortion with linear range considerably extended than other FET VCR’s. In this paper an extremely simple configuration is proposed that achieves the same objectives with only two op-amps and four resistors along with a JFET. Moreover, the new proposition is more versatile than previously known configurations in that it facilitates realization of voltage-controlled inductance and voltage-controlled capacitance from the same circuit. Possible variations of the schemes suitable for CMOS implementation have also been suggested.

I. INTRODUCTION

Voltage-controlled filters, voltage-controlled oscillators, and voltage-to-time period/frequency converters find applications in many instrumentation and measurement situations. A simple way to realize such circuits is to start from known circuits and then to replace some resistor(s) by voltage-controlled resistor(s). Some schemes to realize such voltage-controlled-resistors using FET’s are known in literature. Nay and Budak [1], [2] have presented a technique of obtaining a linear positive/negative voltage-controlled resistance (VCR) having wide dynamic range and low distortion with linear range considerably more extended than other FET VCR’s. The configuration of [1] and [2] employs two op-amps (one connected as a noninverting amplifier, the other connected as a unity gain buffer (UGB)) and six resistors, along with a JFET. Here, we present an alternative configuration, with similar properties, which is simpler than [1] and [2] and yet is more versatile than an NB-circuit in that it not only provides a VCR but can provide voltage-controlled inductance (VCL) and voltage-controlled capacitance (VCC) elements, too, which cannot be realized in the NB-circuits.

II. PROPOSED CONFIGURATION

The proposed configuration is shown in Fig. 1. A routine analysis of the circuit, when FET is confined to operate in nonsaturated region, under the same constraints as in [1] and [2], yields

\[ R_{eq} = R_1 \left[ 1 + \frac{V_c^2}{I_{DSS}} \frac{Z_2}{(V_c - 2V_p)R_2} \right] \]

(1)

which represents a VCR whose value is controllable through \( V_c \). Note that since we have made \( V_{GS} = (V_c + V_{DS})/2 \), substitution in

\[ i_D = 2I_{DSS}V_{GS} \left[ V_{GS} - V_p - V_{DS}/2 \right] / V_p^2 \]

(2)

results in the cancellation of the nonlinear term in the \( V_{DS} - i_D \) characteristic of the FET, represented by (2). This, together with the potential divider action constituted by \( R_1 \) and resistance of the FET (due to which the FET has to handle a small fraction of the input voltage \( V_i \)), is responsible for the extension of the linear range for the input resistance realized by the proposed circuit.

III. GENERALIZATION

Fig. 2 shows the generalization of the proposed configuration from which one can realize VCL and VCC elements by appropriate choice (resistive/capacitive) of impedances \( Z_1 \) and \( Z_2 \). For this circuit

\[ Z_{eq} = Z_1 \left[ 1 + \frac{V_c^2}{I_{DSS}} \frac{Z_2}{(V_c - 2V_p)} \right] \]

(3)

Therefore, with \( Z_1 = 1/xC_1 \) and \( Z_2 = R_2 \), the circuit realizes a VCC with

\[ C_{eq} = C_1 \left[ 1 + \frac{V_c^2}{I_{DSS}} \frac{V_c - 2V_p}{R_2} \right] \]

(4)

On the other hand, with \( Z_1 = R_1 \) and \( Z_2 = 1/xC_2 \), the circuit realizes a VCL with associated \( L_{eq} \) and \( r_{eq} \) given by

\[ L_{eq} = C_2 R_1 V_c^2 \]

(5a)

\[ r_{eq} = R_1 \]

(5b)

The implementation of the voltage-tunable resistor Fig. 1 can be simplified by eliminating the input buffer in which case \( R_{eq} \) would be

\[ R_{eq} = \frac{R_2 + R_{DS}}{1 + \frac{R_2}{R_1}} \]

(6a)

An alternative simplification is to eliminate the input buffer together with resistor \( R_1 \) (or \( R_1 \) and \( R_2 \)) in which case the resulting simplified voltage-tunable resistor reduces to the circuit of fig. 3 of [1].

A detailed nonideal analysis of the proposed configuration is presented in Appendix A, whereas a distortion analysis based upon a more general model equation for the FET is carried out in Appendix B.

IV. EXPERIMENTAL RESULTS

The practically obtained \( v-i \) characteristics of the VCR of Fig. 1 using 741 type op-amps (biased with \( \pm 15 \) V), n-channel JFET

1 It may be pointed out that with the FET replaced by a resistor and the two resistors connected at the gate of the FET eliminated, the circuit becomes similar to the lossy grounded inductance circuit of [3].