An alternative coefficient for sound absorption

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Abstract
The acoustic absorption coefficient is a number that indicates which fraction of the incident acoustic power impinging on a surface is being absorbed. The incident acoustic power is obtained by spatial integration of the incident intensity, which is (classically) defined as the time-averaged intensity associated with the incident sound field. The measurement of the effective, in situ, sound absorption coefficient is problematic as the determination thus requires a decomposition of the sound field in an incident and reflected field which, generally, is virtually impossible to do.

This paper introduces an alternative coefficient with which the effective acoustic absorption can be expressed. This coefficient is based on an alternative definition of the incident intensity; the time average of the positive values of the instantaneous intensity. The alternative coefficient is much easier to use in a sense that it follows directly from an in situ, instantaneous intensity measurement. The coefficient does not rely on any assumptions other than the assumption that the linearized wave equation is satisfied (and thus the acoustic energy corollary). As a result, one does not need to decompose the sound field in incident and reflected waves. Hence, one does not need to have prior information about the incident sound field. Accordingly, one does not need to have prior information about the source. The coefficient can be determined in any sound field, either transient or stationary, free field and diffuse/(semi-)reverberant sound fields. The alternative coefficient is illustrated by means of several numerical examples.

1 Introduction
The capability of a surface to absorb sound when exposed to a certain sound field is typically expressed in terms of its sound absorption coefficient. This coefficient $\alpha$ is the ratio of the active sound power and the incident sound power on the surface under investigation. Knowledge of the sound absorption coefficient is required in many fields of application, such as room acoustics, noise control engineering and more.

Well-established laboratory methods for the experimental determination of the sound absorption coefficient are available. For normal incident sound fields, the Kundt’s tube method according ISO 10534-1 [20] can be used. The transfer function method as described in ISO 10534-2 [21] is less time-consuming and nowadays describes the state-of-the-art. Although the method is standard, influences of mounting of the sample, sample size and structural coupling of the impedance tube to the driving loudspeaker have been addressed by various authors, see e.g. [22, 5]. For diffuse sound fields, as typically assumed in building acoustics, the diffuse incidence sound absorption coefficient, $\alpha_S$, is of interest. $\alpha_S$ is determined by measuring the change in reverberation time in a reverberation room due to placement of a significant area of the material under investigation, as described in ISO 354 [19]. Influences of sample orientation, sample edge effects and the diffuseness of the sound field on the accuracy and applicability of the method have been reported.

Clearly, laboratory methods will only yield coefficients that are valid for the specific acoustic conditions used in the test. In practice however, a material will typically be mounted differently from the test-configuration
and it will be exposed to a different sound field. Consequently, the sound absorption coefficient will be different from the sound absorption coefficient obtained with a laboratory method. Therefore, significant research efforts have been put in the development of in situ methods. In order to avoid possible confusion, we would like to stress that the *in situ* sound absorption coefficient is normally defined as the sound absorption coefficient for a specified angle of incidence for the *in situ* structure in an otherwise *free field*. In this paper however we are interested in the *effective* sound absorption coefficient which is a measure for sound absorption of a structure in the *actual* sound field\(^1\).

This paragraph is restricted to a brief discussion of existing and well-established in situ methods and the reader is referred to more detailed overviews given by Garai [13], Kruse [15], Tamura [3] and Nocke [17]. So-called 2-microphone transfer function methods, see [7, 8, 11, 14, 15, 4] can be applied if there are no reflections from structures other than the surface of interest. Consequently, measurements based on this method have to be carried out in open spaces or rooms that are sufficiently anechoic. The only kind of in situ methods that have found their way into standardization are pulse-based methods, see Garai [13], Mommertz [12] and Nocke [16]. These methods separate, by means of a time window, the reflected waves from the surface of interest, from those that are reflected by other surfaces. In a number of papers, [18, 10], methods that use a PU-sensor to measure the surface impedance for normal and oblique incidence have been presented.

As stated above, the absorption coefficient is defined as:

\[
\alpha = \frac{P_{ac}}{P_{in}},
\]

where \(P_{ac}\) is the active sound power flowing through a specified surface and \(P_{in}\) is the incident sound power flowing through that same surface. For a given surface and a given sound field, the active sound power is obtained by spatial integration of the normal component of the active intensity (i.e. the component normal to the given surface) over that surface. The incident sound power should be obtained by integration of the normal component of the incident intensity, as illustrated in figure 1. The determination of the incident intensity is (classically) accomplished by decomposing the sound field in an incident and reflected field. The incident intensity is then set equal to the *active intensity of the incident sound field*. This approach is practically problematic as one can only accurately measure the incident intensity in 'ideal' acoustic fields, such as acoustic free-fields or semi-free-fields with a single sound source. Apart from the pulse-based methods as described in [12, 13, 16], such fields can only be realized in (semi-)anechoic rooms or outdoors, making the methods hardly applicable in situ for general acoustic environments.

In a recent paper [2], we proposed to measure the incident intensity based on a local plane wave assumption. This assumption enabled the measurement of the absorption coefficient for the in situ sound field without the need to assume and therefore impose an ideal sound field. In this paper, an alternative *effective* coefficient is proposed that can be measured without making any assumptions about the sound field other than the assumption that the linearized wave equation (or Helmholtz equation) is satisfied. It nevertheless is a coefficient that reflects the absorbing capabilities of an absorbing material in the given (arbitrary) sound field.

The idea of the alternative coefficient is simply to set the incident intensity equal to the time average of only the positive values of the instantaneous intensity, see the definition below. As the instantaneous intensity can be measured at the actual site near the surface of interest, there is no need to decompose the sound field in an incident and reflected field; the incident intensity follows directly from the measurement data. In addition, the alternative coefficient shows the dependency of absorption on the in situ sound source, as we will show later.\(^2\)

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\(^1\)The absorption coefficient based on reverberation time measurements is therefore also an effective sound absorption coefficient.

\(^2\)The absorption coefficient is sometimes erroneously identified as a material property. However, as the well known dependency of the angle of incidence on the absorption coefficient illustrates, the absorption coefficient is a property of the absorbing surface and the associated impinging sound field. For arbitrary sound fields, this generalizes to a dependency of the absorption coefficient on the sound source (position and radiation characteristics) and the entire acoustical domain (e.g. the presence of additional reflecting boundaries).
Figure 1: The incident \((I^+_n)\) and reflected intensity \((I^-_n)\) in the direction normal to the absorbing surface \(\partial \Omega\).

In the current paper, the theoretical background for the alternative coefficient will be given and the coefficient is illustrated by various numerical examples.

2 Theory

2.1 General sound fields

The instantaneous intensity vector at a position \(x\) and time \(t\), describing the acoustic energy flux, is defined as:

\[
I(x, t) = p(x, t)u(x, t),
\]

where \(p(x, t)\) denotes the acoustic pressure and \(u(x, t)\) the acoustic velocity vector. The component of the intensity vector in a direction \(n\) \((I(x, t) \cdot n)\) is the energy flux flowing in that direction. The direction vector \(n\) is chosen to be perpendicular to the surface of interest, pointing away from the source. Hence, this component equals the energy flux flowing into that specific surface. The active intensity is the time averaged intensity over a certain period \(T\):

\[
I_{ac}(x) = \frac{1}{T} \int_0^T I(x, t) dt,
\]

and integration of the normal component of the active intensity \((I_{ac} \cdot n)\) over the surface \(\partial \Omega\) yields the active power flowing into the surface \(\partial \Omega\):

\[
P_{ac} = \int_{\partial \Omega} I_{ac} \cdot nd\Gamma,
\]

which is the numerator in equation (1).

To circumvent the requirement of imposing a certain impinging sound field, as was indicated in the previous paragraph, it is proposed to set the incident intensity equal to the time average of the positive values of the normal component of the instantaneous intensity. The reflected intensity is then defined as the time average of only the negative values. This concept is illustrated in figure 2. To avoid confusion in terminology between the classical and alternative concept, the incident intensity for the classical definition will be denoted by \(I_{in}\) and associated power by \(P_{in}\), the incident intensity used in the alternative definition will be denoted by \(I^+_n\) and the associated power by \(P^+\). The incident intensity \(I^+_n(x)\) and reflected intensity \(I^-_n(x)\) are thus defined
as, respectively:

\[ I_n^+(x) \equiv \frac{1}{T} \int_0^T i_n^+(x,t)dt, \]  

(5)

and

\[ I_n^-(x) \equiv \frac{1}{T} \int_0^T i_n^-(x,t)dt. \]  

(6)

\( i_n^+(x,t) \) and \( i_n^-(x,t) \) denote, respectively, the instantaneous incident and instantaneous reflected intensity and are defined as:

\[ i_n^+(x,t) = \begin{cases} I(x,t) \cdot n & \text{for } I(x,t) \cdot n \geq 0 \\ 0 & \text{for } I(x,t) \cdot n < 0 \end{cases} \]  

(7)

and

\[ i_n^-(x,t) = \begin{cases} 0 & \text{for } I(x,t) \cdot n \geq 0 \\ -I(x,t) \cdot n & \text{for } I(x,t) \cdot n < 0 \end{cases} \]  

(8)

Note the minus sign in the definition of the reflected intensity, as we would like the reflected intensity to be positive as the instantaneous intensity is negative. The subscripts \( n \) in the definition of \( I_n^+, I_n^-, i_n^+ \) and \( i_n^- \) have been added to emphasize the dependency on the direction \( n \).

\[
\beta = \frac{P_{ac}}{P^+} = \frac{\int_{\partial \Omega} I_{ac}(x) \cdot n d\Gamma}{\int_{\partial \Omega} I_n^+(x) d\Gamma}.
\]  

(9)

Figure 2: Instantaneous incident intensity \( i_n^+(x,t) \) (white bars) and instantaneous reflected intensity \( i_n^-(x,t) \) (black bars) versus time at some point \( x \) in direction \( n \).

After spatial integration of \( I_n^+ \) over the surface \( \partial \Omega \), one thus obtains the incident power \( P^+ \) flowing through \( \partial \Omega \). The proposed alternative for the classical absorption coefficient \( \alpha \) is therefore given by:

\[
\beta = \frac{P_{ac}}{P^+} = \frac{\int_{\partial \Omega} I_{ac}(x) \cdot n d\Gamma}{\int_{\partial \Omega} I_n^+(x) d\Gamma}.
\]  

(9)

Obviously, the alternative definitions for the incident and reflected intensity can be used for any transient sound field and can be calculated directly from a measured instantaneous intensity.

An additional benefit of the new definition is that the active intensity \( I_{ac} \cdot n \), by definition, equals the difference between the incident and reflected intensity for any sound field:

\[ I_{ac}(x) \cdot n \equiv I_n^+(x) - I_n^-(x). \]  

(10)
This is in contrast to the classical definition of incident intensity, where, for general sound fields, the difference between the incident and reflected intensity is not necessarily equal to the active intensity\(^3\). For the 1D sound field, the difference between the, classically defined, incident and reflected intensity is equal to the active intensity, but this is the exception.

### 2.2 Harmonic sound fields

If the sound field is harmonic, i.e. \(p(t) = \Re(\Re o t)\) and \(u(t) = \Re(\Re o t)\), where \(\Re\) denotes the real part, it is easy to show that the instantaneous intensity vector is given by, see \[1\]:

\[
\mathbf{I}(x, t) = \mathbf{I}_{ac}(x) + \mathbf{I}_{ac}(x) \cos(2\omega t - \phi) + \mathbf{I}_{re}(x) \sin(2\omega t - \phi),
\]

where \(\mathbf{I}_{ac} = \frac{1}{2} \Re(\mathbf{P}\mathbf{U})\) is the active intensity and \(\mathbf{I}_{re} = \frac{1}{2} \Im(\mathbf{P}\mathbf{U})\) is the reactive intensity. \(\Im\) denotes the imaginary part, the overbar \(\bar{\cdot}\) denotes the complex conjugate and \(\phi\) denotes a phase shift. Hence, the normal component of the intensity vector equals:

\[
\mathbf{I}(x, t) \cdot \mathbf{n} = \mathbf{I}_{ac}(x) \cdot \mathbf{n} + \mathbf{I}_{ac}(x) \cdot \mathbf{n} \cos(2\omega t - \phi) + \mathbf{I}_{re}(x) \cdot \mathbf{n} \sin(2\omega t - \phi)
\]

\[
= \mathbf{I}_{ac}(x) \cdot \mathbf{n} + \sqrt{(\mathbf{I}_{ac}(x) \cdot \mathbf{n})^2 + (\mathbf{I}_{re}(x) \cdot \mathbf{n})^2} \cos(2\omega t - \phi_{\alpha}),
\]

where \(\phi_{\alpha}\) is a phase shift. The energy flux in direction \(\mathbf{n}\) thus consists of a constant, time independent part and an oscillatory part, oscillating at frequency \(2\omega\) with amplitude \(\sqrt{(\mathbf{I}_{ac} \cdot \mathbf{n})^2 + (\mathbf{I}_{re} \cdot \mathbf{n})^2}\).

Figure 3 shows an example of the instantaneous intensity \(\mathbf{I}(\tau) \cdot \mathbf{n}\) versus time for a 1000 Hz signal, showing the time segments for which the intensity is incident and reflected (\(\mathbf{I}_{ac} \cdot \mathbf{n} = 1[\text{W/m}^2]\) and \(\mathbf{I}_{re} \cdot \mathbf{n} = 2[\text{W/m}^2]\)).

Figure 3 shows an example of the instantaneous intensity (as a function of time) of a 1000 Hz signal for one period \(T = 2\pi/\omega\) of the sound pressure. Without loss of generality, a time shift \(\Delta t = \phi_{\alpha}/(2\omega)\) has been applied such that \(t = \tau + \Delta t\) and:

\[
\mathbf{I}(x, \tau) \cdot \mathbf{n} = \mathbf{I}_{ac}(x) \cdot \mathbf{n} + \sqrt{(\mathbf{I}_{ac}(x) \cdot \mathbf{n})^2 + (\mathbf{I}_{re}(x) \cdot \mathbf{n})^2} \cos(2\omega \tau).
\]

The zero crossings \(\tau_1\) and \(\tau_2\), as shown in figure 3, are then easily evaluated as \(\tau_1 = \arccos(-IR_n)/(2\omega)\) and \(\tau_2 = (2\pi - \arccos(-IR_n))/(2\omega)\), where \(IR_n\) is the intensity ratio between the active intensity and the

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\(^3\) Decomposing the sound field in an incident and reflected field yields: \(\mathbf{I} = p\mathbf{u} = (p_{in} + p_{ref})(u_{in} - u_{ref}) = p_{in}u_{in} - p_{ref}u_{ref} - p_{in}u_{ref} + p_{ref}u_{in}\). Hence the active intensity is only equal to the incident intensity minus the reflected intensity if the time average of \(-p_{in}u_{ref} + p_{ref}u_{in}\) equals 0.
amplitude of the intensity oscillation:

\[ IR_n(x) = \frac{I_{ac}(x) \cdot n}{\sqrt{(I_{ac}(x) \cdot n)^2 + (I_{re}(x) \cdot n)^2}}. \]  

(15)

It is easy to show that this intensity ratio also equals the cosine of the phase difference between the sound pressure and particle velocity. From a numerical point of view however, it is more convenient to leave the expression as is given above. The incident intensity, according to the definition (5), can, for the harmonic case, be calculated based on the time average over one period \( T \), i.e.

\[ I^+_n = \frac{2\omega}{\pi} \int_0^T I(\tau) \cdot n d\tau. \]

The reflected intensity can be calculated based on the negative intensity values, i.e. \( I^-_n = \frac{\omega}{\pi} \int_T^{2T} -I(\tau) \cdot n d\tau. \)

The resulting time averaged incident and reflected intensities can be shown to equal, respectively:

\[ I^+_n(x) = \left\{ 1 - \frac{\arccos(IR_n(x))}{\pi} \right\} I_{ac}(x) \cdot n + \frac{1}{\pi} |I_{re}(x) \cdot n| \]

and \( I^-_n(x) = \left\{ -\frac{\arccos(IR_n(x))}{\pi} \right\} I_{ac}(x) \cdot n + \frac{1}{\pi} |I_{re}(x) \cdot n|, \]

(16)

(17)

where \(| \cdot |\) denotes the absolute value.

3 Numerical Examples

3.1 Open ended tube

To illustrate the proposed method, the example of an tube with a flanged, open end, see figure 4 is considered. Below the cut-on frequency, the impedance of the open end can well be approximated by

\[ Z = \rho c \left[ (ka)^2 + i(8ka/3\pi) \right], \]

where \( \rho \) is the density, \( c \) the speed of sound, \( k = \omega/c \) is the wavenumber and \( a \) the radius of the tube. For frequencies below the cut-on frequency, the sound field can be described by 2 plane waves; an incident and reflected plane wave. Ignoring viscous and thermal effects, the complex pressure amplitude is given by

\[ P = Ae^{-ikx} + Be^{ikx}, \]

with \( A \) the amplitude of the incident wave (propagating in the positive \( x \)--direction) and \( B \) the reflected wave (propagating in the negative \( x \)--direction). In

\[ \text{Replacing the expression by the cosine of the phase difference would, for the evaluation of the alternative coefficient, require an 'unwrapping' function for the phase which is cumbersome.} \]
the classical definition of the absorption coefficient $\alpha$, the incident intensity is the active intensity associated with the incident wave. For a plane wave of complex amplitude $A$, the active intensity can easily be shown to equal $A\bar{A}/(2\rho c)$ and the absorption coefficient $\alpha$ is then readily obtained as:

$$\alpha = \frac{A\bar{A} - B\bar{B}}{A\bar{A}},$$  \hspace{1cm} (18)

see also [27, 26].

As the absorption is effectively accomplished at the open end of the impedance tube, in this case, it is appropriate to evaluate the alternative coefficient at that location\(^5\). In figure 4, the classical coefficient and the alternative coefficient at the open end are shown. From the figure it can be concluded that both coefficients show the same trend, i.e. the absorption increases with frequency. The alternative coefficient is seen to be a somewhat larger in value compared to the classical absorption coefficient.

### 3.2 2D Example: Louvre door

As a second example, consider the 2-dimensional geometry given in figure 5. This geometry resembles a Louvre door and serves to illustrate the key benefit of the alternative coefficient; it can be determined without any prior knowledge of the location or radiation characteristics of the source. The considered geometry consists of acoustically hard surfaces (indicated by the thick lines and referred to as reflective segments) and surfaces for which the impedance is assumed to equal the plane wave impedance $\rho c$ (indicated by the dotted lines and referred to as the absorbing segments). Two cases are assumed; In case A, a unit power point source is positioned in point $A$, whereas in case B, a unit power point source is positioned in point $B$, see figure 5. Based on a straightforward finite element calculation, the alternative coefficient is evaluated for the line $x = 0.45$. The results are shown in figure 5.

![Figure 5: Absorbing sample (left) and coefficient $\beta$ (right) evaluated for the line $x = 0.45$ for a point source at $A$ ($\beta_A$) and a point source at $B$ ($\beta_B$).](image)

The calculated curves for $\beta$, as shown in figure 5, are as expected. That is, if the point source is positioned at point $A$, the surface normals of the absorbing segments point in the direction of the source. Hence the energy emitted by the source flows into the absorbing segments very effectively and almost all of the incident energy is being absorbed. This results in $\beta$ being very close to unity. On the other hand, if the point source is located

\(^5\)The alternative coefficient, as opposed to the classical definition, does depend on the position in the tube but this is beyond the scope of the current paper.
at point $B$, the surface normals of the reflective segments point in the direction of the source. Accordingly, a large part of the energy flux emitted by the source is reflected back towards the source and the alternative coefficient reduces to values that are substantially less than unity.

Figure 6: Point source located in point A. Left: The incident intensity $I^+_n$ (vectors pointing to the right) and reflected intensity $I^- n$ (vectors pointing to the left) for $n = (1, 0, 0)^T$ on the line $x = 0.45$ at 2000 Hz. Please note that the scale of the incident and reflected vectors are not the same; the reflected intensities are in fact much smaller. Middle: The incident intensity $I^+_n$ distribution for $n = (1, 0, 0)^T$. A black color denotes large values, white denotes small values. Right: The reflected intensity $I^- _n$ distribution for $n = (1, 0, 0)^T$. Note: the color-scale for the incident and reflected intensities is again different in both plots.

Figure 7: Point source located in point B. Left: The incident intensity $I^+_n$ (vectors pointing to the right) and reflected intensity $I^- n$ (vectors pointing to the left) for $n = (1, 0, 0)^T$ on the line $x = 0.45$ at 2000 Hz. Middle: The incident intensity $I^+_n$ distribution for $n = (1, 0, 0)^T$. Right: The reflected intensity $I^- _n$ distribution for $n = (1, 0, 0)^T$. See the remark on the scale and color in the previous figure.

In figures 6 and 7 for the evaluation-line $x = 0.45$, the incident and reflected intensity for $n = (1, 0, 0)^T$, i.e. the $x$-axis, are displayed as vectors. The vectors pointing to the right denote the incident intensity in that direction, the vectors pointing to the left denote the reflected intensity. For clarity reasons, the scale of the arrows of the reflected intensity have been enlarged; especially for case A, the actual reflected intensities
are very small. The figures also show contourplots of the incident intensity $I_+^n$ and reflected intensity $I_-^n$ in the entire domain for both cases. For case A, one observes that the reflected intensity is only larger near the reflective elements near $y = 0$ (bottom section). For locations for which the $y$-values are larger, the wave impinges normal to the absorbing segments and reflection is much less. For case B, one clearly sees larger values of the reflective intensity throughout the domain as, by and large, the wave impinges normal to the reflective segments. One also sees that the incident and reflected intensity vary with the length scale of the absorbing and reflecting surfaces. The method thus nicely show how the incident intensity can change with position. It is clear to see that the incident intensity $I_+^n$ is, as expected, larger near the source. However, contrary to what one might expect from a point source, especially for case B, the distribution of both the incident and reflected intensities is not smooth (although at lower frequencies the distribution is much smoother). This is a result of the irregular shape of the surface.

This example shows that both the incident and reflected intensity clearly depend on both the position of the source in combination with the absorbing sample. Hence, not having to make any assumptions about the sound field and the results directly following from the measured data is a great advantage of the alternative coefficient, even in extreme cases like this.

4 Conclusion

In this paper, an alternative coefficient to measure acoustic absorption is introduced. The alternative coefficient $\beta$ is based on an incident intensity which is defined as the time average of the positive values of the instantaneous intensity, as opposed to the time averaged (active) intensity associated with the incident wave. The advantage of the alternative coefficient is that it does not rely on any assumptions about the source or the sound field near the absorbing surface other than the assumption that the sound field satisfies the wave equation (or, if applicable, the Helmholtz equation). The alternative coefficient directly follows from a measurement of the intensity along the considered surface (either by means of scanning or using a point-by-point method).

References


